Tables of parameters of symmetric configurations v_k*

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Abstract

Tables of the currently known parameters of symmetric configurations are given. Formulas for parameters of the known infinite families of symmetric configurations are presented as well. The results of the recent paper [13] are used. This work can be viewed as an appendix to [13], in the sense that the tables given here cover a much larger set of parameters.

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1 Introduction

Configurations as combinatorial structures were defined in 1876. For an introduction to the problems connected with configurations, see [30–32] and the references therein.

Definition 1.1. [31]

(i) A configuration (v_r, b_k) is an incidence structure of v points and b lines such that each line contains k points, each point lies on r lines, and two distinct points are connected by *at most* one line.

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(ii) If $v = b$ and, hence, $r = k$, the configuration is *symmetric*, and it is referred to as a configuration v_k.

(iii) The *deficiency* d of a configuration (v_r, b_k) is the value $d = v - r(k-1) - 1$.

A symmetric configuration v_k is *cyclic* if there exists a permutation of the set of its points mappings blocks to blocks, and acting regularly on both points and blocks. Equivalently, v_k is cyclic if one of its incidence matrix is circulant.

Steiner systems are configurations with $d = 0$ [31]. The *deficiency* of a symmetric configuration v_k is $d = v - (k^2 - k + 1)$. The deficiency of v_k is zero if and only if v_k is a finite projective plane of order $k - 1$.

A configuration (v_r, b_k) can be viewed as a k-uniform r-regular linear hypergraph with v vertices and b hyperedges [29, 31]. Connections of configurations (v_r, b_k) with numerical semigroups are noted in [9, 55]. Some analogies between configurations (v_r, b_k), regular graphs, and molecule models of chemical elements are remarked in [27]. As an example of a practical application of configurations (both symmetric and non-symmetric), we mention also the problem of user privacy for using database; see [17, 54] and the references therein.

Denote by $\mathbf{M}(v, k)$ an incidence matrix of a symmetric configuration v_k. Any matrix $\mathbf{M}(v, k)$ is a $v \times v$ 01-matrix with k ones in every row and column; moreover, the 2×2 matrix consisting of all ones is not a submatrix of $\mathbf{M}(v, k)$. Two incidence matrices of the same configuration may differ by a permutation on the rows and the columns.

A matrix $\mathbf{M}(v, k)$ can be considered as a biadjacency matrix of the *Levi graph* of the configuration v_k which is a k-regular bipartite graph without multiple edges [31, Sec. 7.2]. Clearly, the graph has girth at least six, i.e. it does not contain 4-cycles. Such graphs are useful for the construction of bipartite-graph codes that can be treated as *low-density parity-check* (LDPC) *codes*. If $\mathbf{M}(v, k)$ is *circulant*, then the corresponding LDPC code is *quasi-cyclic*; it can be encoded with the help of shift-registers with relatively small complexity; see [3, 4, 14, 16, 23, 35, 36] and the references therein.

Matrices $\mathbf{M}(v, k)$ consisting of square circulant submatrices have a number of useful properties, e.g. they are more suitable for LDPC codes implementation. We say that a 01-matrix $\mathbf{A}$ is *block double-circulant* (BDC for short) if $\mathbf{A}$ consists of square circulant blocks whose weights give rise to a circulant matrix (see Definition 3.1). A configuration v_k with a BDC incidence matrix $\mathbf{M}(v, k)$ is called a *BDC symmetric configuration*. Symmetric and non-symmetric configurations with incidence matrices consisting of square circulant blocks are considered, for example, in [14–16, 35, 36, 47].

Cyclic configurations are considered, for instance, in [14–16, 21, 26, 35, 36, 42, 45]. A standard method to construct cyclic configurations (or, equivalently, circulant matrices $\mathbf{M}(v, k)$) is based on *Golomb rulers* [18, 21, 49–51].

