OPTIMAL MULTITHRESHOLD CONTROL FOR BMAP/SM/1 QUEUE WITH MAP-INPUT OF DISASTERS

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ABSTRACT
BMAP/SM/1 queueing system with controllable mode of operation is considered. Operational mode is changed according to a multithreshold strategy. System also has Markovian input of disasters. Disaster is a special kind of negative customer which interrupts the service and causes all the customers to leave the system unserved immediately whenever it arrives to the busy system. The stationary state distribution of embedded Markov chain and performance characteristics (mean queue length, mean interdeparture time, mean number of customers lost per time unit) are obtained under the fixed control strategy. The optimal control strategy is determined numerically.

INTRODUCTION

Queueing systems with dynamic control allow to redistribute the system resources during system operation. In particular, such systems have variable parameters of input flow, service and other processes characterizing the queueing system. The each group of parameters used simultaneously is referred to as operational mode. At the decision making moments the certain operational mode is selected for customer processing correspondingly to some control strategy (rule of mode selection). Queueing models with controllable mode of operation can be effectively applied for modelling the telecommunication network fragments. That is the resource of telecommunication network which hands a mixture of flows having different requirements to the response time; systems which combine transmission of several types of information having different requirements to the quality of service and having a possibility to distribute the bandwidth dynamically, etc.

There are two directions of the controllable queueing systems investigation. The papers of the first direction, e.g. (Rykov 1999) and (Artalejo and Economou 2004), deal with the problem of the optimal strategy existence and properties of such strategies. In the papers of the second direction including the present paper the class of control strategies is chosen in advance (e.g., threshold or hysteresis strategies) and the problem to find the optimal strategy in the given class is dealt with. Threshold strategies to control the queues with two operational modes were considered in (Nishimura and Jiang 1995) and (Nobel and Tijms 1999); queues with hysteresis strategies were analyzed in (Dudin 2002) and (Dudin and Nishimura 2000). Multithreshold control strategies were investigated in (Dudin 1998), (Dudin and Chakravarthy 2003) and (Kim et al. 2006) for the queues with several operational modes. The latter works include analysis of queues having the BMAP (Batch Markovian Arrival Process) input of customers. Nowadays BMAP is the most general customer input model which allows to analyze the queue with such input analytically, (Neuts 1989, Lucantoni 1991).

In this paper the analysis of controllable BMAP/SM/1 queue having n, $n \geq 2$, operation modes and input of disasters is presented. Control strategies are multithreshold that is mode of operation can be change at definite epochs (customer departure epochs) accordingly to the relation between queue length and prespecified integers called the thresholds. The system also has an additional input of disasters which cause all customers to leave the system unserved. Disaster is the special kind of negative arrival that interrupts the service and removes all the customers from the system whenever it enters the busy system. Theory of negative arrivals was originated by E. Gelenbe (Gelenbe 1989) and since is being developed significantly. Detailed review of achieved results is presented in reviews (Artalejo 2000) and (Bocharov and Vishnevskij 2003). BMAP/SM/1 queue with disasters was investigated in (Dudin and Nishimura 1999). Note that queueing models with disasters can be applied to computer networks with virus infections and migration processes with catastrophes.

The present paper generalizes the results of (Semenova 2004) obtained for the queue with two modes of operation to the case of several operation modes.

MODEL

A single-server queue having n, $n \geq 2$, modes of operation and input of disasters is under consideration.

The rth mode is described as follows. Customers arrive to the system accordingly to Batch Markovian Arrival Process (BMAP) which is governed by a stochastic process $\nu_t, t \geq 0$, with a state space $\{0, 1, ..., W\}$. Transitions of the process $\nu_t, t \geq 0$ and arrivals of customers are performed with a matrix generation function $D^{(r)}(z) = \sum_{m=0}^{\infty} D_m^{(r)} z^m, |z| < 1$. The matrix $D_0^{(r)}$ governs the transitions corresponding to no arrivals, and $D_m^{(r)}$ governs the transitions corresponding to arrivals of size m batch of customers, $m \geq 1$.

