STABLE ALGORITHM FOR STATIONARY DISTRIBUTION CALCULATION FOR A BMAP/SM/1 QUEUE WITH MAP-INPUT OF DISASTERS AND NON-INSTANTANEOUS RECOVERY

Olga V. Semenova
Laboratory of Applied Probabilistic Analysis
Department of Applied Mathematics and Computer Sciences
Belarus State University
4, F.Skorina Ave., Minsk-50, 220050, Belarus
semenovaov@bsu.by

Che S. Kim
Department of Industrial Engineering
Sangji University
Wonju, Kongwon
Korea, 220-702
dowoo@sangji.ac.kr

KEYWORDS

BMAP/SM/1 queue, disasters, embedded Markov chain

ABSTRACT

A BMAP/SM/1 queueing system with Markovian arrival input of disasters is considered. After a disaster arrival all customers leave the system instantaneously and a server is recovered during a random period of time. Numerically stable algorithm for calculation of the stationary state distribution of embedded Markov chain is presented.

INTRODUCTION

During real queueing system operation, the appearance of disasters is possible which causes customers loss and system operation disturbances. Simultaneous loss of all customers can be described by a disaster causing all customers to leave the system instantaneously. Such disaster is a special case of a so called negative arrival that removes one customer or a batch of ones of random size from the queueing system. The theory of negative arrivals has been originated and developed significantly by Gelenbe, see, e.g., (Gelenbe 1991).

A BMAP/SM/1 queueing system with MAP input of disasters in the cases of instantaneous and non-instantaneous recovery of a server after a disaster arrival was investigated in (Dudin and Nishimura 1999) and (Dudin and Karolik 2001). The main results obtained for this queueing system (arbitrary time stationary queue length distribution, the performance characteristics) need calculation of the stationary state distribution of the Markov chain embedded at customer departure epochs. The algorithm presented in (Dudin and Karolik 2001) exploits the analyticity of the vector generating function of the stationary distribution in a unit disc of a complex plane. It includes calculation of roots of some function in a unit disc. In the case of high dimensionality of the stationary state probability vectors the roots determination problem can arise. One more shortage is following. The recursion for probability vectors calculation contains subtraction operation that leads to calculation error growth and the probability vectors corresponding to higher states can have negative entries.

In this paper we present the alternative algorithm for stationary state distribution calculation for the system BMAP/SM/1 with disasters in the case of non-instantaneous recovery of a server. Presented algorithm is stable in numerical realization and doesn't have mentioned shortages. The similar algorithm is obtained in (Dudin and Semenova 2004) for the case of instantaneous recovery of a server after a disaster arrival.

MODEL

We consider a single-server queueing system with unlimited waiting space.

The input into the system is a BMAP (Batch Markovian Arrival Process). This input is directed by a continuous-time Markov chain $\nu_t, t \geq 0$ with a state space $\{0, 1, \ldots, W\}$. The transitions of process $\nu_t, t \geq 0$ and arrivals of customers are performed accordingly to a matrix generating function $D(z) = \sum_{k=0}^{\infty} D_k z^k, |z| \leq 1$. A more detailed description of BMAP and assumptions about the matrix function $D(z)$ are given by (Lucantoni 1991).

Denote by $\vec{\varphi}$ the stationary probability row vector of the Markov chain $\nu_t, t \geq 0$. It is defined by equations:

$$\vec{\varphi} D(1) = \mathbf{0}, \qquad \vec{\varphi}\mathbf{1} = 1.$$

Here $\mathbf{0}$ is a zero row vector and $\mathbf{1}$ is a unit column vector.

The intensity λ of BMAP-input (the fundamental rate) is calculated as $\lambda = \vec{\varphi} D'(1)\mathbf{1}$.

We assume that service process is of SM-type. It means that successful service times are the sojourn times of a semi-Markovian process $m_t, t \geq 0$. This process has a state space $\{1, ..., M\}$ and a semi-Markovian kernel $B(x) = \|B_{m,m'}(x)\|_{m,m'=\overline{1,M}}$. The function $B_{m,m'}(x)$ is the conditional distribution function of the sojourn time of the process $m_t, t \geq 0$ in a state m under the condition that the next state is m', $m, m' = \overline{1,M}$.

We use the same assumptions about the kernel $B(x)$ which are given in (Lucantoni and Neuts 1994) and (Neuts 1989).

