Asymptotic control and stabilization of nonlinear oscillators with non isolated equilibria

Hedy ATTOUCH and Marc-Olivier CZARNECKI

Let $\Phi : H \to R$ be a $\mathcal{C}^1$ function on a real Hilbert space H and let $\gamma > 0$ be a positive (damping) parameter. For any control function $\varepsilon R_+ \to R_+$ which tends to zero as $t \to +\infty$, we study the asymptotic behavior of the trajectories of the damped nonlinear oscillator

$$(HBFC) \quad \ddot{x}(t) + \gamma \dot{x}(t) + \nabla \Phi(x(t)) + \varepsilon(t) x(t) = 0.$$

We show that, if $\varepsilon(t)$ does not tend to zero too rapidly as $t \to +\infty$, then the term $\varepsilon(t)x(t)$ asymptotically acts as a Tikhonov regularization, which forces the trajectories to converge to a particular equilibrium. Indeed, in the main result of this paper, it is established that, when Φ is convex and $S = argmin \Phi \neq \emptyset$, under the key assumption that ε is a "slow" control, i.e., $\int_0^{+\infty} \varepsilon(t) dt = +\infty$, then each trajectory of the $(HBFC)$ system strongly converges, as $t \to +\infty$, to the element of minimal norm of the closed convex set S