Denote by $\boldsymbol{\gamma}^{(r)}$ the stationary probability row vector of the Markov chain $\nu_t, t \geq 0$. It is given by the system of equations: $\boldsymbol{\gamma}^{(r)} D^{(r)}(1) = \mathbf{0}, \quad \boldsymbol{\gamma}^{(r)}\mathbf{1} = 1$. Here and bellow $\mathbf{0}$ is a null column-vector of appropriate size, $\mathbf{1}$ is a row-vector consisting of ones. The intensity $\lambda^{(r)}$ of BMAP is given by

$$\lambda^{(r)} = \boldsymbol{\gamma}^{(r)} \frac{dD^{(r)}(z)}{dz}\bigg|_{z=1} \mathbf{1}.$$

The variation coefficient c_{var} of intervals between successive group arrivals is given by the formula $c_{var}^2 = 2\lambda_g^{(r)} \boldsymbol{\gamma}^{(r)} (-D_0^{(r)})^{-1}\mathbf{1} - 1$, where $\lambda_g^{(r)} = \boldsymbol{\gamma}^{(r)}(-D_0^{(r)})\mathbf{1}$ is the intensity of group arrivals. The correlation coefficient c_{cor} of intervals between customer group arrivals is given by $c_{cor} = (\lambda_g^{(r)} \boldsymbol{\gamma}^{(r)} (-D_0^{(r)})^{-1} (D^{(r)}(1) - D_0^{(r)})(-D_0^{(r)})^{-1}\mathbf{1})/c_{var}^2$.

More detailed description of BMAP and assumptions about the matrix function $D^{(r)}(z)$ can be found e.g. in (Lucantoni 1991).

The service process is of SM-type. It means that successful service times are the sojourn times of a semi-Markovian process $m_t, t \geq 0$. This process has a state space $\{1, ..., M\}$ and a semi-Markovian kernel $B^{(r)}(x) = \left(B_{m,m'}^{(r)}(x)\right)_{m,m'=\overline{1,M}}$ when system works under the rth mode. The function $B_{m,m'}^{(r)}(x)$ is the conditional distribution function of the sojourn time of the process $m_t, t \geq 0$ in a state m under the condition that the next state will be m', $m, m' = \overline{1, M}$.

For the system under consideration the service is interrupted at a disaster arrival epoch. It is assumed that the states of the service directing process $m_t, t \geq 0$ are changed at service completion epochs accordingly to the matrix $P^{(r)} = B^{(r)}(\infty)$ in the rth operational mode regardless of whether service is completed successfully or is cancelled by disaster appearance.

Denote by $b^{(r)}$ the mean service time which is not interrupted by a disaster arrival in the rth mode. It's value is given by the formula $b^{(r)} = \boldsymbol{\delta} \int_0^{\infty} t dB^{(r)}(t)\mathbf{1}$, where $\boldsymbol{\delta}$ is invariant row vector of the matrix $P^{(r)}$.

As (Lucantoni and Neuts 1994) and (Neuts 1989) we assume that the matrix $P^{(r)}$ is indecomposable, $B^{(r)}(+0) = 0$ and $b^{(r)} < \infty$.

Disaster arrival process is MAP (Markovian Arrival Process) which is the particular case of BMAP allowing the ordinary arrivals only. Disaster arrival to the busy system interrupts the service and causes all customers to leave the system instantaneously. If disaster arrives to the empty system it is ignored. We suppose that input of disasters is governed by the process $\eta_t, t \geq 0$, having a state space $\{0, 1, ..., N\}$ and a matrix generating function $F^{(r)}(z) = F_0^{(r)} + F_1^{(r)} z, |z| \leq 1$. Description of the rth operation mode is completed, $r = \overline{1, n}$.

The operation mode can be switched at the service completion epochs according to multithreshold strategy.

The aim of the system control is to minimize the cost criterion

$$C = a\Lambda L + \sum_{r=1}^{n} c_r \Phi_r + dR, \tag{1}$$

where L is the mean queue length at customer departure epoch; Λ^{-1} is the mean interdeparture time; Φ_r s the average fraction of time, when the r-th mode is in use, $r = \overline{1, n}$; R is the average number of customers lost per time unit due to disaster arrival; a, c_r, $r = \overline{1, n}$, and d are the cost coefficients.