For the system under consideration the service is interrupted at a disaster arrival epoch. We assume that the states of the service directing process $m_t, t \geq 0$ are changed at service completion epochs accordingly the matrix $P = B(\infty)$ regardless of whether service is completed successfully or is cancelled by a disaster appearance.

Denote by b_1 the mean service time which is not interrupted by a disaster arrival. The value b_1 is defined by the formula $b_1 = \vec{\delta} \int_0^\infty t dB(t)\mathbf{1}$, where $\vec{\delta}$ is invariant row vector of the matrix P.

The input of disasters is MAP (Markovian Arrival Process). MAP is a partial case of BMAP when ordinary arrivals are allowed only. We assume that MAP is directed by a continuous-time Markov chain $\eta_t, t \geq 0$ with a state space $\{0, 1, \ldots, N\}$ and a matrix generating function $F(z) = F_0 + F_1 z, |z| \leq 1$.

Following (Jain and Sigman 1996) we suppose that arrival of disaster at a busy period interrupts the service and immediately removes all customers from the system. Then the server is recovered during a period having distribution function $G(t)$. If disaster arrives to the empty system or during a recovery period it's ignored by the system. We consider two cases of customers admission during a recovery period:

a) arriving batch of customers is admitted to the queue with probability q_a and is ignored with complementary probability $1 - q_a$;

b) each customer of arriving batch is admitted to the queue with probability q_b and is ignored with complementary probability $1 - q_b$.

The paper (Dudin 1999) contains the detailed description of customers input that is thinned by the ways a) and b).

EMBEDDED MARKOV CHAIN

Let t_n be the n-th epoch of customers departure from the system, $n \geq 1$. It's a service completion epoch or a disaster arrival epoch at a busy period.

Introduce into consideration the following five-dimensional Markov chain:

$$\xi_n = \{i_n, c_n, \nu_n, \eta_n, m_n\},\ n \geq 1,$$

where i_n is a queue length at the epoch $t_n + 0$, $i_n \geq 0$; ν_n is the state of arrival directing process $\nu_t, t \geq 0$ at the epoch t_n, $\nu_n = \overline{0,W}$; η_n is the state of disaster directing process $\eta_t, t \geq 0$ at the epoch $t_n + 0$, $\eta_n = \overline{0,N}$; m_n is the state of service directing process $m_t, t \geq 0$ at the epoch $t_n + 0$, $m_n = \overline{1,M}$;

$$c_n = \begin{cases} 0, & \text{if } t_n \text{ is a successful service completion epoch,} \\ 1, & \text{if } t_n \text{ is a disaster arrival epoch, } n \geq 1. \end{cases}$$

Denote by

$$P\{(i, c, \nu, \eta, m) \to (l, c', \nu', \eta', m')\} = P\{i_{n+1} = l, c_{n+1} = c', \nu_{n+1} = \nu', \eta_{n+1} = \eta', m_{n+1} = m | i_n = i, c_n = c, \nu_n = \nu, \eta_n = \eta, m_n = m\}$$

the one step transition probabilities of the Markov chain $\xi_n, n \geq 1$. Let these probabilities be listed in lexicographic order of the components $\{\nu, \eta, m\}$ increasing.

Introduce into consideration the matrices

$$P_{i,l} = \begin{pmatrix} P_{i,l}^{(0,0)} & P_{i,l}^{(0,1)} \\ P_{i,l}^{(1,0)} & P_{i,l}^{(1,1)} \end{pmatrix}, \quad i, l \geq 0,$$

where the block $P_{i,l}^{(c,c')}$ is the matrix formed by the probabilities $P\{(i, c, \nu, \eta, m) \to (l, c', \nu', \eta', m')\}$.

The non-zero matrices $P_{i,l}, i, l \geq 0$ have the form

$$P_{0,0} = \begin{pmatrix} \Psi_1 \Omega_0 & \Psi(1) S \\ H_0 \Psi_1 \Omega_0 & H_0 \Psi(1) S + \sum\limits_{i=1}^{\infty} H_i S \end{pmatrix},$$

$$P_{1,0} = \begin{pmatrix} \Omega_0 & S \\ O & O \end{pmatrix},$$

$$P_{i,0} = \begin{pmatrix} O & S \\ O & O \end{pmatrix}, \quad i > 1,$$

$$P_{0,l} = \begin{pmatrix} \sum\limits_{i=1}^{l+1} \Psi_i \Omega_{l-i+1} & O \\ \sum\limits_{i=1}^{l+1} (H_0 \Psi_i + H_i) \Omega_{l-i+1} & O \end{pmatrix},$$

