Dessins d'enfants and the arithmetic of moduli spaces of curves

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According to Mumford-Harer-Witten-Kontsevich-Penner-..., there exists a cell decomposition

$$\mathcal{M}_{g,N}(\mathbb{C}) \times \mathbb{R}^{N}_{>0} \simeq \coprod_{D \in \mathrm{DESS}_{g,N}} \mathbb{R}^{\mathrm{E}(D)}_{>0},$$

where $\mathcal{M}_{g,N}$ is the moduli space of genus g curves with N marked points, D runs over the (finite) set of dessins d'enfants of genus g with N faces and 0-valencies ≥ 3 and $\mathrm{E}(D)$ is the set of edges of a dessin. The N "extra" positive real numbers can be treated as the perimeters of faces of dessins; fixing them, one gets the family of quasitriangilations of the moduli spaces, and the corresponding volumes as functions of these perimeters contain lots of information on the cohomologies of moduli spaces.

According to Belyi-Grothendieck and Mulase-Penkava, replacing $\mathbb{R}^{\mathrm{E}(D)}_{>0}$ by $\mathbb{N}^{\mathrm{E}(D)}$ allows to replace $\mathcal{M}_{g,N}(\mathbb{C})$ by $\mathcal{M}_{g,N}(\overline{\mathbb{Q}})$. Calculation of the volumes can be (asymptotically) reduced to counting integer points in certain convex bodies; the corresponding tools were elaborated in a recent paper by Norbury. Thus the intersection theory of Witten-Kontsevich is reduced to an elementary problem that will be completely explained in the talk.

Another approach through point-counting in $\mathcal{M}_{g,N}(\mathbb{F}_{\mathbb{q}})$ will be mentioned and the problem of related the two approaches posed.