Multithreshold control strategy is determined as follows. The nonnegative integers $(j_1, j_2, \ldots, j_{n-1})$, $-1 = j_0 < j_1 \leq j_2 \leq \cdots \leq j_{n-1} < j_n = \infty$, which are called the thresholds are fixed. If a queue length i at a given customer departure epoch satisfies the inequality $j_{r-1} + 1 \leq i \leq j_r$, the rth mode is selected for the next customer service, $r = \overline{1, N}$.

To obtain the performance characteristics involved in (1) under the fixed set of thresholds we use the method of embedded Markov chains and derive the stationary state distribution of the embedded chain describing the system's behavior at service completion epochs.

EMBEDDED MARKOV CHAIN

Let t_k be the kth epoch of customers departure from the system, $k \geq 1$. Note that it is a service completion epoch or a disaster arrival epoch at a busy period.

Introduce into consideration the following five-dimensional Markov chain:

$$\xi_k = \{i_k, u_k, \zeta_k\}, \quad k \geq 1,$$

where $\zeta_k = \{\nu_k, \eta_k, m_k\}$, ν_k is the state of arrival directing process ν_t, $t \geq 0$, at the epoch t_k, $\nu_k = \overline{0, W}$, η_k is the state of disaster directing process η_t, $t \geq 0$, at the epoch $t_k + 0$, $\eta_k = \overline{0, N}$; m_k is the state of the service directing process m_t, $t \geq 0$, at the epoch t_k+0, $m_k=\overline{1,M}$. The component u_k possesses the following values:

— $u_k=0$, if t_k is the epoch of successful service completion, in this case i_k means the queue length at the epoch t_k+0, $i_k\ge 0$;

— $u_k=1$, if t_k is a disaster arrival epoch, in this case i_k means the number of customers that leave the system at epoch t_k, $i_k\ge 1$, $k\ge 1$.

Let the states of process ζ_k, $k\ge 1$, be listed in lexicographic order of the components $\{\nu_k,\eta_k,m_k\}$ increase and be numbered from 1 to $H=(W+1)(N+1)M$. For convenience the state $\{\nu,\eta,m\}$ of the triple $\{\nu_k,\eta_k,m_k\}$ will be further replaced with it's serial number.

Introduce into consideration one-step transition probabilities

$$P\{(i,u,\zeta)\to(l,u',\zeta')\}=P\{i_{k+1}=l,u_{k+1}=u',\\ \zeta_{k+1}=\zeta'|i_k=i,u_k=u,\zeta_k=\zeta\},\\ i,l\ge 0,\ u,u'=\overline{0,1},\ \zeta,\zeta'=\overline{1,H},$$

and form the matrices

$$P_{i,l}^{(u,u')}=\big(P\{(i,u,\zeta)\to(l,u',\zeta')\}\big)_{\zeta,\zeta'=\overline{1,H}},\\ i,l\ge 0,u,u'=\overline{0,1}.$$

Introduce also the matrices Ψ_k, $k\ge 1$, and $S_l^{(r)}$, $\Omega_l^{(r)}$, $l\ge 0$, $r=\overline{1,n}$, having the following probabilistic sense.

The entry $[\Omega_k^{(r)}]_{s,m}$ of the matrix $\Omega_k^{(r)}$ is the probability that k customers arrive but no disaster arrives to the system during the customer processing in the r-th mode, and the process ζ_l, $l\ge 1$, transits from the state s into the state m, $s,m=\overline{1,H}$, $k\ge 0$, $r=\overline{1,n}$.

The entry $[S_k^{(r)}]_{s,m}$ of the matrix $S_k^{(r)}$ is the probability that disaster appears during customer service in the r-th mode, k customers arrive to the queue and the process ζ_l, $l\ge 1$, transits from the state s into the state m during the uncompleted service time, $s,m=\overline{1,H}$, $k\ge 0$, $r=\overline{1,n}$.