$l > 0$,

$$P_{i,l} = \begin{pmatrix} \Omega_{l-i+1} & O \\ O & O \end{pmatrix}, l \geq max\{1, i-1\}, i > 0.$$

Here the matrices $\Omega_l, H_l, l \geq 0$ are defined by the matrix expansions

$$\sum_{l=0}^{\infty} \Omega_l z^l = \beta(z) = \int_0^\infty e^{D(z)t} \otimes e^{F_0 t} \otimes dB(t), \qquad (1)$$

$$\sum_{l=0}^{\infty} H_l z^l = H(z) = \int_0^\infty e^{(R(z) \oplus F(1))t} dG(t) \otimes I_M, \quad (2)$$

$$R(z) = \begin{cases} q_a D(z) + (1-q_a)D(1), & \text{in the case a)}, \\ D(q_b(z-1)+1), & \text{in the case b)}, \end{cases}$$

the matrices S and $\Psi_k, k \geq 1$ are calculated as

$$S = \int_0^\infty e^{D(1)t} \otimes (e^{F_0 t}F_1) \otimes (P - B(t))dt, \quad (3)$$

$$\Psi_k = -[(D_0 \oplus F(1))^{-1}(D_k \otimes I_{N+1})] \otimes I_M, \quad (4)$$
$$k \geq 1,$$

where $\otimes$ and $\oplus$ are the symbols of the Kronecker product and the Kronecker sum, $I_\bullet$ denotes an identity matrix of corresponding size, O is a zero matrix of size $K = (W+1)(N+1)M$.

The matrices Ω_l and S describe transitions of the process $\zeta_n = \{\nu_n, \eta_n, m_n\}, n \geq 1$ during the service time. The matrix Ω_l corresponds to the l customers arrival and no disaster arrival during the service time, $l \geq 0$. The matrix S corresponds to the disaster arrival during the service time. The matrix Ψ_k describes transitions of the process $\zeta_n, n \geq 1$ during the idle period (excluding the recovery period) which finishes by arrival of batch of k customers, $k \geq 1$. The matrix H_l means accumulation of l customers during a recovery period, $l \geq 0$. The detailed description of the matrices involved in (1)–(4) is given in (Dudin and Karolik 2001).

STABLE ALGORITHM

Consider the following stationary state probabilities:

$$\begin{aligned} & p(i,\nu,\eta,m) = \\ & \lim_{n\to\infty} P\{i_n = i, c_n = 0, \nu_n = \nu, \eta_n = \eta, m_n = m\}, \\ & k(\nu,\eta,m) = \quad (5) \\ & \lim_{n\to\infty} P\{i_n = 0, c_n = 1, \nu_n = \nu, \eta_n = \eta, m_n = m\}, \\ & i \geq 0, \nu = \overline{0,W}, \eta = \overline{0,N}, m = \overline{1,M}. \end{aligned}$$

The limits (5) exist for any finite positive arrival and service rates due to the presence of disasters.

Define the following vectors:

$$\vec{p}(i,\nu,\eta) = (p(i,\nu,\eta,1), \dots, p(i,\nu,\eta,M)),$$

$$\vec{p}(i,\nu) = (\vec{p}(i,\nu,0), \dots, \vec{p}(1,\nu,N)),$$

$$\vec{p}_i = (\vec{p}(i,0), \dots, \vec{p}(i,W)),$$

$$\vec{k}(\nu,\eta) = (k(\nu,\eta,1), \dots, k(\nu,\eta,M)),$$

$$\vec{k}(\nu) = (\vec{k}(\nu,0), \dots, \vec{k}(\nu,N)),$$

$$\vec{k} = (\vec{k}(0), \dots, \vec{k}(W)).$$

Bellow we obtain the algorithm for calculating the stationary state probabilities in the form of vectors

$$\vec{\pi}_0 = (\vec{p}_0, \vec{k}), \quad \vec{\pi}_i = (\vec{p}_i, \mathbf{0}), \ i \geq 1. \quad (6)$$

Let $G^{(k)}$ be the matrix which describes transitions of components $\{\nu_n, \eta_n, m_n\}$ of the Markov chain $\xi_n, n \geq 1$ in the time interval during which the state of the component i_n changes from $k+1$ to k and no disaster arrives, $k \geq 0$. The matrices $G^{(k)}, k \geq 0$ satisfy the following equation:

$$G^{(k)} = P_{k+1,k} + \sum_{i=k+1}^{\infty} P_{k+1,i} \times \quad (7)$$
$$G^{(i-1)}G^{(i-2)} \cdots G^{(k)}, \ k \geq 0.$$

Here we assume that $P_{1,0} = \begin{pmatrix} \Omega_0 & O \\ O & O \end{pmatrix}$.