Definition 1.2. [21, 49]

(i) A *Golomb ruler* G_k of *order* k is an ordered set of k integers $(a_1, a_2, \ldots, a_k)$ such that $0 \le a_1 < a_2 < \ldots < a_k$ and all the differences $\{a_i - a_j \mid 1 \le j < i \le k\}$ are distinct. The *length* $L_\mathrm{G}(k)$ of the ruler G_k is equal to $a_k - a_1$. Let $L_{\overline{\mathrm{G}}}(k)$ be the length of the *shortest known* Golomb ruler $\overline{\mathrm{G}}_k$.

(ii) A (v, k) *modular Golomb ruler* is an ordered set of k integers $(a_1, a_2, \ldots, a_k)$ such that $0 \leq a_1 < a_2 < \ldots < a_k$ and all the differences $\{a_i - a_j \,|\, 1 \leq i, j \leq k; \, i \neq j\}$ are distinct and nonzero modulo v.

For any $\delta \geq 0$, Golomb rulers $(a_1, a_2, \ldots, a_k)$ and $(a_1 + \delta, a_2 + \delta, \ldots, a_k + \delta)$ have the same properties. Usually, $a_1 = 0$ is assumed. We say that a 0,1-*vector* $\mathbf{u} = (u_0, u_1, \ldots, u_{v-1})$ *corresponds* to a (modular) Golomb ruler if the increasing sequence of integers $j \in \{0, 1, \ldots, v - 1\}$ such that $u_j = 1$ form a (modular) Golomb ruler.

Recall that *weight* of a *circulant* 0, 1-*matrix* is the number of ones in each its row.

Theorem 1.3. [26, Sec. 4], [43]

(i) *Any Golomb ruler* G_k *of length* $L_\mathrm{G}(k)$ *is a* (v, k) *modular Golomb ruler for all* v *such that* $v \geq 2L_\mathrm{G}(k) + 1$.

(ii) *A circulant* $v \times v$ *0,1-matrix of weight* k *is an incidence matrix* $\mathbf{M}(v, k)$ *of a cyclic symmetric configuration* v_k *if and only if the first row of the matrix corresponds to a* (v, k) *modular Golomb ruler.*

(iii) *For all* v *such that* $v \geq 2L_{\overline{\mathrm{G}}}(k) + 1$, *there exists a cyclic symmetric configuration* v_k.

We call the value $G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1$ the *Golomb bound*. On the other hand, we call $P(k) = k^2 - k + 1$ the *projective plane bound*.

Let $v_\delta(k)$ be the smallest possible value of v for which a (v, k) modular Golomb ruler (or, equivalently, a cyclic symmetric configuration) exists.

In [13], two bounds are considered. The *existence bound* $E(k)$ is the least integer such that for any $v \geq E(k)$, there exists a symmetric configuration v_k. Similarly, the *cyclic existence bound* $E_c(k)$ is the least integer such that for any $v \geq E_c(k)$, there exists a cyclic v_k. Clearly, for a fixed k, we have

$$k^2 - k + 1 \;=\; P(k) \leq E(k) \leq E_c(k) \leq G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1. \tag{1.1}$$

$$k^2 - k + 1 \;=\; P(k) \leq v_\delta(k) \leq E_c(k) \leq G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1. \tag{1.2}$$

The aim of this work is to give tables of the currently known parameters of symmetric configurations v_k, including those arising from the recent work [13]. We consider the *spectrum* of possible parameters of v_k (with special attention to parameters of cyclic symmetric configurations) in the interval

$$k^2 - k + 1 = P(k) \leq v < G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1. \tag{1.3}$$

Also, we pay attention to parameters of circulant and block double-circulant incidence matrices $\mathbf{M}(v, k)$. Some upper bounds on $E(k)$ and $E_c(k)$ are pointed out.

From the stand point of applications, including Coding Theory, it is sometimes useful to have different matrices $\mathbf{M}(v, k)$ for the same v and k. This is why we remark situations when *different constructions* provide configurations with *the same parameters*.

