THE APPLICATION OF Q-ARY LDPC-CODES FOR FIBER OPTIC LINES

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Abstract

The structure of LDPC-code over $GF(16)$ is described. An iterative decoding algorithm based on adding and correcting of erasures is suggested. The numerical results of data transmission over the channel given by the dependencies of error and erasure probabilities on SNR are introduced (suggested decoding is used).

I. INTRODUCTION

The standard **ITU-T G.975** defines the following method of data transmission over fiber optic lines. The data is transmitted on the only frequency with the use of intersymbol intervals. Intersymbol intervals are needed as the impulse shape is changed while going through a fiber optic line. The transmission process of a (0111) word is shown in Fig. 1.

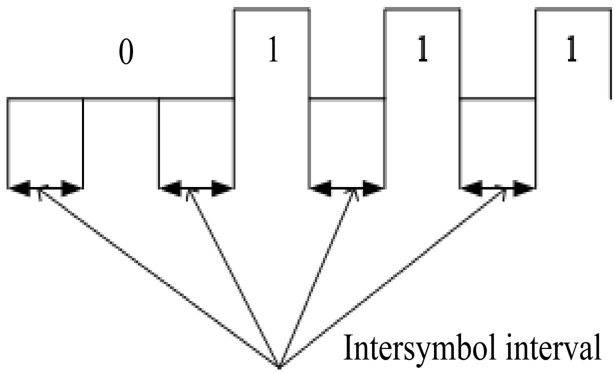

Fig. 1. The transmission of a (0111) word over a fiber optic line with the use of the method defined in the standard

The standard provides the use of Reed-Solomon $(255, 239)$ codes as a noise combating codes.

Disadvantages:

1) The transmission rate is reduced because of the use of intersymbol intervals
2) Reed-Solomon $(255, 239)$ code corrects at most 8 errors in the general case

II. DATA TRANSMISSION METHOD DESCRIPTION

The transmission of 8 bits on 32 frequencies is the main difference of the used method from the method described in the standard. In fact two 4-bit symbols are transmitted. Multi-frequency results in appearance of erasures in the received vector (errors may only be in the received vector while using the previous transmission method). The description of the new transmission method is not the aim of the article [1]. We will just use the calculated dependencies of error and erasure probabilities on SNR to study the new noise combating coding method.

Possible variants for each symbol:

1) Correct
2) Error
3) Erasure

The dependencies of error and erasure probabilities on SNR are shown in Fig. 2. The bit error probability dependence on SNR for the method described in standard is also shown in Fig. 2.

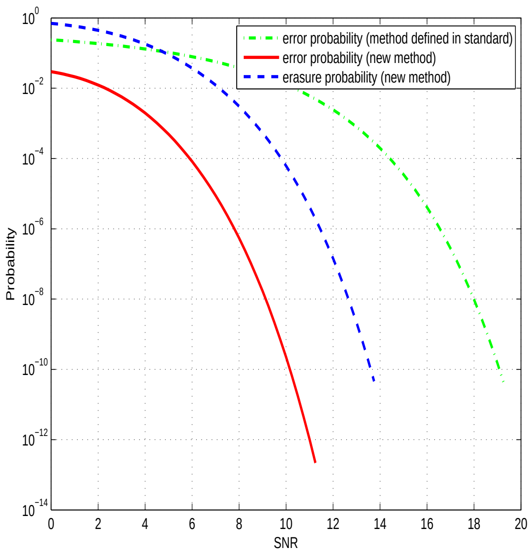

Fig. 2. Dependencies of error and erasure probabilities on SNR

III. LDPC CODE STRUCTURE

A method for constructing long codes from short constituent codes, based on bipartite graphs, was introduced by Tanner in [2]. In his method, one of the two sets of nodes in a bipartite graph is associated with code symbols, while the other set is associated with constituent block codes whose length is equal to the node degree. These two sets of nodes are hereafter referred to as variable nodes and constraint nodes, respectively. Tanner's general code construction unifies many known code families that can be obtained by choosing different underlying bipartite graphs and associating different constituent codes with their constraint nodes.