For the system under consideration the matrix $G^{(k)}$ does not depend on k, $k \geq 0$, and is equal to

$$G = \begin{pmatrix} \hat{G} & O \\ O & O \end{pmatrix}, \quad (8)$$

where $\hat{G}$ is the solution of the matrix equation

$$\hat{G} = \beta(\hat{G}) = \sum_{l=0}^{\infty} \Omega_l \hat{G}^l. \quad (9)$$

Formulas (8) –(9) follows from (7) and the block form of matrices $P_{k,i}, i \geq k-1$. Algorithm for solving the equation (9) can be found in (Neuts 1989).

Let $X^{(k)}$ be the matrix which describes the transitions of components $\{\nu_n, \eta_n, m_n\}$ of the Markov chain $\xi_n, n \geq 1$ in the time interval that starts from the the state k of the component i_n and finishes by reaching the state 0 due to a disaster arrival without visiting the state $k-1$, $k \geq 1$.

The matrices $X^{(k)}, k \geq 1$ are defined by the equation

$$X^{(k)} = P_{k,0} + P_{k,k}X^{(k)} + \sum_{n=k+1}^{\infty} P_{k,n} \times$$
$$\left(\sum_{i=k}^{n-1} G^{(n-1)}G^{(n-2)} \cdots G^{(i)}X^{(i)} + X^{(n)} \right), \ k \geq 1.$$

Here we assume that $P_{1,0} = \begin{pmatrix} O & S \\ O & O \end{pmatrix}$.

As $G^{(k)}$ the matrix $X^{(k)}$ does not depend on k and is equal to

$$X = \begin{pmatrix} O & (\hat{G} - I)(\beta(1) - I)^{-1}S \\ O & O \end{pmatrix}.$$

Theorem. Stationary probability vectors $\vec{\pi}_i, i \geq 0$ are calculated by the following way:

$$\vec{\pi}_i = \vec{\pi}_0 \Phi_i, \quad i \geq 1, \quad (10)$$

where the matrices $\Phi_i, i \geq 0$ are calculated recurrently

$$\Phi_0 = I, \quad \Phi_k = \sum_{i=0}^{k-1} \Phi_i Y_k^{(i)} (I - Y_k^{(k)})^{-1}, k \geq 1, \ (11)$$

the matrices $Y_k^{(i)}$ are defined as

$$Y_k^{(i)} = \sum_{l=k}^{\infty} P_{i,l} G^{l-k},\ i = \overline{0,k}, k \geq 0, \quad (12)$$

vector $\vec{\pi}_0$ satisfies the system

$$\begin{aligned} &\vec{\pi}_0 \left(I - Y_0^{(0)} - \sum_{k=1}^{\infty} P_{0,k} \sum_{i=0}^{k-1} G^i X \right) = \mathbf{0}, \\ &\vec{\pi}_0 \sum_{i=0}^{\infty} \Phi_i \mathbf{1} = 1. \end{aligned} \quad (13)$$

Theorem is proved as Theorem 1 in (Dudin and Semenova 2004). The proof is based on the theory of censoring Markov chains, see (Kemeni et al. 1966).

Nonsingularity of the matrix $I - Y_k^{(k)}$ in (11) follows from the Hadamard Theorem. It follows from the Lederman theorem (Bellman 1960) that entries of the matrix $(I - Y_k^{(k)})^{-1}$ are nonnegative.

Formulas (11) involve only sum and product of the matrices with nonnegative entries. So the recursion (11) is numerically stable.

For numerical realization we calculate the first J vectors $\vec{\pi}_i$. Level J is defined from the inequality

$$\|\vec{\pi}_{J+1} - \vec{\pi}_J\| < \varepsilon, \quad (14)$$

where ε is a given accuracy of calculations.

