A Multiple Access System for Disjunctive Vector Channel

DMITRY OSIPOV, ALEXEY FROLOV AND VICTOR ZYABLOV
{d_osipov, alexey.frolov, zyablov}@iitp.ru
Inst. for Information Transmission Problems
Russian Academy of Sciences
Moscow, Russia

Abstract. This paper addresses the problem of constructing a multiple access system for a disjunctive vector channel, similar to multiuser channel without intensity information, as described in [1]. To solve this problem a signal-code construction based on the q-ary codes is proposed. It is shown that the proposed signal-code construction allows to obtain the asymptotic value of the total relative rate arbitrarily close to $\ln 2$.

1 Introduction

In [1] a multiuser channel model without intensity information (A-channel) was introduced. In this model it is assumed that the channel consists of a q independent non-overlapping frequency subchannels. In addition, it is assumed that multiple users can simultaneously transmit information over this channel (we denote the number of users by S and as the authors [1] assume that $S \geqslant 2$). At each time a user selects one of the frequency subchannels for transmission. The output of this channel is a list of all the frequencies at which the transfer was carried out. Note that the information about how many users are transmitting at a frequency is not available at the output of the channel (the value is referred as the intensity information). In addition, the channel is assumed to be noiseless.

Note that this channel model allows a broader interpretation. The channel inputs can be viewed as binary vectors of length q, where each element of the vector can be associated with a certain frequency (as is done in [1]) or with a time slot. The value of each element of the vector associated with a certain user, depends on whether the user transmits in this subchannel. In this case, the channel output can be viewed as an elementwise disjunction of the vectors at input. Thus, A-channel is in fact a disjunctive vector channel.

In [1] there were suggested some examples of signal-code constructions for identification. In [2–5] estimates of the capacity of A-channel were derrived. In the paper we introduce a signal-code sequence using a channel described above and study the properties of multiple access system, built on the basis of this construction.

2 A multiple access model description

Assume that all the users use the same alphabet – symbols of $GF(q)$.

Transmission. Each user encodes the information transmitted by q-ary (n, k, d) code C (all users use the same code). Consider the process of sending the message by i-th user. Let us denote the transmitted codeword by c_i, each character c_i is associated with a binary vector of length q and weight 1, the unit is in a position corresponding to the element of $GF(q)$ to be transmitted (we assume that the elements of the vector indexed by elements of the field, and this order fixed and equal for all users). We denote the matrix constructed in this way by bfC_i. Transmission occurs character by character. Before sending a binary vector a random permutation is performed. The permutations used are selected with equal probability and independently.

Reception. The base station sequentially receives messages from all users. Let us consider the process of receiving a message from the i-th user. We assume that the base station synchronized with a transmitter of each user. This means that n columns that correspond to the codeword passed i-th user are known at the receiver. At receiving of each column the reverse permutation is performed. Thus, we obtain the matrix

$$\mathbf{Y}_i = \mathbf{C}_i \vee \left(\bigvee_{m=1:S, m \neq i} \mathbf{X}_m \right),$$

where $\mathbf{C}_i$ is a matrix corresponding to c_i and matrixes $\mathbf{X}_m, m = 1 : S, m \neq i$ are the results of another users activity.

Consider the codeword $c_t \in C$. We need to construct a matrix $\mathbf{C}_t$ corresponding to c_t in the manner described above. Since the system uses the disjunctive multiple access channel all the elements of the channel output corresponding to the codeword of the i-th will be nonzero. Therefore, the assumption that the codeword $c_t \in C$ was transmitted by i-th user is true only if the condition follows

$$\mathbf{C}_t \wedge \mathbf{Y}_i = \mathbf{C}_t. \tag{1}$$

We need to check the condition (1) for all the codewords of C and create a list of possible codewords. In case of only one word in the list we output the word, else we output denial of decoding.

Remark 1. *Note that we do not assume a block synchronization.*

Remark 2. *In a real system it is advisable to use pseudorandom number generators to create pseudorandom permutations (the generators are a part of any system with frequency hopping [7]).*

3 An upper bound for the probability of denial

Let us estimate the probability of denial (p_*) for i-th user.

Theorem 1.

$$\begin{aligned} p_* \quad &\leqslant \quad \sum_{W=d}^{n} \left[A\left(W\right) \left(1 - \left(1 - \frac{1}{q}\right)^{S-1}\right)^{W} \right] < \\ &< \quad q^k \left(1 - \left(1 - \frac{1}{q}\right)^{S-1}\right)^{d}, \end{aligned} \tag{2}$$

where $A(W)$ is a number of codewords of weight W in a code C.

Corollary 1. *Let q, k, S and p_r be fixed, than if the condition*

$$d \geqslant \frac{k - \log_q p_r}{\beta}, \tag{3}$$

follows, where $\beta = -\log_q \left(1 - \left(1 - \frac{1}{q}\right)^{S-1}\right)$, than $p_* < p_r$

4 The choice of the code length

Statement 1. *If the condition*

$$n \geqslant \frac{\log_2 q}{\log_2 q - 1} \left(k + d_* \log_q \left(q - 1\right)\right),$$

follows than there is a code C such that $d(C) \geqslant d_*$.