The *Generalized Martinetti Construction* (Construction GM), proposed in [20] and considered later in [3, 4], plays a key role for the investigation of the *spectrum* of possible parameters of symmetric configurations v_k, as it provides intervals of values of v for

a fixed k. To be successfully applied, Construction GM needs a convenient starting incidence matrix. In this work we use both the original starting matrices from [3, 4] and some new ones obtained in [13].

We remark that new cyclic configurations provide new modular Golomb rulers, i.e. new deficient cyclic difference sets.

The work is organized as follows. In Section 2, we briefly summarize some constructions and parameters of configurations v_k. Preliminaries on BDC matrices are given in Section 3. In Section 4, parameters of block double-circulant incidence matrices $\mathbf{M}(v, k)$ are reported, according to some results from [13]. In Section 5, parameters of configurations v_k obtained by the Construction GM from the new starting matrices proposed in [13] are given. In Sections 6 and 7, results on the spectra of parameters of cyclic and non-cyclic configurations are reported. Finally, Section 8 contains the tables of parameters of symmetric configurations which are the main object of the paper. In particular, tables of parameters of BDC configurations v_k based on projective planes and punctured affine planes are given, as well as tables of values v for which a cyclic symmetric configuration v_k exists. Finally, aggregated tables on the existence of symmetric configurations are given. In the tables, the new parameters obtained from [13] are written in bold font.

2 Some known constructions and parameters of configurations v_k with $P(k) \leq v < G(k)$

The aim of this section is to provide a list of pairs (v, k) for which a (cyclic) symmetric configuration v_k is known to exist, see Equations (2.1)–(2.12). Infinite families of configurations v_k given in this section are considered in [1–4, 8, 12, 14–16, 20–22, 24, 26–32, 42, 45]; see also the references therein.

Throughout the work, q is a prime power and p is a prime. Let F_q be Galois field of q elements. Let $F_q^* = F_q \backslash \{0\}$. Let $\mathbf{0}_u$ be the zero $u \times u$ matrix. Denote by $\mathbf{P}_u$ a permutation matrix of order u.

We recall that several pairs $(v, k - \delta)$ can be actually obtained from a given v_k.

Theorem 2.1. [45] *If a (cyclic) configuration v_k exists, then for each δ with $0 \leq \delta < k$ there exists a (cyclic) configuration $v_{k-\delta}$ as well.*

We note that from a cyclic configuration v_k, a cyclic configurations $v_{k-\delta}$ can be obtained by dismissing δ ones in the 1-st row of its incidence matrix. For the general case, Theorem 2.1 is based on the fact that an incidence matrix $\mathbf{M}(v, k)$ can be represented as a sum of k permutations $v \times v$ matrices (in different ways). This fact follows from the results of Steinits (1894) and König (1914), see e.g. [29, Sec. 5.2] and [32, Sec. 2.5].

The value δ appearing in Equations (2.1)–(2.12) is connected with Theorem 2.1. When a reference is given, it usually refers to the case $\delta = 0$.

The families giving rise to pairs (2.1)–(2.3) below are obtained from (v, k) modular

Golomb rulers [18, Ch. 5], [19], [26, Sec. 5], [49, Sec. 19.3]; see Theorem 1.3(ii).

$$\text{cyclic } v_k \quad : \quad v = q^2 + q + 1, \; k = q + 1 - \delta, \; q + 1 > \delta \geq 0; \tag{2.1}$$

$$\text{cyclic } v_k \quad : \quad v = q^2 - 1, \; k = q - \delta, \; q > \delta \geq 0; \tag{2.2}$$

$$\text{cyclic } v_k \quad : \quad v = p^2 - p, \; k = p - 1 - \delta, \; p - 1 > \delta \geq 0. \tag{2.3}$$

The configurations giving rise to (2.1) use the incidence matrix of the cyclic projective plane $PG(2, q)$ [18, Sec. 5.5], [19], [49, Th. 19.15], [53]. The family with parameters (2.2) is obtained from the *cyclic punctured affine plane* $AG(2, q)$ [8], [18, Sec. 5.6], [19, 21], [49, Th. 19.17]; see also [16, Ex. 5] and [22] where the configurations are called *anti-flags*. We recall that the punctured plane $AG(2, q)$ is the affine plane without the origin and the lines through the origin. Finally, the configurations with parameters (2.3) follow from Ruzsa's construction [18, Sec. 5.4], [19, 48], [49, Th. 19.19].