The single parity check codes associated with the constraint nodes in the Tanner graph of an LDPC code can be replaced with other constituent block codes (e.g., BCH codes, or Reed-Solomon codes [3]), which yields alternative constructions of LDPC codes, often referred to as generalized LDPC codes. The parity-check matrix of such an LDPC code is obtained by replacing every 1 in the graph's adjacency matrix with a column of the constituent code's parity-check matrix, and every 0 with the all-zero column.

We will use a single parity check code over $GF(16)$ as a constituent code. A parity-check matrix of the code consists of n_0 non-zero elements of $GF(16)$. The code has a minimum distance $d_0 = 2$ and is able to correct at most one erasure.

Let $\mathbf{H}_b$ denote a block-diagonal matrix with the b constituent parity-check matrices $\mathbf{H}_0$ on the main diagonal, that is,

$$\mathbf{H}_b = \begin{pmatrix} \mathbf{H}_0 & 0 & \cdots & 0 \\ 0 & \mathbf{H}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H}_0 \end{pmatrix} \tag{1}$$

where b is very large. The matrix $\mathbf{H}_b$ is of size $b \times bn_0$. Let $\pi(\mathbf{H}_b)$ denote a random column permutation of $\mathbf{H}_b$. Then the matrix constructed using $l \geqslant 2$ such permutations as layers,

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_l \end{pmatrix} = \begin{pmatrix} \pi_1(\mathbf{H}_b) \\ \pi_2(\mathbf{H}_b) \\ \vdots \\ \pi_l(\mathbf{H}_b) \end{pmatrix} \tag{2}$$

$\mathbf{H}$ is a sparse $lb \times bn_0$ matrix. Let us define an ensemble of LDPC codes in such a way: the elements of the ensemble $E(n_0, l, b)$ are obtained by sampling independently the permutations π_i , $i = 1, 2, ..., l$, which are all equiprobable. The rate of a code $C \in E(n_0, l, b)$ is lower-bounded by [2].

$$R \geqslant 1 - \frac{lb(n_0 - k_0)}{n} = 1 - l(1 - R_0) \tag{3}$$

with equality if the matrix $\mathbf{H}$ has full rank.

IV. DECODING ALGORITHM

The main difference of suggested algorithm from existing decoding algorithms for LDPC codes is in replacing of error-suspicious symbols with erasures. In each iteration error-suspicious symbols are replaced with erasures and then only the erasure correcting is performed within the iteration. The erasures which were added and were not corrected after the iteration finished are removed. These two operations are repeated until the syndrome is changing.

It is significant that the syndrome is changed only two times within the iteration: after all additional erasures are added and after all erasures are tried to be corrected. This means that in each iteration the result for the current symbol does not depend on the results for the previous and next ones. That is why the algorithm is good for parallelizing. One more advantage of the algorithm is that we are in no need of constraining the maximal number of iterations. It never gets into an infinite loop.

Let us introduce the formal algorithms description:

1) **Syndrome calculation**
The syndrome of the received vector is calculated. It consists of syndromes of constituent-codes. If a constituent code contains erased symbols then its syndrome is not calculated and considered to be erased.

2) **Error-suspicious symbols erasing**
For each non-erased symbol the syndromes of L constituent codes containing the symbol are considered (hereafter decisions). The subset of maximal cardinality (L') containing equal decisions is chosen. If L' is more than the sum of the number of zero decisions and the number of erased decisions, than the symbol is replaced with erasure. The syndromes of constituent codes containing the symbol are erased. The position of the symbol is placed to the list of added erasures.
3) **Correcting of erasures**
For each symbol from the erased symbols list the subset of constituent codes containing the symbol is considered. Only the codes containing one erasure belong to the subset. For each code from this subset we can correct the erasure and form a list of possible symbol values. Then the most often value is found and the erased symbol is replaced with this value.
4) **Stop criterion**
Added erasures are removed. The syndromes before and after iteration are compared. In case of equal syndromes the syndrome weight is calculated. If the weight is equal to zero then the decoded vector is returned. If the weight is not equal to zero the two cases are possible:
   a) If it is the first decoding attempt then erase the symbol belonging to the most part of constituent codes with not null syndromes and return to Step 2.
   b) If it is the second attempt then it is a denial of decoding.