Proof. We need to use the Varshamov–Gilbert bound

$$n \geqslant k + \log_q \left[\sum_{i=0}^{d_*-2} \binom{n-1}{i} (q-1)^i \right].$$

Loosen this inequality, we obtain

$$n \geqslant \frac{\log_2 q}{\log_2 q - 1} \left(k + d_* \log_q \left(q - 1\right)\right).$$

□

Let us choose the length in the following way

$$n\left(q, S, k, p_r\right) = \left\lceil \frac{\log_2 q}{\log_2 q - 1} \left(k + \left\lceil \frac{k - \log_q p_r}{\beta} \right\rceil \log_q \left(q - 1\right)\right) \right\rceil. \tag{4}$$

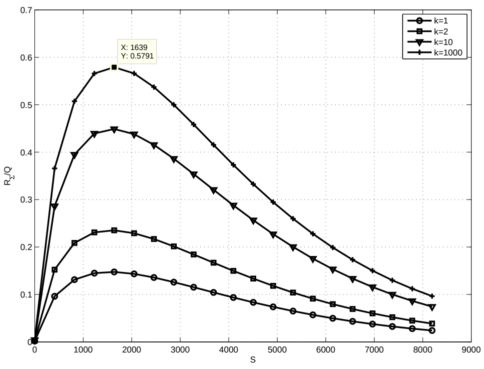

Figure 1: $q = 2^{11}$, $p_r = 10^{-10}$

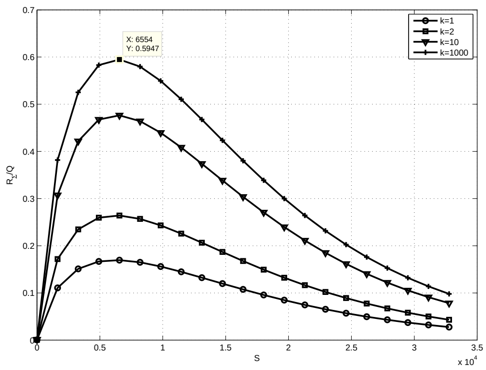

Figure 2: $q = 2^{13}$, $p_r = 10^{-10}$

5 An asymptotic estimate of the relative group rate

The transmission rate for one user can be calculated as follows

$$R_i(q, S, k, p_r) = \frac{k}{n\left(q, S, k, p_r\right)} \log_2 q$$

Group rate can be calculated as follows

$$R_\Sigma(q, S, k, p_r) = \sum_{i=1}^{S} R_i(q, S, k, p_r) = S \frac{k}{n\left(q, S, k, p_r\right)} \log_2 q$$

In Fig. 1 and 2 the dependencies of R_Σ/q on S, plotted with $q = 2^{11}$ and $q = 2^{13}$ respectively. At each figure we see a family of dependencies with different k, $p_r = 10^{-10}$.

Let $S = \gamma q$, $p_r = 2^{-cn}, c > 0$, if $c < -\log_2\left(1 - e^{-\gamma}\right)$ and $q \to \infty$ we obtain

$$n \sim \frac{k \log_2 q}{-\log_2\left(1 - e^{-\gamma}\right) - c}. \tag{5}$$

Let us introduce a notion

$$\rho(\gamma, k, c) = \lim_{q \to \infty} \frac{R_\Sigma\left(q, \gamma q, k, 2^{-cn}\right)}{q}, \tag{6}$$

Theorem 2. *If $\gamma < -\ln\left(1 - 2^{-c}\right)$ than the following inequality follows*

$$\rho\left(\gamma, k, c\right) \geqslant \gamma \left(\log_2\left(\frac{1}{1 - e^{-\gamma}}\right) - c\right). \tag{7}$$

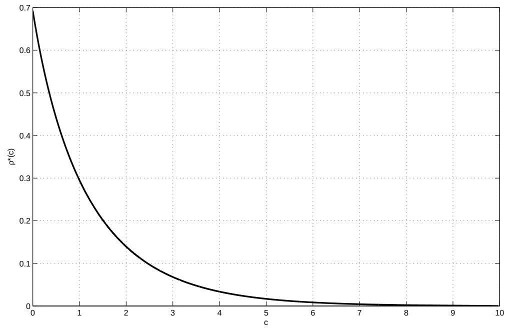

Figure 3: $\rho^*(c)$

Remark 3. *Let us denote a lower bound on $\rho(\gamma, k, c)$ obtained by $\underline{\rho}(\gamma, c)$. Note that it does not depend on k.*

Let us introduce a notion

$$\rho^*(c) = \max_{\gamma} \left[\underline{\rho}(\gamma, c)\right].$$

In Fig. 3 the dependency of $\rho^*(c)$ on c is shown.

Remark 4. *Note that*

$$\rho^*(\varepsilon) \geqslant \ln 2(1-\varepsilon),$$

we just need to substitute $\gamma = \ln 2$ to (7).

Remark 5. *In [2] it is shown that in the case of uncoordinated transfer and in case of a uniform probability distribution at the input $\rho(\gamma, k, c) \leqslant -\gamma \log_2 (1 - e^{-\gamma})$. The signal-code sequence introduce allows us to provide $\rho(\gamma, k, c)$ very close to the upper bound when $c = \varepsilon$.*

6 Conclusion

Hereinafter a novel multiple access system for for disjunctive vector channel has been introduced. The analysis of the obtained results has shown that the system under consideration has a great potential (it enables to adapt a very great number of users enabling at the same time to maintain relatively high rates and a very low probability of erasure) and flexibility (the length of the code in use can be changed in order to either keep the probability of erasure at the desired level (if the system load increases) or increase the transmission rate (if the system load decreases)). Therefore the proposed system model can be considered as a promising candidate for future investigation and possible basis for the development of a whole class of new standards for real-life applications.

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