The non-cyclic families with parameters (2.4) and (2.5) are given in [1, Constructions (i),(ii), p. 126] and [24, Constructions 3.2, 3.3, Rem. 3.5]; see also the references therein and [3], [4, Sec. 3], [16, Sec. 7.3], [22, 26, 45].

$$v_k \quad : \quad v = q^2 - qs, \; k = q - s - \delta, \; q > s \geq 0, \; q - s > \delta \geq 0; \tag{2.4}$$

$$v_k \quad : \quad v = q^2 - (q - 1)s - 1, \; k = q - s - \delta, \; q > s \geq 0, \; q - s > \delta \geq 0. \tag{2.5}$$

For q a *square*, in [1, Conjec. 4.4, Rem. 4.5, Ex. 4.6] and [24, Construction 3.7, Th. 3.8], families of non-cyclic configuration v_k with parameters (2.6) are provided; see also [16, Ex. 8]. Taking $c = q - \sqrt{q}$, we obtain (2.7), see also [14, Ex. 2(ii)], [22].

$$v_k \quad : \quad v = c(q + \sqrt{q} + 1), k = \sqrt{q} + c - \delta, \; c = 2, 3, \ldots, q - \sqrt{q}, \; \delta \geq 0; \tag{2.6}$$

$$v_k \quad : \quad v = q^2 - \sqrt{q}, \; k = q - \delta, \; q > \delta \geq 0. \tag{2.7}$$

In [22, Th. 1.1], a non-cyclic family with parameters

$$v_k : v = 2p^2, \; k = p + s - \delta, \; 0 < s \leq q + 1, \; q^2 + q + 1 \leq p, \; p + s > \delta \geq 0 \tag{2.8}$$

is given. In [16, Sec. 6], a construction of non-cyclic configuration based on the cyclic punctured affine plane, is provided with parameters

$$v_k \quad : \quad v = c(q - 1), \; k = c - \delta, \; c = 2, 3, \ldots, b, \; b = q \text{ if } \delta \geq 1, \tag{2.9}$$
$$b \; = \; \left\lceil \frac{q}{2} \right\rceil \text{ if } \delta = 0, \; c > \delta \geq 0.$$

In [14, Sec. 2], [16, Sec. 3], the following geometrical construction which uses point orbits under the action of a collineation group is described.

Construction A. Take any point orbit $\mathcal{P}$ under the action of a collineation group in an affine or projective space of order q. Choose an integer $k \leq q + 1$ such that the set $\mathcal{L}(\mathcal{P}, k)$ of lines meeting $\mathcal{P}$ in precisely k points is not empty. Define the following incidence structure: the points are the points of $\mathcal{P}$, the lines are the lines of $\mathcal{L}(\mathcal{P}, k)$, the incidence is that of the ambient space.

Theorem 2.2. *In Construction A the number of lines of $\mathcal{L}(\mathcal{P}, k)$ through a point of $\mathcal{P}$ is a constant r_k. The incidence structure is a configuration (v_{r_k}, b_k) with $v_{r_k} = |\mathcal{P}|$, $b_k = |\mathcal{L}(\mathcal{P}, k)|$.*

By Definition 1.1, if $r_k = k$ then Construction A produces a symmetric configuration v_k. It is noted in [14, 16] that Construction A works for any 2-$(v, k, 1)$ design D and for any group of automorphism of D. The size of any block in D plays the role of $q + 1$.

