TRANSIENTS IN QUASI–CONTROLLABLE SYSTEMS. OVERSHOOTING, STABILITY AND INSTABILITY

V.S. KOZYAKIN, N.A. KUZNETSOV and A.V. POKROVSKII*

Institute of Information Transmission Problems, Russian Academy of Sciences, 19 Bolshoi Karetny lane, Moscow 101447, Russia, e-mail kozyakin@iitp.ru

Abstract. Families of regimes for control systems are studied possessing the so called quasi–controlłability property that is similar to the Kalman controlłability property. A new approach is proposed to estimate the degree of transients overshooting in quasi–controllable systems. This approach is conceptually related with the principle of bounded regimes absence in the absolute stability problem. Its essence is in obtaining of constructive a priori bounds for degree of overshooting in terms of the so called quasi–controlłability measure. It is shown that relations between stability, asymptotic stability and instability for quasi–controllable systems are similar to those for systems described by linear differential or difference equations in the case when the leading eigenvalue of the corresponding matrix is simple. The results are applicable for analysis of transients, classical absolute stability problem, stability problem for desynchronized systems and so on.

Key Words. Controllability; convergence; mathematical system theory; stability; robustness

1 INTRODUCTION

Stability is recognized as the most important intrinsic attribute of a real control system. Traditionally, when designing a system one tries to make it as stable as possible. That is why sometimes one neglect the fact that a large amount of stability of a system is not a guarantee of its "good" behavior. The reason is that the stability property characterizes only the asymptotic behavior of a system and doesn't take into account system behavior during the so called "transient interval". As a result, a stable system can have large overshooting or "peaks" in the transient process that can result in complete failure of a system. As was noted in Izmaylov (1989); Mita and Yoshida (1980); Olbrot and Cieslik (1988) when the regulator in feedback links is chosen to guarantee as large a degree of stability as possible then, simultaneously, overshooting of system state during the transient process grows, i.e., the peaking effect is heightened.

Currently, there are a growing number of cases in which systems are described as operating permanently as if in the transient regime. Examples are flexible manufacturing systems, adaptive control systems with high level of external noises, so called desynchronized or asynchronous discrete event systems Asarin *et al.* (1990); Bertsekas and Tsitsiklis (1988); Kleptsyn *et al.* (1984); Kozyakin (1991) and many others. In connection with this, it is necessary to develop effective and simple criteria to estimate the state vector amplitude within the whole time interval of the system's functioning including the interval of transient regime and an infinite interval when the state vector is "close to equilibrium".

In this paper a new approach is developed presenting the means to solve effectively the problem of estimation the state vector amplitude within the whole time interval. The key concept used is a quasi–controłlability property of a system that is similar, but weaker than the Kalman controłlability property. The degree of quasi–controłlability can be characterized by a numeric value. The main result of the paper is in proving the following: if a quasi–controllable system is stable then the amplitudes of all its state trajectories starting from the unit ball are bounded by the value reciprocal of the quasi–controłlability measure. Due to the fact that the measure of quasi–controłlability can be easily computed, this fact becomes an efficient tool for analysis of transients. Is shown also that for quasi–controllable systems the properties of stability or instability are robust with respect to small perturbation of system's parameters.

2 QUASI-CONTROLLABLE $\mathcal{O}$-FAMILIES

The results of this Section generalize part of results obtained by Pyatnitskii and Rappoport in Pyatnitskii and Rappoport (1991). Let $\mathcal{T}$ be either the real half-axes $t \geq 0$ or the set of all nonnegative integers. The collection $\mathcal{X}$ is considered, elements of which are the functions $x(t)$ $(t \in \mathcal{T})$ taking values

*Current address: Mathematical Department, University of Queensland, Qld 4072 Australia; Pokrovskii has been supported by the Australian Research Council Grant A 89132609.

in $\mathbb{R}^N$. In applications functions from $\mathcal{X}$ often play the role of output signals for some family of dynamic control systems.

