Martin boundary with large deviation technique for partially homogeneous random random walks.

To identify the Martin boundary for a transient Markov chain having Green's function $G(x, y)$ one have to identify all possible limits $\lim_{n\to\infty} G(x, y_n)/G(x_0, y_n)$ with y_n "tending to infinity". For homogeneous random walks, these limits are usually obtained from the exact asymptotics of the Green's function $G(x, y)$. For non-homogeneous random walks, the exact asymptotics of the Green's function is an extremely difficult problem. We discuss several examples of partially homogeneous random walks, where Martin boundary can be identified by using large deviation technique. The minimal Martin boundary is in general not homeomorphic to the "radial" compactification obtained by Ney and Spitzer for the homogeneous random walks in $\mathbb{Z}^d$: convergence of a sequence of points y_n to a point of on the Martin boundary does not imply convergence of the sequence $y_n/|y_n|$ on the unit sphere. Such a phenomenon is a consequence of non-linear optimal large deviation trajectories.