Families of non-cyclic configuration v_k obtained by Construction A with the following parameters are given in [16, Exs 2, 3].

$$v_k \quad : \quad v = \frac{q(q-1)}{2}, \quad k = \frac{q+1}{2} - \delta, \quad \frac{q+1}{2} > \delta \geq 0, \quad q \text{ odd.} \tag{2.10}$$

$$v_k \quad : \quad v = \frac{q(q+1)}{2}, \quad k = \frac{q-1}{2} - \delta, \quad \frac{q-1}{2} > \delta \geq 0, \quad q \text{ odd.} \tag{2.11}$$

$$v_k \quad : \quad v = q^2 + q - q\sqrt{q}, \quad k = q - \sqrt{q}, \quad q - \sqrt{q} > \delta \geq 0, \quad q \text{ square.} \tag{2.12}$$

A classical construction by V. Martinetti for configurations v_3, going back to 1887 [44], is described in detail, e.g. in [4,7,10,20,27], [32, Sec. 2.4, Fig. 2.4.1]. In [20] a *Generalized Martinetti Construction* (Construction GM) for configurations v_k, $k \geq 3$, is proposed. Later, in [3,4] Construction GM is considered too. Moreover, in [3,4] it is emphasized that Construction GM is useful for obtaining a wide spectrum of parameters v, k.

Construction GM can be described as follows.

Definition 2.3. Let $\mathbf{M}(v, k)$ be an incidence matrix of a symmetric configuration v_k. In $\mathbf{M}(v, k)$, we consider an aggregate $\mathcal{A}$ of $k - 1$ rows corresponding to pairwise disjoint lines of v_k and $k - 1$ columns corresponding to pairwise non-collinear points of v_k. If a $(k - 1) \times (k - 1)$ submatrix $\mathbf{C}(\mathcal{A})$ formed by the intersection of the rows and columns of $\mathcal{A}$ is a permutation matrix $\mathbf{P}_{k-1}$ then $\mathcal{A}$ is called an *extending aggregate* (or *E-aggregate*). The matrix $\mathbf{M}(v, k)$ *admits an extension* if it contains at least one E-aggregate. The matrix $\mathbf{M}(v, k)$ *admits θ extensions* if it contains θ E-aggregates that do not intersect each other.

Procedure E (*Extension Procedure*). Let $\mathbf{M}(v, k) = [m_{ij}]$ be an incidence matrix of a symmetric configuration $v_k = (\mathcal{P}, \mathcal{L})$. Assume that $\mathbf{M}(v, k)$ admits an extension. A matrix $\mathbf{M}(v + 1, k)$ can be obtained by two steps.

1. To the matrix $\mathbf{M}(v, k)$, add a new row from below and a new column to the right. Denote the new $(v + 1) \times (v + 1)$ matrix by $\mathbf{B} = [b_{ij}]$, and let $b_{v+1, v+1} = 1$ whereas $b_{v+1,1} = \ldots = b_{v+1,v} = 0$, $b_{1,v+1} = \ldots = b_{v,v+1} = 0$.

2. One of E-aggregates of $\mathbf{M}(v, k)$, say $\mathcal{A}$, is chosen. In the matrix $\mathbf{B}$, we "clone" all $k - 1$ ones of the submatrix $\mathbf{C}(\mathcal{A})$ writing their "projections" to the new row and column. Finally, the ones cloned are changed by zeroes. In other words, let the aggregate $\mathcal{A}$ consist of rows with indexes i_u, $u = 1, 2, \ldots, k - 1$, and columns with indexes j_d, $d = 1, 2, \ldots, k - 1$. Then the ones of $\mathbf{C}(\mathcal{A})$ are as follows: $m_{i_u j_{\pi(u)}} = 1$, $u = 1, 2, \ldots, k - 1$, for some permutation π of the indexes $1, \ldots, k - 1$. Then $\mathbf{B}$ arising from Step 1 is changed as follows: $b_{i_u, v+1} = 1$, $b_{v+1, j_d} = 1$, $b_{i_u j_{\pi(u)}} = 0$, $u = 1, 2, \ldots, k - 1$, $d = 1, 2, \ldots, k - 1$.