   Return to Step 2 in case of not equal syndromes.

V. NUMERICAL RESULTS

Over than 6 million tests are carried out within the modeling process. The calculated dependencies of initial/final probabilities are shown in Fig. 4, Fig. 5, Fig. 6. The comparison of RS $(255, 239)$ coding and coding by means of studied LDPC codes is shown Fig. 3.

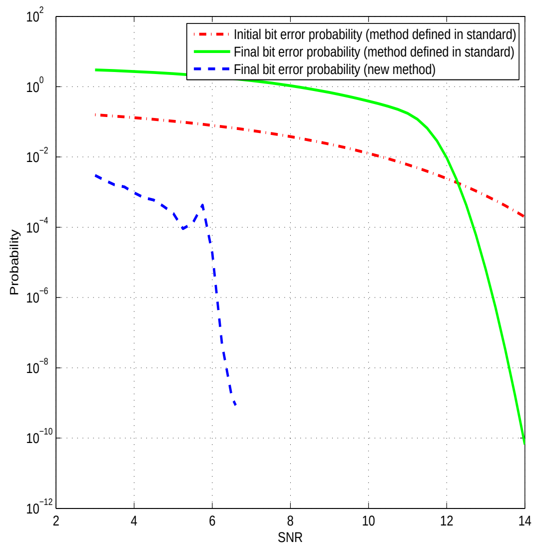

Fig. 3. The comparison of final bit error probabilities for coding defined in standard and coding by means of studied LDPC codes

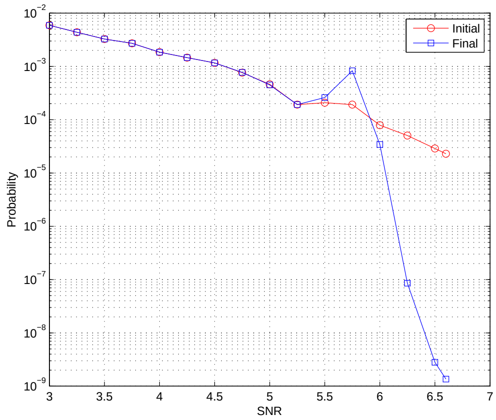

Fig. 4. The comparison of initial and final bit error probabilities for the new method

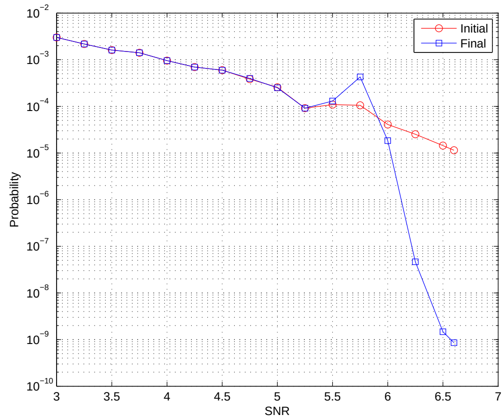

Fig. 5. The comparison of initial and final symbol error probabilities for the new method

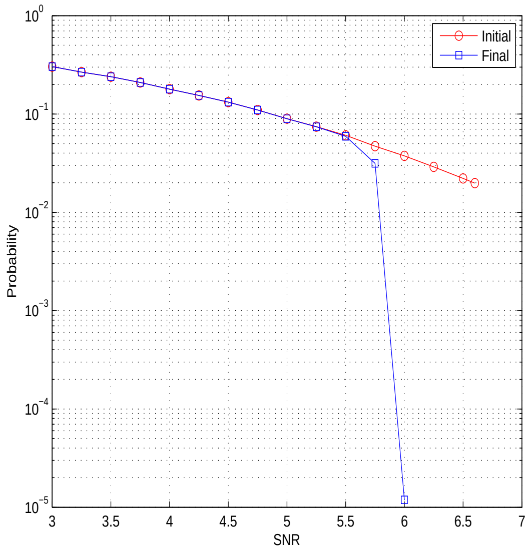

Fig. 6. The comparison of initial and final symbol erasure probabilities for the new method (All the channel erasures are corrected if SNR is greater than 6 dB)

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