



**Pokrovskii: Economics  
via Asynchronous  
Systems**

**VICTOR KOZYAKIN**

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# **One Idea of Pokrovskii**

## **How to Link Economic Problems with Asynchronous Systems?**

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Nonlinear Dynamics Conference in Memory of Alexei Pokrovskii  
University College Cork, Ireland  
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Being in the summer of 2009 in Moscow, Alexei has asked me *to think a bit* about one problem.

He added: *It seems, it is a kind of problems you like.*<sup>1</sup>

Indeed, the formulation of the problem was so simple that I was not able to get rid of it...

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<sup>1</sup>Everybody knows that Alexei was a great master in ***Posing the Right Question to the Right Person at the Right Time.***



Problem: consider a full, weighted, oriented graph...

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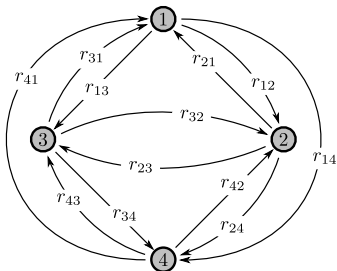
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$$r_{ij} = \frac{1}{r_{ji}} > 0$$

Given a triplet  $\omega = (i, j, k)$  with  $i \neq j$ ,  $k \neq i, j$ , let us update the weights in accordance with the following rule:

$$r_{ij}^{\text{new}} = \max\{r_{ij}, r_{ik} \cdot r_{kj}\}, \quad r_{ji}^{\text{new}} = 1/r_{ij}^{\text{new}}.$$

## Conjecture

For any sequence of triplets  $\{\omega_n\}$ , the updated weights *converge to an equilibrium*.



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The screenshot shows a Wikipedia article titled "Triangular arbitrage". The article is redirected from "Triangle arbitrage". The main text explains that triangular arbitrage is the act of exploiting an arbitrage opportunity resulting from a pricing discrepancy among three different currencies in the foreign exchange market. A triangular arbitrage strategy involves three trades, exchanging the currencies in a cycle. The article also discusses cross exchange rate discrepancies and provides the equation  $S_{a/\$} = S_{a/b} S_{b/\$}$ , which is circled in red. The text explains that  $S_{a/\$}$  is the implicit cross exchange rate for dollars in terms of currency a,  $S_{a/b}$  is the quoted market cross exchange rate for b in terms of currency a,  $S_{b/\$}$  is the quoted market cross exchange rate for dollars in terms of currency b, and  $S_{\$/b}$  is merely the reciprocal exchange rate for currency b in dollar terms, in which case the equation becomes  $S_{a/\$} = S_{a/b} / S_{\$/b}$ .

Figure: Transform rules  $r_{ij}^{\text{new}} = \max\{r_{ij}, r_{ik} \cdot r_{kj}\}$  are motivated by economic reasons.



# Example of a realistic triangular arbitrage scenario

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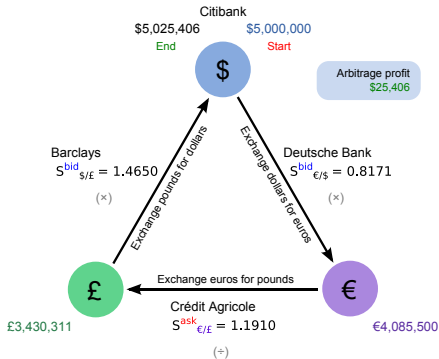
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**Figure:** A visual representation of a realistic triangular arbitrage scenario, using sample bid and ask prices quoted by international banks



The screenshot shows the Wikipedia article for "Arbitrage". The page layout includes a navigation bar at the top with "Log in / create account" and a search box. Below the navigation bar are tabs for "Article", "Discussion", "Read", "Edit", and "View history". The main heading is "Arbitrage", followed by the text "From Wikipedia, the free encyclopedia". A note states "For the upcoming film, see Arbitrage (film)." and another note says "Not to be confused with Arbitration." The main text defines arbitrage in economics and finance, mentioning market prices and cash flow. A red box highlights the sentence: "Arbitrage has the effect of causing prices in different markets to converge." Below this, the text discusses price convergence and market efficiency. The left sidebar contains a table of contents with links to various sections like "Main page", "Contents", "Featured content", "Current events", "Random article", "Donate to Wikipedia", "Interaction", "Help", "About Wikipedia", "Community portal", "Recent changes", "Contact Wikipedia", "Toolbox", "Print/export", "Languages", "العربية", and "Česky".