As the Golomb bound $2L_{\overline{G}}(k) + 1$ is important for studying parameters v, k, we note that for sufficiently large orders k, relatively short Golomb rulers are constructed and are available online, see [18, 50, 51] and the references therein. For $k \leq 150$, the order of magnitude of the lengths $L_{\overline{G}}(k)$ of the shortest known Golomb rulers is ck^2 with $c \in [0.7, 0.9]$, see [18, 21, 26, 31, 49, 50]. Moreover, $L_{\overline{G}}(k) < k^2$ for $k < 65000$, see [18]. Constructions of Golomb rulers for large k can be found in [19]. Remind also that Sidon sets are equivalent to Golomb rulers, see [18, 46] and the references therein.

3 Preliminaries on block double-circulant incidence matrices

All results of this section are taken from [13].

Recall that the weight of a circulant binary is the number of 1's in each its rows.

Definition 3.1. Let $v = td$. A binary $v \times v$ matrix $\mathbf{A}$ is said to be a *block double-circulant matrix* (*BDC matrix* for short) if

$$
\mathbf{A} = \left[\begin{array}{cccc}
\mathbf{C}_{0,0} & \mathbf{C}_{0,1} & \cdots & \mathbf{C}_{0,t-1} \\
\mathbf{C}_{1,0} & \mathbf{C}_{1,1} & \cdots & \mathbf{C}_{1,t-1} \\
\vdots & \vdots & \vdots & \vdots \\
\mathbf{C}_{t-1,0} & \mathbf{C}_{t-1,1} & \cdots & \mathbf{C}_{t-1,t-1}
\end{array} \right],
\tag{3.1}
$$

where $\mathbf{C}_{i,j}$ is a *circulant* $d \times d$ binary matrix for all i, j, and submatrices $\mathbf{C}_{i,j}$ and $\mathbf{C}_{l,m}$ with $j - i \equiv m - l \pmod{t}$ have the same weight. The matrix

$$
\mathbf{W}(\mathbf{A}) = \left[\begin{array}{ccccccc}
w_0 & w_1 & w_2 & w_3 & \cdots & w_{t-2} & w_{t-1} \\
w_{t-1} & w_0 & w_1 & w_2 & \cdots & w_{t-3} & w_{t-2} \\
w_{t-2} & w_{t-1} & w_0 & w_1 & \cdots & w_{t-4} & w_{t-3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
w_1 & w_2 & w_3 & w_4 & \cdots & w_{t-1} & w_0
\end{array} \right]
\tag{3.2}
$$

is a *circulant* $t \times t$ matrix whose entry in position i, j is the *weight* of $\mathbf{C}_{i,j}$. $\mathbf{W}(\mathbf{A})$ is called the *weight matrix* of $\mathbf{A}$. The vector $\overline{\mathbf{W}}(\mathbf{A}) = (w_0, w_1, \dots, w_{t-1})$ is called the *weight vector* of $\mathbf{A}$.

Let $\mathbf{A}$ be as in Definition 3.1. In addition, assume that $\mathbf{A}$ is the incidence matrix of a symmetric configuration v_k with $k = \sum_{i=0}^{t} w_i$. From $\mathbf{A}$ one can obtain BDC incidence $v' \times v'$ matrices $\mathbf{A}'$ of symmetric configurations $v'_{k'}$ by the following way.

(i) For each $h \in \{0, 1, \dots, t - 1\}$, in each row of every submatrix $\mathbf{C}_{i,j}$ with $j - i \equiv h$ (mod t) replace $\delta_h \geq 0$ values of 1 with zeros, in such a way that the obtained submatrix is still circulant. As a result, a BDC incidence matrix of a configuration $v'_{k'}$ with

$$
v' = v, \; k' = k - \sum_{h=0}^{t-1} \delta_h, \; 0 \leq \delta_h \leq w_h, \; w'_h = w_h - \delta_h, \; \overline{\mathbf{W}}(\mathbf{A}') = (w'_0, \dots, w'_{t-1})
\tag{3.3}
$$

is obtained.