NUMERICAL RESULTS

Let algorithm A be the algorithm elaborated in (Dudin and Karolik 2001) and algorithm B be one based on formulas (8)–(13).

The aim of numerical experiments is comparison of algorithms A and B. It's assumed in (Dudin and Karolik 2001) that all customers either are accumulated or are lost during the recovery period. So we can consider the cases $q_a = q_b = 1$ or $q_a = q_b = 0$ only.

We compare the following characteristics of algorithms:

- the stability of the both algorithms. In our experiments the stability is characterized by the number $\hat{N}$ of probability vectors calculated correctly. The correctness means that calculated vector has nonnegative entries.
- the values of corresponding vectors calculated by the algorithms A and B correctly.
- the running time.

We will compare the vectors $\vec{p}_i, i \geq 0$. For algorithm B the vector $\vec{p}_i$ is determined from (6) as the first K entries of the vector $\vec{\pi}_i$.

Example 1. Let us consider the case $q_a = q_b = 1$. It means that all arriving customers are accepted to the queue during a recovery period.

BMAP-input is defined by the matrices

$$D_0 = \begin{pmatrix} -2.2 & 1.2 \\ 4.8 & -7.8 \end{pmatrix}, D_1 = D_2 = \begin{pmatrix} 0.5 & 0 \\ 0 & 1.5 \end{pmatrix}.$$

The intensity of BMAP is 1.2 and correlation coefficient is 0.013.

The MAP-input of disasters is characterized by the matrices

$$F_0 = \begin{pmatrix} -0.17 & 0.16 \\ 0.27 & -0.35 \end{pmatrix}, F_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.08 \end{pmatrix}$$

with intensity 0.092 and correlation coefficient $3.2 \cdot 10^{-4}$.

For SM-service we assume that the kernel $B(x)$ has the form

$$B(x) = \begin{pmatrix} 0.65\, B_1(x) & 0.35\, B_1(x) \\ 0.45\, B_2(x) & 0.55\, B_2(x) \end{pmatrix}, \quad (15)$$

where

$$B_i(t) = \int_0^t \frac{\gamma_i (\gamma_i \tau)^{k_i - 1}}{(k_i - 1)!} e^{-\gamma_i \tau} d\tau, \quad i = \overline{1,2}, \quad (16)$$

$\gamma_1 = 15$, $\gamma_2 = 20$, $k_1 = 3$, $k_2 = 4$. The mean service time is 0.20.

Recovery period is distributed exponentially with the rate 0.5.

Denote by ρ the product of the BMAP fundamental rate and mean service time. In our example $\rho = 0.42$.

Table 1 contains the values $\vec{p}_i \mathbf{1}$ calculated by the algorithms A and B, the number $\hat{N}$ of vectors calculated correctly and running time T (on PC Pentium II). We set $\varepsilon = 10^{-5}$ in condition (14) of algorithm B. We need to calculate non less 42 vectors $\vec{p}_i$ to provide the given accuracy ε.

Table 1

	Algorithm A	Algorithm B
i	$\vec{p}_i \mathbf{1}$	$\vec{p}_i \mathbf{1}$
0	0.334410031	0.334412237
1	0.276703645	0.276705467
2	0.143837023	0.143837968
3	0.087357481	0.087358054
4	0.047631483	0.047631795
5	0.028496848	0.028497033
6	0.017399227	0.017399327
7	0.011347005	0.011346807
8	0.007778331	0.007772587
$\hat{N}$	8	≥ 50
T	00:15:36	00:10:40

It follows from Table 1 that the corresponding values $\vec{p}_i \mathbf{1}$ coincide with accuracy 10^{-5}.

If the service process is recurrent with distribution function $B_1(t)$ defined by the equation (16), we can calculate the first 17 vectors $\vec{p}_i$ correctly by algorithm A.

Example 2. Now let q_a and q_b be equal to 0. It means that all arriving customers are lost during a recovery period.

The BMAP-input, distribution or recovery period and ε are the same as in example 1. The time interval between disaster arrivals is distributed exponentially with the rate 0.9. The service times have exponential distribution with the rate 0.8.

In this case $\rho = 2.625$. Note that the system BMAP/SM/1 without disasters has no stationary distribution for $\rho \geq 1$, see (Lucantoni and Neuts 1994). As mentioned above the stationary distribution can be calculated for the system under consideration due to disaster arrivals.