Figure: Arbitrage has the effect of *causing prices to converge*



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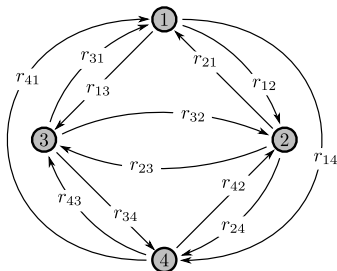
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We have the graph



$$r_{ij} = \frac{1}{r_{ji}} > 0$$

and the set of multiplicative updating rules:

$$r_{ij}^{\text{new}} = \max \{ r_{ij}, r_{ik} \cdot r_{kj} \}, \quad r_{ji}^{\text{new}} = 1 / r_{ij}^{\text{new}}.$$

To simplify Problem, let us set

$$a_{ij} := \log r_{ij} \quad \forall i, j.$$



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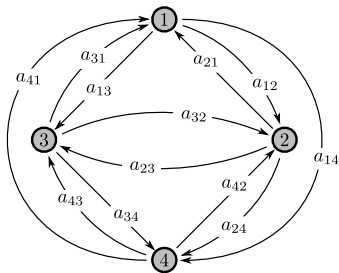
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We obtain the graph



$$a_{ij} = -a_{ji}$$

and the set of additive updating rules:

$$a_{ij}^{\text{new}} = \max \{ a_{ij}, a_{ik} + a_{kj} \}, \quad a_{ji}^{\text{new}} = -a_{ij}^{\text{new}}.$$





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Each updating rule

$$a_{ij}^{\text{new}} = \max\{a_{ij}, a_{ik} + a_{kj}\}, \quad a_{ji}^{\text{new}} = -a_{ij}^{\text{new}}.$$

means the following 'timing' operations:

- 1 given indices  $i$  and  $j$  we first update  $a_{ij}$  to  $a_{ij}^{\text{new}}$ ;
- 2 then, knowing  $a_{ij}^{\text{new}}$  we update  $a_{ji}$  to  $a_{ji}^{\text{new}}$ ;
- 3 as a result, we obtain the updated pair  $(a_{ij}^{\text{new}}, a_{ji}^{\text{new}})$ .

## Question

How will look updating rules if we start updating from the pair of indices  $(j, i)$  ?



# 'Linearization' of Problem (cont.)

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$$a_{ji}^{\text{new}} = \max \{a_{ji}, a_{jk} + a_{ki}\}$$



$$-a_{ij}^{\text{new}} = \max \{-a_{ij}, -a_{kj} - a_{ik}\}$$



$$-a_{ij}^{\text{new}} = -\min \{a_{ij}, a_{ik} + a_{kj}\}$$



$$a_{ij}^{\text{new}} = \min \{a_{ij}, a_{ik} + a_{kj}\}$$



## 'Linearization' of Problem (cont.)

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If we don't care which of the weights  $a_{ij}$  or  $a_{ji}$  is updated first, then we obtain that *there are valid both of the following updating rules:*

$$a_{ij}^{\text{new}} = \mathbf{max} \{a_{ij}, a_{ik} + a_{kj}\}, \quad a_{ji}^{\text{new}} = -a_{ij}^{\text{new}}.$$

OR

$$a_{ij}^{\text{new}} = \mathbf{min} \{a_{ij}, a_{ik} + a_{kj}\}, \quad a_{ji}^{\text{new}} = -a_{ij}^{\text{new}}.$$

### Conclusion

**max** and **min** in the above updating rules are irrelevant and *may be discarded.*



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The rule of updating may be rewritten as follows:

Either  $a_{ij}$  is not changed during update or it is changed and then it is updated as follows:

$$a_{ij}^{\text{new}} = a_{ik} + a_{kj},$$

Wow!

Updating rules became **linear**!



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## Remark

Weights  $a_{ji}$  'mimic'  $a_{ij}$  in the skew-symmetric way.

Then instead of the full set of weights  $\{a_{ij}\}$  it suffices to consider the set of weights  $\{a_{ij}\}$  with  $i < j$ .