The entry $[\Psi_k]_{s,m}$ of the matrix Ψ_k is the probability that system's busy period starts by the arrival of customers batch of size k and the process ζ_l, $l\ge 1$, transits from the state s into the state m during the idle period, $s,m=\overline{1,H}$, $k\ge 1$.

Analyzing the one-step transition probabilities of the Markov chain ξ_k, $k\ge 1$, the following statement can be proved.

Lemma 1. Nonzero matrices $P_{i,l}^{(u,u')}$, $i,l\ge 0$, $u,u'=\overline{0,1}$, of one-step transition probabilities of the Markov chain ξ_k, $k\ge 1$, are given as

$$P_{0,l}^{(0,0)}=P_{i,l}^{(1,0)}=\sum_{k=1}^{l+1}\Psi_k\Omega_{l-k+1}^{(1)},\ i>0,\ l\ge 0,$$

$$P_{0,l}^{(0,1)}=P_{i,l}^{(1,1)}=\sum_{k=1}^{l}\Psi_k S_{l-k}^{(1)},\ i,l\ge 1,\quad (2)$$

$$P_{i,l}^{(0,0)}=\Omega_{l-i+1}^{(r)},\quad i>0,\ l\ge i-1,\ j_{r-1}<i\le j_r,$$

$$P_{i,l}^{(0,1)}=S_{l-i}^{(r)},\quad i>0,\ l\ge i,\ j_{r-1}<i\le j_r,\quad r=\overline{1,n},$$

where the matrices Ψ_k, $k\ge 1$, и $S_l^{(r)}$, $\Omega_l^{(r)}$, $l\ge 0$, $r=\overline{1,n}$, can be determined from the following matrix expansions:

$$\Psi(z)=\sum_{k=1}^{\infty}\Psi_k z^k=-[(D_0^{(1)}\oplus F^{(1)}(1))^{-1}\otimes I_M]\times\\ \times((D^{(1)}(z)-D_0^{(1)})\otimes I_{(N+1)M}),$$

$$S^{(r)}(z)=\sum_{k=0}^{\infty}S_k^{(r)}z^k=\int_0^{\infty}e^{D^{(r)}(z)t}\otimes\big(e^{F_0^{(r)}t}F_1^{(r)}\big)\otimes\\ \otimes(P^{(r)}-B^{(r)}(t))\,dt,$$

$$\Omega^{(r)}(z)=\sum_{k=0}^{\infty}\Omega_k^{(r)}z^k=\int_0^{\infty}e^{D^{(r)}(z)t}\otimes e^{F_0^{(r)}t}\otimes\\ \otimes dB^{(r)}(t),\ r=\overline{1,n},$$

$\otimes$ and $\oplus$ are the symbols of the Kronecker product and the Kronecker sum, $I_\bullet$ denotes an identity matrix of corresponding size.

Markov chain $\xi_k, k\ge 1$ is ergodic under any parameters of input and disaster flows, service and mode switching processes if the matrices $F_1^{(r)}$, $r=\overline{1,N}$, are nonzero. If one of these matrices is zero matrix the embedded Markov chain is ergodic if and only if the inequality $\lambda^{(r)}b^{(r)}<1$ holds (Lucantoni and Neuts 1994), where r is the number of mode having zero matrix $F_1^{(r)}$.

Introduce into consideration the stationary state probabilities

$$p(i,\zeta)=\lim_{l\to\infty}P\{i_l=i,u_l=0,\zeta_l=\zeta\},\ i\ge 0,$$
$$k(i,\zeta)=\lim_{l\to\infty}P\{i_l=i,u_l=1,\zeta_l=\zeta\},\ i\ge 1,\zeta=\overline{1,H},$$

vectors $\boldsymbol{p}_i=(p(i,1),\dots,p(i,H))$, $i\ge 0$, $\boldsymbol{k}_i=(k(i,1),\dots,k(i,H))$, $i\ge 1$, and their generating functions

$$\boldsymbol{P}_r(z)=\sum_{i=j_{r-1}+1}^{j_r}\boldsymbol{p}_i z^i,\ r=\overline{1,n},\ \boldsymbol{K}(z)=\sum_{i=1}^{\infty}\boldsymbol{k}_i z^i,\ |z|\le 1.$$