Let us introduce the following notation

$$\mathcal{X}(u,t,s) = \{y = x(t) : \ x(\cdot) \in \mathcal{X}, \ x(s) = u\},$$

where $u \in \mathbb{R}^N$, $s,t \in \mathcal{T}$, $s \le t$; a norm in $\mathbb{R}^N$ will be denoted by $\|\cdot\|$.

Suppose that the family $\mathcal{X}$ possesses the following properties naturally arising in applications:

- it satisfies the *sewing condition*, i.e. $\forall\{t_1,t_2,t_3 \in \mathcal{T} : t_1 \le t_2 \le t_3\}$, $\forall\{x,y \in \mathcal{X} : x(t_2) = y(t_2)\}$ $\{\exists z \in \mathcal{X} : z(t) = x(t)$ for $t_1 \le t \le t_2, \ z(t) = y(t)$ for $t_2 \le t \le t_3\}$;
- it is *homogeneous*, i.e., $\mathcal{X}(\lambda u,t,0) = \lambda\mathcal{X}(u,t,0)$ for $u \in \mathbb{R}^N$, $t \in \mathcal{T}$, $\lambda \in \mathbb{R}^1$;
- it is *time invariant*, i.e., $\mathcal{X}(u,t,s) = \mathcal{X}(u,t-s,0)$ for $u \in \mathbb{R}^N$, $s,t \in \mathcal{T}$, $s \le t$;
- it is *concave*, i.e., $\mathcal{X}(\lambda u + (1-\lambda)v,t,0) \subseteq \lambda\mathcal{X}(u,t,0) + (1-\lambda)\mathcal{X}(v,t,0)$ for $u,v \in \mathbb{R}^N$, $t \in \mathcal{T}$, $\lambda \in [0,1]$;
- it is *locally bounded*, i.e., there exists such a continuous function $\rho(t)$ that the inequality $\|x\| \le \rho(t)$ is true for $x \in \mathcal{X}(u,t,0)$, $\|u\| = 1$, $t \in \mathcal{T}$;
- it is *continuous*, i.e., from $x_n \in \mathbb{R}^N$, $x_n \to x_0$ for any $t \in \mathcal{T}$ follows the equality $\lim_{n\to\infty} H\{\mathcal{X}(x_n,t,0), \mathcal{X}(x_0,t,0)\} = 0$, where H is the Hausdorff distance between sets in $\mathbb{R}^N$.

Definition 2.1 *A homogeneous, time invariant, concave, locally bounded and continuous collection of functions satisfying the sewing condition will be called an $\mathcal{O}$-family (output family).*

Example 2.2 *Consider the collection $\mathcal{X}$ of all solutions of the class of differential equations of the form*

$$\frac{dx}{dt} = Ax + bu, \qquad |u| \le \gamma|\langle c,x\rangle|, \tag{1}$$

where $b,c \in \mathbb{R}^N, c \ne 0$, A is an $N{\times}N$ matrix and γ is a positive constant. The collection $\mathcal{X}$ is an $\mathcal{O}$-family.

Denote by $\mathrm{co}(W)$ and $\mathrm{span}(W)$ respectively the convex hull and the linear span of a vector set $W \subseteq \mathbb{R}^N$; $\mathrm{absco}(W) = \mathrm{co}(W \bigcup -W)$ denotes the absolute convex hull of W.

Example 2.3 *Let $\mathcal{A}$ be a bounded collection of linear operators acting in $\mathbb{R}^N$. Let us set $\mathrm{co}\mathcal{A}(x) = \mathrm{co}\{\bigcup Ax : \ A \in \mathcal{A}\}$, $x \in \mathbb{R}^N$. Consider the collection $\mathcal{X}$ of all solutions of the differential inclusion $dx/dt \in \mathrm{co}\mathcal{A}(x)$. The collection $\mathcal{X}$ is an $\mathcal{O}$-family.*

The last Example covers variety of cases, including Example 2.2, occurring in the absolute stability theory (see, e.g., Pyatnitskii and Rappoport (1991)).