Using algorithm A the first 13 vectors $\vec{p}_i$ are calculated correctly. About 145 vectors must be calculated to provide the accuracy 10^{-5} in algorithm B.

The values $\vec{p}_i\mathbf{1}$ calculated by algorithms A and B, the characteristics $\hat{N}$ and T are given in Table 2.

Table 2

	Algorithm A	Algorithm B
i	$\vec{p}_i\mathbf{1}$	$\vec{p}_i\mathbf{1}$
0	0.024153855	0.024153876
1	0.032344791	0.032344799
2	0.031966088	0.031966095
3	0.034250833	0.034250839
4	0.034330425	0.034330431
5	0.034635679	0.034635684
6	0.034226661	0.034226667
7	0.033672918	0.033672923
8	0.032828344	0.032828349
$\hat{N}$	13	≥ 300
T	00:04:17	01:13:15

Analyzing the numerical results the following conclusions can be made.

1. The stability of algorithm A depends on dimensions of vectors, the value ρ and the type of service process (SM or recurrent). The algorithm B provides the stable calculations for all considered arrival, service, disasters and recovery parameters.

2. The values of vector $\vec{p}_i$ calculated correctly by the both algorithms coincide with accuracy 10^{-k} when $\varepsilon = 10^{-k}$ in (14).

3. If dimension of vectors $\vec{p}_i$ is up to 8 the algorithm A is more preferable than algorithm B because of less running time.

The algorithms were realized as the modules of software "SIRIUS+" developed in Laboratory of Applied Probabilistic Analysis of Belarus State University, see (Dudin et al. 2000).

REFERENCES

Bellman, R. 1960. "Introduction to Matrix Analysis". McGraw Hill, New York.

Dudin, A.N. 1999. "The BMAP/SM/1 models with impatient customers". *Queues: Flows, Systems, Networks* 15. BSU, Minsk, 22-25.

Dudin, A.N. and A.V. Karolik. 2001. "BMAP/SM/1 queue with Markovian input of disasters and non-instantaneous recovery". *Performance Evaluation* 45, No.1, 19-32.

Dudin, A.N.; V.I. Klimenok; I.A. Klimenok; et al. 2000. "Software "SIRIUS+"for evaluation and optimization of queues with the BMAP-input". In *Matrix Analytic Methods for Stochastic Models*, eds. G. Latouche and P. Taylor. Notable Publications, N.J., 115-133.

Dudin, A.N. and S. Nishimura. 1999. "A BMAP/SM/1 queueing system with Markovian arrival of disasters". *Journal of Applied Probability* 36, No.3, 868-881.

Dudin, A. and O. Semenova. 2004. "Stable algorithm for stationary distribution calculation for a BMAP/SM/1 queueing system with Markovian arrival input of disasters". *Journal of Applied Probability* 41, No.2.

Gelenbe, E. 1991. "Product form networks with negative and positive customers". *Journal of Applied Probability* 28, 655-663.

Jain, G. and K.A. Sigman. 1996. "A Pollaczek-Khinchine formula for M/G/1 queues with disasters". *Journal of Applied Probability* 36, No.4, 1191-1200.

Kemeni, J.; J. Snell; and A. Knapp. 1966. "Denumerable Markov Chains". Van Nostrand, New York.

Lucantoni, D.M. 1991. "New results on the single server queue with a batch Markovian arrival process". *Communications in Statistical and Stochastical Models* 7, No.1, 1-46.

Lucantoni, D.M. and M.F. Neuts. 1994. "Some steady-state distributions for the BMAP/SM/1 queue". *Communications in Statistical and Stochastical Models* 10, 575-598.

Neuts, M.F. 1989. "Structured Stochastic Matrices of M/G/1 Type Applications". Marcel Dekker, New York.

AUTHOR BIOGRAPHIES

OLGA V. SEMENOVA has got Master degree in Mathematics in 2001 from Gomel State University (Belarus). Now she is a Ph.D. student in Belarus State University. Her e-mail address is: semenovaov@bsu.by.

CHE S. KIM. Professor Che-Soong Kim received his Master degree and Doctor degree in Engineering from Department of Industrial Engineering at Seoul National University in 1989 and 1993. He is currently professor of the Department of Industrial Engineering at Sangji University. His current research interests include various aspects of system modeling and performance analysis, reliability analysis, stochastic process and their application. His e-mail address is: dowoo@sangji.ac.kr.