Theorem 1. Probability generating functions $\boldsymbol{P}_r(z)$, $r=\overline{1,n}$, and $\boldsymbol{K}(z)$ satisfy the functional equations

$$\sum_{r=1}^{n} \boldsymbol{P}_r(z)(zI - \Omega^{(r)}(z)) = (\boldsymbol{\pi}_0(\Psi(z) - I) + \quad (3)$$

$$+\boldsymbol{K}(1))\Omega^{(1)}(z),$$

$$\boldsymbol{K}(z) = (\boldsymbol{\pi}_0(\Psi(z) - I) + \boldsymbol{K}(1))S^{(1)}(z) + \quad (4)$$

$$+\sum_{r=1}^{n} \boldsymbol{P}_r(z)S^{(r)}(z),$$

where $\boldsymbol{\pi}_0 = \boldsymbol{p}_0 + \boldsymbol{K}(1)$.

Proof. Using the formula of total probability and Lemma 1, we derive the following system of equations:

$$\boldsymbol{p}_l = (\boldsymbol{p}_0 + \sum_{m=1}^{\infty} \boldsymbol{k}_m) \sum_{k=1}^{l+1} \Psi_k \Omega^{(1)}_{l-k+1} +$$

$$+ \sum_{i=j_{r-1}+1}^{l+1} \boldsymbol{p}_i \Omega^{(r)}_{l-i+1} + \sum_{m=1}^{r-1} \sum_{i=j_{m-1}+1}^{j_m} \boldsymbol{p}_i \Omega^{(m)}_{l-i+1},\ l \geq 0,$$

$$\boldsymbol{k}_l = (\boldsymbol{p}_0 + \sum_{m=1}^{\infty} \boldsymbol{k}_m) \sum_{k=1}^{l} \Psi_k S^{(1)}_{l-k} + \sum_{i=j_{r-1}+1}^{l} \boldsymbol{p}_i S^{(r)}_{l-i} +$$

$$+ \sum_{m=1}^{r-1} \sum_{i=j_{m-1}+1}^{j_m} \boldsymbol{p}_i S^{(m)}_{l-i},$$

$$l > 0,\ j_{r-1} \leq l < j_r,\ r = \overline{1,n}.$$

Multiplying the equations by a corresponding degree of z and summing them up we obtain the theorem statement. Theorem 1 is proved.

Equation (3) involves n unknown functions $\boldsymbol{P}_r(z)$, $r = \overline{1,n}$, and vectors $\boldsymbol{\pi}_0$, $\boldsymbol{K}(1)$. Generating functions $\boldsymbol{P}_r(z)$, $r = \overline{1,n-1}$, can be determined by expanding the both sides of (3) in series at the point $z = 0$:

$$\boldsymbol{P}_r(z) = \boldsymbol{\pi}_0 Y_r(z) + \boldsymbol{K}(1) Q_r(z), \quad r = \overline{1,n-1}, \quad (5)$$

where

$$Y_r(z) = \sum_{i=j_{r-1}+1}^{j_r} Y_i z^i,\ Q(z) = \sum_{i=j_{r-1}+1}^{j_r} Q_i z^i,$$

and matrices Y_i, Q_i, $i = \overline{0, j_{n-1}}$, are calculated recurrently $Y_0 = I$, $Q_0 = -I$,

$$Y_{i+1} = \left(Y_i - \sum_{k=1}^{i+1} \Psi_k \Omega^{(1)}_{i-k+1} - \sum_{k=j_{r-1}+1}^{i} Y_k \Omega^{(r)}_{i-k+1} - \right.$$

$$\left. - \sum_{m=1}^{r-1} \sum_{k=j_{m-1}+1}^{j_m} Y_k \Omega^{(m)}_{i-k+1} \right) \left(\Omega^{(r)}_0\right)^{-1},$$

$$Q_{i+1} = \left(Q_i - \sum_{m=1}^{r-1} \sum_{k=j_{m-1}+1}^{j_m} Q_k \Omega^{(m)}_{i-k+1} - \right.$$