Example 2.4 *Let the collection $\mathcal{A}$ be the same as in the previous example. Denote by $\mathcal{X}$ the collection of piecewise smooth continuous functions $x(t)$ $(t \ge 0)$ for each of which there exists a numeric sequence t_n $(t_0 = 0, \ t_n < t_{n+1}, \ n = 0,1,2,\ldots)$ such that the function $x(t)$ satisfies on each interval $[t_n,t_{n+1}]$ one of the differential equations of the form $dx/dt = Ax$ $\ (A \in \mathcal{A})$. The collection $\mathcal{X}$ is an $\mathcal{O}$-family.*

Example 2.5 *Let $\mathcal{T}$ be the set of nonnegative integers and let $\mathcal{A}$ be a bounded collection of $N{\times}N$ matrixes. Let us set $\mathcal{A}(x) = \bigcup\{Ax : \ A \in \mathcal{A}\}$ for each $x \in \mathbb{R}^N$. Consider the collection $\mathcal{X} = \mathcal{X}(\mathcal{A})$ of all solutions of difference inclusion $x(t+1) \in \mathcal{A}(x(t))$. The collection $\mathcal{X}$ is an $\mathcal{O}$-family.*

This example contains as particular cases a variety of families arising in the theory of desynchronized and vertex systems Asarin *et al.* (1990); Bertsekas and Tsitsiklis (1988); Kleptsyn *et al.* (1984); Kozyakin (1991), two of which will be discussed in more details in Examples 3.3 and 3.4.

2.1 Quasi–controllability

A subspace $E \subseteq \mathbb{R}^N$ is said to be *invariant* with respect to the family $\mathcal{X}$ if for any $x \in E$ the inclusion $\mathcal{X}(x,t,0) \subseteq E$ is valid.

Definition 2.6 *An $\mathcal{O}$-family $\mathcal{X}$ will be called quasi–controllable one if there is no nonzero proper subspace $E \subset \mathbb{R}^N$ invariant with respect to $\mathcal{X}$.*

Denote by $\mathcal{S}$ the set of all $s \in \mathcal{T}$ such that the amount of elements of the set $\{t \in \mathcal{T} : \ 0 \le t \le s\}$ is not less than N. Then one can state that the family $\mathcal{X}$ is quasi–controllable if and only if for any nonzero $x \in \mathbb{R}^N$ and for any $s \in \mathcal{S}$

$$\mathrm{span}\{\mathcal{X}(x,t,0) : \ t \in \mathcal{T}, \ 0 \le t \le s\} = \mathbb{R}^N. \tag{2}$$

In the important for the control theory cases of Examples 2.2-2.5, the quasi–controllability property occurs rather often and it can be verified efficiently. So, in Example 2.2 for quasi–controllability of $\mathcal{X}$ it is necessary and sufficient that the pair $\{A,b\}$ be completely controllable and that the pair $\{A,c\}$ be completely observable by Kalman. In Examples 2.3-2.5 for quasi–controllability of $\mathcal{X}$ it is necessary and sufficient that the matrixes from the collection $\mathcal{A}$ would not have a common invariant subspace.

2.2 Quasi–controllability measure

In order to verify the quasi–controllability of a family $\mathcal{X}$ it is preferable to use in some situations the criteria (2) rather than Definition 2.6. However, in a variety of situations the most suitable tool for verification of quasi–controllability of a family $\mathcal{X}$ is the numerical measure of quasi–controllability proposed below. Denote the ball of the radius t in a norm $\|\cdot\|$ by $\mathbf{S}(t)$.