As a result, the updating rules  $a_{ij}^{\text{new}} = a_{ik} + a_{kj}$  take the following form:

$$a_{ij}^{\text{new}} = \begin{cases} -a_{ki} + a_{kj} & \text{if } k < i < j, \\ a_{ik} + a_{kj} & \text{if } i < k < j, \\ a_{ik} - a_{jk} & \text{if } i < j < k. \end{cases}$$

It is convenient to represent these last relations  
in matrix form.



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By introducing the column-vector

$$\vec{a} = \{a_{12}, a_{23}, a_{34}, a_{13}, a_{24}, a_{14}\}^T$$

the update rules take the 'matrix' form:

$$\vec{a}^{\text{new}} = A_{(ijk)} \vec{a}, \quad i < j, k \neq i, j,$$

where twelve ( $6 \times 6$ )-matrices  $A_{(ijk)}$  are as follows:

$$A_{(123)} = \begin{pmatrix} 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_{(124)} = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$A_{(231)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_{(234)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

etc.





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Now the problem of investigation of the *weights dynamics* may be posed in the following form:

*Given an initial vector  $\vec{a}(0) = \vec{a}_0$  and a sequence  $\{\omega_n\}$  of triplets  $\omega_n = (i_n j_n k_n)$  such that  $i_n < j_n$ ,  $k_n \neq i_n, j_n$ , we need to study the dynamics of the sequence*

$$\vec{a}(n+1) = A_{\omega_n} \vec{a}(n)$$

*or, what is the same, behavior of the vectors*

$$\vec{a}(n+1) = A_{\omega_n} A_{\omega_{n-1}} \cdots A_{\omega_0} \vec{a}_0$$

Conjecture (Matrix Reformulation)

For any sequence of triplets  $\{\omega_n\}$ , the sequence of matrix products  $A_{\omega_n} A_{\omega_{n-1}} \cdots A_{\omega_0}$  **is convergent.**



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*or, what is the same, behavior of the vectors*

$$\vec{a}(n+1) = A_{\omega_n} A_{\omega_{n-1}} \cdots A_{\omega_0} \vec{a}_0$$

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## Observation

All the matrices  $\{A_{(ijk)}\}$  have a **common invariant subspace of fixed points** determined by the relations

$$\begin{cases} a_{13} &= a_{12} + a_{23}, \\ a_{14} &= a_{13} + a_{34}, \\ a_{24} &= a_{23} + a_{34}. \end{cases}$$

## Corollary

*There exists a change of variables  $Q$  such that each of the matrices  $Q^{-1}A_{(ijk)}Q$  takes the block-triangle form:*

$$B_{(ijk)} := Q^{-1}A_{(ijk)}Q = \left\| \begin{array}{cc} I & C_{(ijk)} \\ 0 & D_{(ijk)} \end{array} \right\|,$$

*where  $C_{(ijk)}$  and  $D_{(ijk)}$  are  $(3 \times 3)$ -matrices.*



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Each matrix product  $B_{\omega_n} B_{\omega_{n-1}} \cdots B_{\omega_0}$  has the following form

$$B_{\omega_n} B_{\omega_{n-1}} \cdots B_{\omega_0} = \left\| \begin{array}{c} I & & * \\ 0 & D_{\omega_n} D_{\omega_{n-1}} \cdots D_{\omega_0} & \end{array} \right\|.$$

## Corollary

*The matrix product  $B_{\omega_n} B_{\omega_{n-1}} \cdots B_{\omega_0}$  is convergent **only if** the matrix product  $D_{\omega_n} D_{\omega_{n-1}} \cdots D_{\omega_0}$  is convergent.*



We need to investigate the convergence  
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*The matrix product  $B_{\omega_n} B_{\omega_{n-1}} \cdots B_{\omega_0}$  is convergent **only if** the matrix product  $D_{\omega_n} D_{\omega_{n-1}} \cdots D_{\omega_0}$  is convergent.*



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Matrices  $\{D_{(ijk)}\}$  are of the form:

$$D_{(123)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad D_{(124)} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_{(132)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$D_{(134)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D_{(142)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_{(143)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$D_{(231)} = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad D_{(234)} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}, \quad D_{(241)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

$$D_{(243)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D_{(341)} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix}, \quad D_{(342)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix},$$



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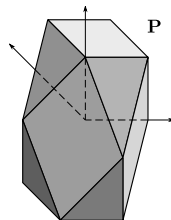
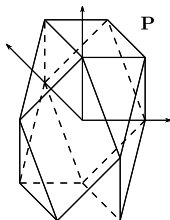
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## Observation