$$\left. - \sum_{k=j_{r-1}+1}^{i} Q_k \Omega^{(r)}_{i-k+1} \right) \left(\Omega^{(r)}_0\right)^{-1},$$

$$i = \overline{j_{r-1}, j_r - 1},\ r = \overline{1, n-1}.$$

Then using (3) and (5) we can derive the generating function $\boldsymbol{P}_n(z)$. So we have only two vectors $\boldsymbol{\pi}_0$ and $\boldsymbol{K}(1)$ unknown. To get the value $\boldsymbol{K}(1)$ from (4), the value $\boldsymbol{P}_n(1)$ needs to be calculated. Substitution $z = 1$ into equations (3)–(4) and their summation give the relation

$$\sum_{r=1}^{n} \boldsymbol{P}_r(1)(I - A_r) + \boldsymbol{K}(1)(I - A_1) = \boldsymbol{\pi}_0(\Psi(1) - I)A_1, \quad (6)$$

where $A_r(1) = \Omega^{(r)}(1) + S^{(r)}(1)$, $r = \overline{1,n}$. Note that matrices $A_r(1)$, $r = \overline{1,n}$, are stochastic, so the matrices $I - A_n + \mathbf{1}\boldsymbol{\rho}$, $r = \overline{1,n}$, are nonsingular, where $\boldsymbol{\rho}$ is row eigenvector of the matrix A_n corresponding to the eigenvalue 1.

Adding the expression

$$\left(\sum_{r=1}^{n} \boldsymbol{P}_r(1) + \boldsymbol{K}(1) \right) \mathbf{1}\boldsymbol{\rho} = \boldsymbol{\rho}$$

to the both sides of (6), we obtain

$$\boldsymbol{P}_n(1) = \boldsymbol{\rho} + \boldsymbol{\pi}_0(\Psi(1) - I)A_1 Z + \sum_{r=1}^{n-1} \boldsymbol{P}_r(1)(A_r - \quad (7)$$

$$-I - \mathbf{1}\boldsymbol{\rho})Z + \boldsymbol{K}(1)(A_1 - I - \mathbf{1}\boldsymbol{\rho})Z,$$

where $Z = (I - A_n + \mathbf{1}\boldsymbol{\rho})^{-1}$. Then using (4), (5) and (7), the dependence of the vector $\boldsymbol{K}(1)$ on $\boldsymbol{\pi}_0$ is derived

$$\boldsymbol{K}(1) = \boldsymbol{\rho} S^{(n)}(1) V + \boldsymbol{\pi}_0 T V, \quad (8)$$

where

$$T = (\Psi(1) - I)A^*_1 + \sum_{r=1}^{n-1} Y_r(1) A^*_r,$$

$$V = (I - A^*_1 - \sum_{r=1}^{n-1} Q_r A^*_r)^{-1},$$

$$A^*_r = S^{(r)}(1) + (A_r - I - \mathbf{1}\boldsymbol{\rho}) Z S^{(n)}(1),\ r = \overline{1, n-1}.$$

Finally, to calculate unknown vector $\boldsymbol{\pi}_0$ we exploit the functional equation (3) and the property of the equation $det(zI - \beta^{(n)}(z)) = 0$ to have exactly $H = (W+1)(N+1)M$ roots inside a unit disc $|z| < 1$ of a complex plane (Theorem 3 in (Gail et al. 1997)). Denote these roots as z_k with corresponding multiplicities l_k, $k = \overline{1,J}$, $\sum_{k=1}^{J} l_k = H$, where J is the number of different root. Using the analyticity property of the function $\boldsymbol{P}_n(z)$ inside the disc $|z| < 1$ (see e.g. (Dudin 1998)) we obtain the following system of equations for vector $\boldsymbol{\pi}_0$ entries:

$$\boldsymbol{\pi}_0 \frac{d^l}{dz^l}\Big\{\Big[(\Psi(z) - I + TV)\Omega^{(1)}(z) + \sum_{r=1}^{n-1}(Y_r(z) + +TVQ_r(z))(\Omega^{(r)}(z) - zI)\Big] \times \times adj(zI - \Omega^{(n)}(z))\Big\}\Big|_{z=z_k} = = -\boldsymbol{\rho} S^{(n)} V \frac{d^l}{dz^l}\Big\{\Big[\Omega^{(1)}(z) + \sum_{r=1}^{n-1} Q_r(z)(\Omega^{(r)}(z) - zI)\Big] \times \times adj(zI - \Omega^{(n)}(z))\Big\}\Big|_{z=z_k},$$

$$l = \overline{0, l_k - 1},\ k = \overline{1, J}, \sum_{k=1}^{J} l_k = H.$$

Having known the value of vector $\boldsymbol{\pi}_0$, we calculate the value of $\boldsymbol{K}(1)$. Then substituting the values of these vectors into (3), (4) and (5) we get the final expressions for the generating functions $\boldsymbol{P}_r(z)$, $r = \overline{1, n}$, and $\boldsymbol{K}(z)$.

PERFORMANCE CHARACTERISTICS AND THE COST CRITERION VALUE

As some customers can leave the system unserved, the important performance characteristic is the probability of an arbitrary customer successful service. Denote this probability as P_+. Using the ergodic theorems for functionals defined on the Markov chains (Skorohod 1980) the formula for P_+ can be obtained

$$P_+ = \frac{\sum_{r=1}^{n} \boldsymbol{P}_r(1)\mathbf{1}}{\sum_{r=1}^{n} \boldsymbol{P}_r(1)\mathbf{1} + \boldsymbol{K}'(1)\mathbf{1}}.$$

and the performance characteristics involved in cost criterion (1) can be determined

$$L = \sum_{r=1}^{n} \boldsymbol{P}_r'(1)\mathbf{1},$$

$$\Lambda = \frac{\lambda^{(1)}}{\sum_{r=1}^{n} \boldsymbol{P}_r(1)\mathbf{1} + \boldsymbol{K}'(1) + \sum_{r=2}^{n}(\lambda^{(1)} - \lambda^{(r)})\boldsymbol{P}_r(1)B_r\mathbf{1}},$$

$$\Phi_r = \Lambda \boldsymbol{P}_r(1)B_r\mathbf{1},\ r = \overline{2, n}, \quad \Phi_1 = 1 - \sum_{r=2}^{n}\Phi_r,$$

$$R = \sum_{r=1}^{n} \lambda^{(r)}\Phi_r(1 - P_+).$$

where $B_r = \int_0^\infty e^{D^{(r)}(1)t} \otimes e^{F_0^{(r)}t} \otimes dB^{(r)}(t) + \int_0^\infty e^{D^{(r)}(1)t} \otimes \big(te^{F_0^{(r)}t}F_1^{(r)}\big) \otimes (P^{(r)} - B^{(r)}(t))\,dt$, $r = \overline{2, n}$.

Having calculated performance characteristics we get the value of the cost criterion under the fixed set of thresholds $(j_1, j_2, \dots, j_{n-1})$. Having the algorithm for calculation of the cost criterion value for any fixed set of thresholds we can find the optimal value of thresholds $(j_1^*, j_2^*, \dots, j_{n-1}^*)$ minimizing the cost criterion on the bounded region $A_J = \{(j_1, j_2, \dots, j_{n-1}) : 0 \le j_1 \le j_2 \le \dots \le j_{n-1} \le J\}$.

NUMERICAL EXAMPLE

To illustrate the obtained results we present bellow simple numerical example. Consider the queue with four modes of operation, $a = 0.5$, $c_1 = 2$, $c_2 = 100$, $c_3 = 20$, $c_4 = 35$, $d = 25$. BMAP-input of customers is given by the matrices

$$D_0^{(1)} = \begin{pmatrix} -5.52 & 0.52 \\ 0.35 & -0.35 \end{pmatrix}, D_1^{(1)} = \begin{pmatrix} 4 & 0 \\ 0 & 4.8 \end{pmatrix},$$

$$D_2^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 1.2 \end{pmatrix}, D_0^{(2)} = \begin{pmatrix} -4.36 & 0.36 \\ 0.21 & -3.2 \end{pmatrix},$$