Definition 2.7 *Let $s \in \mathcal{T}$. The number $\mathrm{qcm}_s(\mathcal{X}) = \inf_{x\in\mathbb{R}^N,\|x\|=1} \sup\{\rho : \mathbf{S}(\rho) \subseteq \mathrm{absco}(\mathcal{X}(x,t,0) : t \in \mathcal{T}, \ 0 \le t \le s)\}$ will be called the s-measure of quasi–controllability of the family $\mathcal{X}$. The number*

$$\mathrm{qcm}(\mathcal{X}) = \sup_{s\in\mathcal{S}} \mathrm{qcm}_s(\mathcal{X})$$

will be called the quasi–controllability measure of the family $\mathcal{X}$.

Lemma 2.8 *If the family $\mathcal{X}$ is quasi–controllable, then $\mathrm{qcm}_s(\mathcal{X}) > 0$ for any $s \in \mathcal{S}$. If $\mathrm{qcm}_s(\mathcal{X}) > 0$ for some $s \in \mathcal{S}$, then the family $\mathcal{X}$ is quasi–controllable.*

In some situations of interest it is not very difficult to find lower bounds for quasi–controllability measure (see Examples 3.3, 3.4 below and Kozyakin and Pokrovskii (1992)).

3 OVERSHOOTING

Below an efficient and conceptually simple way for estimation of norms of functions from a given quasi–controllable $\mathcal{O}$-family is proposed. The corresponding estimates are valid within the whole time interval $t \in \mathcal{T}$, including the initial time interval of transient regime and the following infinite time interval when the state vector of a system is close to the equilibrium. The principal result may be described as follows: if quasi–controllable $\mathcal{O}$-family is stable then the amplitudes of all its state trajectories starting from the unit ball are bounded by the reciprocal of the quasi–controllability measure. Due to the fact that the measure of quasi–controllability can be easily computed in a variety of cases, this criterion is an efficient tool for analysis of transients in control systems.

3.1 Theorem on an a priori bound

Definition 3.1 *An $\mathcal{O}$-family $\mathcal{X}$ is said to be Lyapunov stable at the point x if for some $\mu > 0$*

$$\sup\{\|u\| : \ u \in \bigcup(\mathcal{X}(x,t,0), \ t \in \mathcal{T})\} \le \mu.$$

An $\mathcal{O}$-family $\mathcal{X}$ is said to be by Lyapunov stable if for some $\mu > 0$

$$\sup\{\|u\| : \ u \in \bigcup(\mathcal{X}(x,t,0), \ t \in \mathcal{T})\} \le \mu\|x\|. \quad (3)$$

An $\mathcal{O}$-family $\mathcal{X}$ is said to be exponentially Lyapunov stable if for some $\epsilon, \mu > 0$ it is true the inequality

$$\sup\{\|u\| : \ u \in \bigcup \mathcal{X}(x,t,0)\} \le \mu e^{-\epsilon t}\|x\|.$$

The greatest lower bound of those μ for which the inequality (3) holds is called the *overshooting measure* of the family $\mathcal{X}$ and is denoted by $\mathrm{ovm}(\mathcal{X})$. Note that the overshooting measure formally characterizes "the value of peaking effect".

Theorem 3.2 *If the quasi–controllable $\mathcal{O}$-family $\mathcal{X}$ is stable then* $\mathrm{ovm}(\mathcal{X}) \le (\mathrm{qcm}(\mathcal{X}))^{-1}$.

One application of Theorem 3.2 to the estimation of peaking effect of classical control systems (1) immediately follows from Kozyakin and Pokrovskii (1992). Another application is discussed below.