All the matrices  $\{D_{(ijk)}\}$

- have a **common invariant symmetric body set P** (an elongated *cubeoctahedron*),
- **transform the vertices of P either to other vertices of P or to the origin.**





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Let  $\{\omega_n\}$  be the 12-periodic sequence such that

$$\begin{aligned}\omega_0 &= (1, 4, 3), & \omega_1 &= (3, 4, 1), & \omega_2 &= (3, 4, 2), & \omega_3 &= (1, 4, 2), \\ \omega_4 &= (1, 2, 4), & \omega_5 &= (2, 3, 1), & \omega_6 &= (1, 3, 2), & \omega_7 &= (2, 4, 3), \\ \omega_8 &= (1, 3, 4), & \omega_9 &= (2, 4, 1), & \omega_{10} &= (1, 2, 3), & \omega_{11} &= (2, 3, 4),\end{aligned}$$

then the sequence of matrix products

$$D_{\omega_n} D_{\omega_{n-1}} \cdots D_{\omega_0}, \quad n = 0, 1, \dots,$$

is 12-periodic while the sequence of matrix products

$$A_{\omega_n} A_{\omega_{n-1}} \cdots A_{\omega_0}, \quad n = 0, 1, \dots,$$

is divergent!

**Conjecture of Pokrovskii is false !**





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then the sequence of matrix products

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## Observation 2

Let  $\{\omega_n\}$  be the 16-periodic sequence such that

$$\begin{aligned} \omega_0 &= (1, 4, 2), & \omega_1 &= (1, 2, 3), & \omega_2 &= (3, 4, 1), & \omega_3 &= (1, 4, 2), \\ \omega_4 &= (1, 3, 4), & \omega_5 &= (2, 4, 3), & \omega_6 &= (2, 3, 1), & \omega_7 &= (3, 4, 2), \\ \omega_8 &= (2, 4, 1), & \omega_9 &= (1, 3, 4), & \omega_{10} &= (3, 4, 2), & \omega_{11} &= (1, 4, 3), \\ \omega_{12} &= (2, 3, 4), & \omega_{13} &= (1, 3, 2), & \omega_{14} &= (1, 2, 4), & \omega_{15} &= (1, 4, 3), \end{aligned}$$

then both sequences of matrix products

$$D_{\omega_n} D_{\omega_{n-1}} \cdots D_{\omega_0}, \quad n = 0, 1, \dots,$$

and

$$A_{\omega_n} A_{\omega_{n-1}} \cdots A_{\omega_0}, \quad n = 0, 1, \dots,$$

are 16-periodic!



# Question

Pokrovskii: Economics  
via Asynchronous  
Systems

VICTOR KOZYAKIN

## Introduction

Problem Formulation

Economic Background

## Problem Solving

Step 1: Additive Reformulation

Step 2: Linearization

Step 3: Dimensionality Reduction

Step 4: Matrix Representation

Step 5: Further Dimensionality  
Reduction

Step 6: Final Implications

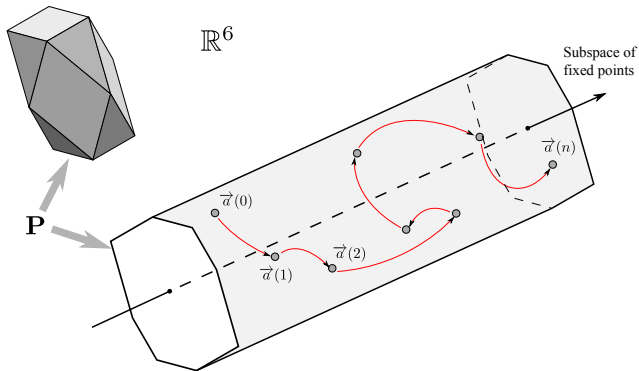
Step 7: Disproof of Conjecture

## Conclusion

A Question

Further Work

Any trajectory  $\{\vec{a}(n)\}$  belongs to a **tube** around the space of common fixed points of the matrices  $A_{(ijk)}$ .



**Does this make any economic sense?**



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Step 6: Final Implications  
Step 7: Disproof of Conjecture

### Conclusion

A Question  
Further Work

On returning to Cork in 2009, Alexei brought to this work  
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