$$D_1^{(2)} = \begin{pmatrix} 2.8 & 0 \\ 0 & 2.1 \end{pmatrix}, D_2^{(2)} = \begin{pmatrix} 1.2 & 0 \\ 0 & 0.09 \end{pmatrix}.$$

$$D_0^{(3)} = \begin{pmatrix} -3.36 & 0.36 \\ 0.21 & -2.21 \end{pmatrix}, D_2^{(3)} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix},$$

$$D_0^{(4)} = \begin{pmatrix} -1.25 & 0.25 \\ 0.04 & -2.04 \end{pmatrix}, D_1^{(4)} = D_2^{(4)} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}.$$

The semi-Markovian kernel characterizing the service process in the rth mode has the form

$$B^{(r)}(t) = \begin{pmatrix} 0.6B_1^{(1)}(t) & 0.4B_2^{(1)}(t) \\ 0.35B_1^{(2)}(t) & 0.65B_2^{(2)}(t) \end{pmatrix},$$

where $B_i^{(r)}(t) = \int_0^t \frac{b_i^{(r)}(b_i^{(r)}\tau)^{k_i^{(r)}-1}}{(k_i^{(r)}-1)!} e^{-b_i^{(r)}\tau} d\tau$, $b_1^{(1)} = 18$, $b_1^{(2)} = b_2^{(2)} = 20$, $b_2^{(1)} = 6$, $b_1^{(3)} = 24$, $b_2^{(3)} = 30$, $b_1^{(4)} = 10$, $b_2^{(4)} = 15$, $k_1^{(1)} = k_2^{(2)} = k_1^{(4)} = 2$, $k_2^{(1)} = k_2^{(4)} = 3$, $k_1^{(2)} = 1$, $k_1^{(3)} = 4$, $k_2^{(3)} = 5$.

MAP-input of disasters is given by the matrices

$$F_0^{(1)} = \begin{pmatrix} -0.61 & 0.21 \\ 0.32 & -0.68 \end{pmatrix}, F_1^{(1)} = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.36 \end{pmatrix},$$

$$F_0^{(2)} = \begin{pmatrix} -0.26 & 0.16 \\ 0.27 & -0.35 \end{pmatrix}, F_1^{(2)} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.08 \end{pmatrix},$$

$$F_0^{(3)} = \begin{pmatrix} -0.26 & 0.21 \\ 0.32 & -0.4 \end{pmatrix}, F_1^{(3)} = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.08 \end{pmatrix},$$

$$F_0^{(4)} = \begin{pmatrix} -0.25 & 0.21 \\ 0.32 & -0.33 \end{pmatrix}, F_1^{(4)} = \begin{pmatrix} 0.04 & 0 \\ 0 & 0.01 \end{pmatrix}.$$

Table contains the values of customer input intensity $\lambda^{(r)}$, disaster flow intensity $\varphi^{(r)}$ and mean service time $b^{(r)}$ when rth mode is used, $r = \overline{1, n}$.

Table: Characteristics of operation modes

number of mode	$\lambda^{(r)}$	$\varphi^{(r)}$	$b^{(r)}$
1	6.71	0.384	0.131
2	4.37	0.092	0.184
3	4.73	0.062	0.166
4	2.79	0.028	0.20

Denote by C_r the cost criterion value when the rth operation mode is used only, $r = \overline{1,4}$. For our system $C_1 = 39.09$, $C_2 = 116.14$, $C_3 = 37.39$, $C_4 = 38.67$. The values of C_r, $r = \overline{1,4}$, are calculated by the formulas from (Dudin and Nishimura 1999).

When all operation modes are in use the optimal criterion value is $C^* = 27.780$ and optimal thresholds are $j_1^* = j_2^* = 1$, $j_3^* = 3$. Note that the second mode is not used under optimal system operation. Comparing the value C^* and $min\{C_1, C_2, C_3, C_4\}$ we can conclude that optimal control allows to reduce the system operation cost more than 27%.

The algorithm was realized as the module of software "SIRIUS+" developed in Laboratory of Applied Probabilistic Analysis of Belarus State University, see (Dudin et al. 2000, Dudin et al. 2004).

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