3.2 Desynchronized systems

Let $\mathcal{X} = \mathcal{X}(\mathcal{A})$ be class of $\mathcal{O}$-families as in Example 2.5, induced by a bounded collection of matrices $\mathcal{A}$. In desynchronized systems theory (see Asarin *et al.* (1990); Kleptsyn *et al.* (1984); Kozyakin (1991)) the collection of matrixes $\mathcal{A}$ is often defined as follows. Given a scalar $N{\times}N$ matrix $A = (a_{ij})$, then denote $\mathcal{A} = \{A_1, A_2, \ldots, A_N\}$ where each A_i is obtaned from the identity matric by replasing i-th string by the corresponding string of the matrix A. The matrix A is said to be *irreducible*, if by any renumeration of the basis elements in $\mathbb{R}^N$ it cannot be represented in a block triangle form. Let the norm $\|\cdot\|$ in $\mathbb{R}^N$ be $\|x\| = |x_1| + |x_2| + \ldots + |x_N|$. Define the values $\alpha = \frac{1}{2N}\min\{\|(A-I)x\| : \ \|x\| = 1\}$, $\beta = \frac{1}{2}\min\{|a_{ij}| : i \ne j, a_{ij} \ne 0\}$.

Example 3.3 *The $\mathcal{O}$-family $\mathcal{X}(\mathcal{A})$ is quasi–controllable if and only if the number 1 is not an eigenvalue of the matrix A and A is irreducible. In this case* $\mathrm{qcm}_N[\mathcal{X}(\mathcal{A})] \ge \alpha\beta^{N-1}$.

3.3 Vertex systems

Let $\mathcal{X} = \mathcal{X}(\mathcal{V})$ be the $\mathcal{O}$-family from Example 2.5 induced by a bounded collection of matrixes $\mathcal{V} = \{D_1A, D_2A, \ldots, D_NA\}$. Here A is an $N{\times}N$ scalar matrix with entries a_{ij} and $D_i = \mathrm{diag}\{d_{1i}, \ldots, d_{ii}, \ldots, d_{Ni}\}$, $i = 1, 2, \ldots, N$, where $d_{ij} = 1$ if $i \ne j$ and $d_{ij} = -1$ if $i = j$. The $\mathcal{O}$-family $\mathcal{X}(\mathcal{V})$ is called *a vertex family.*

Let again the norm $\|\cdot\|$ in $\mathbb{R}^N$ be defined by the equality $\|x\| = |x_1| + |x_2| + \ldots + |x_N|$. Define the values $\tilde{\alpha} = \frac{1}{N}\min\{\|Ax\| : \ \|x\| = 1\}$, $\tilde{\beta} = \min\{|a_{ij}| : i \ne j, a_{ij} \ne 0\}$.

Example 3.4 *The $\mathcal{O}$-family $\mathcal{X}(\mathcal{V})$ is quasi–controllable if and only if the number 0 is not an eigenvalue of the matrix A and matrix A is irreducible. In this case* $\mathrm{qcm}_N[\mathcal{X}(\mathcal{V})] \ge \tilde{\alpha}\tilde{\beta}^{N-1}$.

4 ROBUSTNESS

Below it is shown that the quasi–controllability property, as well as properties of stability or instability for quasi–controllable $\mathcal{O}$-families, are robust with respect to small perturbations. Due to these properties quasi–controllable $\mathcal{O}$-families are very attractive object of study for control theory.

Remind that the symbol H denotes the Hausdorff distance between closed sets from $\mathbb{R}^N$. Say that $\mathcal{O}$-families $\mathcal{X}_n$ converge to the $\mathcal{O}$-family $\mathcal{X}_*$ if for all $x \in \mathbb{R}^N$, $s \in \mathcal{T}$ the following relation is valid

$$\lim_{n\to\infty}\sup_{t\le s} H\{\mathrm{cl}(\mathcal{X}_n(x,t,0)), \mathrm{cl}(\mathcal{X}_*(x,t,0))\} = 0. \quad (4)$$

Let us present a simple auxiliary property of the s-measure of quasi–controllability. For any two $\mathcal{O}$-families $\mathcal{X}$ and $\mathcal{X}_*$ let us define the value

$$\rho_s(\mathcal{X},\mathcal{X}_*) = \sup_{t\le s, \|x\|\le 1} H\{\mathrm{cl}(\mathcal{X}(x,t,0)), \ \mathrm{cl}(\mathcal{X}_*(x,t,0))\},$$

where $s \in \mathcal{S}$. Then $|\mathrm{qcm}_s(\mathcal{X}_*) - \mathrm{qcm}_s(\mathcal{X})| \le \rho_s(\mathcal{X},\mathcal{X}_*)$. This implies the following result:

Theorem 4.1 *Let the sequence of $\mathcal{O}$-families $\mathcal{X}_n$ converge to the quasi–controllable $\mathcal{O}$-family $\mathcal{X}$. Then $\mathcal{X}_n$ is quasi–controllable for sufficiently large n and* $\lim_{n\to\infty}\mathrm{qcm}_s(\mathcal{X}_n) = \mathrm{qcm}_s(\mathcal{X})$ *is valid for each $s \in \mathcal{S}$.*

4.1 Robustness of stability

The next result follows from Theorems 3.2 and 4.1.

Theorem 4.2 *Let the sequence of $\mathcal{O}$-families $\mathcal{X}_n$ converge to the quasi–controllable Lyapunov stable $\mathcal{O}$-family $\mathcal{X}$. Then* $\lim_{n\to\infty}\mathrm{ovm}(\mathcal{X}_n) \le (\mathrm{qcm}(\mathcal{X}))^{-1}$, *and, for sufficiently large n, $\mathcal{X}_n$ is Lyapunov stable.*

Theorem 4.3 *Let the sequence of Lyapunov stable $\mathcal{O}$-families $\mathcal{X}_n$ converges to a quasi–controllable $\mathcal{O}$-family $\mathcal{X}$. Then the family $\mathcal{X}$ is also Lyapunov stable.*

From Theorem 4.2 it follows that any $\mathcal{O}$-family sufficiently close to a stable quasi–controllable $\mathcal{O}$-family (in the sense (4)) is itself stable and quasi–controllable. At the same time a stable but not quasi–controllable $\mathcal{O}$-family can sometimes be approximated by stable $\mathcal{O}$-families with unbounded quasi–controllability measures. As an example consider the sequence of $\mathcal{O}$-families $\mathcal{X}_n$ each of which is the set of solutions of linear difference vector equations $x(k+1) = A_n x(k)$, where A_n are two-dimensional matrixes of the form

$$A_n = \begin{pmatrix} 1-\epsilon_n & \delta_n \\ 0 & 1-\epsilon_n \end{pmatrix},$$

where δ_n tends to zero significantly slower than ϵ_n. The sequence of $\mathcal{O}$-families $\mathcal{X}_n$ converges to the $\mathcal{O}$-family $\mathcal{X}$ consisting of solutions of linear vector equation $x(k+1) = x(k)$, which is stable but not quasi–controllable.

4.2 Robustness of instability

An $\mathcal{O}$-family $\mathcal{X}$ will be called *absolutely exponentially unstable with the exponent* $\epsilon > 0$, if there exists a constant $\gamma > 0$ such that for each $u \in \mathbb{R}^N$, $\|u\| = 1$, there corresponds a function $x(t,u) \in \mathcal{X}$ satisfying $\|x(t,u)\| \geq \gamma e^{\epsilon t} \quad (t \in \mathcal{T})$.

The property of absolute exponential instability is a strong one but unfortunately it is rather difficult to verify. In order for the quasi–controllable $\mathcal{O}$-family $\mathcal{X}$ to be exponentially unstable, it is necessary and sufficient that for some $\epsilon > 0$ the inequality

$$\liminf_{t\to\infty} \inf_{\|u\|=1} e^{-\epsilon t} \sup\{\|x\| : \; x \in \mathcal{X}(u,t,0)\} > 0$$

be fulfilled.

Theorem 4.4 *Let a quasi–controllable $\mathcal{O}$-family $\mathcal{X}$ be not Lyapunov stable at least at one point x (see Def.* 3.1*). Then the family $\mathcal{X}$ as well as any $\mathcal{O}$-family close to $\mathcal{X}$ in the sense of convergence* (4) *is absolutely exponentially unstable.*

4.3 Structure of quasi–controllable families

The previous theorems can be supplemented by the following two statements on a structure of quasi–controllable $\mathcal{O}$-families.

Theorem 4.5 *Suppose that each function $x(\cdot)$ from a quasi–controllable $\mathcal{O}$-family $\mathcal{X}$ tends to zero when $t \to \infty$. Then the family $\mathcal{X}$ is exponentially stable.*

It should be stressed that there are no any assumptions concerning the closure of the family $\mathcal{X}$ in the condition of the last Theorem. The next classification Theorem follows from Theorems 4.5 and 4.4.

Theorem 4.6 *Every quasi–controllable $\mathcal{O}$-family is either exponentially stable or is absolutely exponentially unstable, or it is Lyapunov stable and contains functions $x(t)$ that do not tend to zero for $t \to \infty$.*

Some ideas used in proving the above statements are similar to those used in proving of the principle of bounded solutions absence Kozyakin (1990); Krasnoselskii and Pokrovskii (1977).

5 ACKNOWLEDGEMENTS

The authors would like to thank Phil Diamond and Mark Krasnoselskii for reading a first draft of the manuscript and making many useful suggestions.

REFERENCES

Asarin, E.A., M.A. Krasnoselskii, V.S. Kozyakin, and N.A. Kuznetsov (1990). Stability analysis of desynchronized systems. *Preprints of the 11th IFAC World Congress*, Vol. 2, pp. 56-60. Tallinn, Estonia, USSR.

Bertsekas, D.P., J.N. Tsitsiklis (1988). *Parallel and distributed computation. Numerical methods.* Englewood Cliffs, NJ

Bongiorno, J., and D. Youla (1970). Discussion of "On observers in multi-variable control systems". *Intern. J. Control*, **12**, 183-190.

Izmaylov, R.N. (1989). A canonical representation of a state identifier and optimization of transient modes. *Avtomatika i telemekhanika*, **4**, 59-64. (in Russian)

Kleptsyn, A.F., M.A. Krasnoselskii, N.A. Kuznetsov, and V.S. Kozjakin (1984). Desynchronization of linear systems. *Mathematics and Computers in Simulation*, **26**, 423-431.

Kozyakin, V.S. (1990). On absolute stability of systems with non-synchronously operating impulse elements. *Avtomatika i telemekhanika*, **10**, 56-63. (in Russian)

Kozyakin, V.S. (1991). Equivalent norms technique for stability analysis of linear discrete event systems. In: *Proceedings of the 1991 IFAC Workshop on Discrete Event System Theory and Applications in Manufacturing and Social Phenomena, June 25-27, 1991*, pp. 290-295. Shenyang, China, International Academic Publishers, A Pergamon - CNPIEC Join Venture.

Kozyakin, V.S., and A.V. Pokrovskii (1992). Role of controllability-type properties in the stability investigation of desynchronized dynamic systems, *Doklady AN SSSR*, **324**, 60-64 (in Russian).

Krasnoselskii, M.A., and A.V. Pokrovskii (1977). Principle of bounded solutions absence in the absolute stability problem. *Doklady AN SSSR*, **233**, 293-296 (in Russian).

Mita, T., and H. Yoshida (1980). Eigenvector assignability and responses of the closed loop systems. *Transact. Soc. Instr. and Contr. Eng.*, **16**, 477-483.

Olbrot, A., and J. Cieslik (1988). A qualitative bound in robustness of stabilization by state feedback. *IEEE Transact. Automat. Control*, **AC-33**, 1165-1166.

Pyatnitskii, E.S., and L.S. Rappoport (1991). Periodic motions and criterion of absolute stability of nonlinear nonstationary systems *Avtomatika i telemekhanika*, **10**, 31-37 (in Russian)