(ii) Fix some non-negative integer $j \leq t - 1$. Let m be such that $w_m \leq w_h$ for all $h \neq j$. Cyclically shift all block rows of $\mathbf{A}$ to the left by j block positions. A matrix $\mathbf{A}^*$ with $\overline{\mathbf{W}}(\mathbf{A}^*) = (w_0^* = w_j, \ldots, w_u^* = w_{u+j \pmod t}, \ldots, w_{t-1}^* = w_{j-1})$ is obtained. By applying (i), construct a matrix $\mathbf{A}^{**}$ with $w_0^{**} = w_0^* = w_j$, $w_h^{**} = w_m$, $h \geq 1$. Now remove from $\mathbf{A}^{**}$ $t - c$ block rows and columns from the bottom and the right. In this way an incidence $cd \times cd$ BDC matrix $\mathbf{A}'$ of a configuration $v'_{k'}$ is obtained with

$$v' = cd, \ k' = w_j + (c-1)w_m, \ c = 1, 2, \ldots, t, \ \overline{\mathbf{W}}(\mathbf{A}') = (w_j, w_m, \ldots, w_m). \qquad (3.4)$$

(iii) Let t be even. Let $\mathbf{A}^*$ be as in (ii). Let $w_{\mathrm{O}}, w_{\mathrm{E}}$ be weights such that $w_{\mathrm{O}} \leq w_h^*$ for odd h and $w_{\mathrm{E}} \leq w_h^*$ for even h. By applying (i), construct a matrix $\mathbf{A}^{**}$ with $w_0^{**} = w_0^* = w_j$, $w_h^{**} = w_{\mathrm{O}}$ for odd h, $w_h^{**} = w_{\mathrm{E}}$ for even $h \geq 2$. Let $f = 1, 2, \ldots, t/2$. From $\mathbf{A}^{**}$ remove $t - 2f$ block rows and columns from the bottom and the right. An incidence $2fd \times 2fd$ BDC matrix $\mathbf{A}'$ of a configuration $v'_{k'}$ is obtained with

$$v' = 2fd, k' = w_j + w_{\mathrm{O}} + (f-1)(w_{\mathrm{E}} + w_{\mathrm{O}}), \overline{\mathbf{W}}(\mathbf{A}') = (w_j, w_{\mathrm{O}}, \underbrace{w_{\mathrm{E}}, w_{\mathrm{O}}, \ldots, w_{\mathrm{E}}, w_{\mathrm{O}}}_{f-1 \text{ pairs}}). \quad (3.5)$$

Other methods for obtaining families of symmetric configurations from $\mathbf{A}$ can be found in [16, Sec. 4].

4 Constructions and parameters of block double-circulant incidence matrices from [13]

All the results of this section are taken from [13], apart from Tables 3.1 and 3.2 which essentially present more examples than the corresponding [13, Tab. 1]. In Subsections 4.1 and 4.2 we give some results based on a general method connected with the action of the automorphism group S of a configuration. The method was originally proposed in [14,16] and then developed in [13].

4.1 BDC incidence matrices from projective planes

In this subsection, the projective plane $PG(2,q)$ is considered as a cyclic symmetric configuration $(q^2 + q + 1)_{q+1}$ [16, Sec. 5], [18, Sec. 5.5], [49, Th. 19.15], [53]. The Singer group of $PG(2,q)$ is used as the automorphism group S.

The following BDC matrices with $d \times d$ circulant submatrices and the corresponding BDC configurations v_k are given in [13, Sec. 4.1]:

$$\text{BDC } v_k : \ d = \frac{q^2 + q + 1}{3}, \ v = 2d, \ k = \frac{2q + \sqrt{q} + 2}{3}, \ q = p^{4m+2}, \ p \equiv 2 \pmod 3;$$
$$(4.1)$$

$$\text{BDC } v_k : \ d = \frac{q^2 + q + 1}{3}, \ v = 2d, \ k = \frac{2q - \sqrt{q} + 2}{3}, \ q = p^{4m}, \ p \equiv 2 \pmod 3.$$

$$\text{BDC } v_k: \ d = \frac{q^2 + q + 1}{t}, \ v = cd, \ k = \frac{q + 1 \pm (1 - t)\sqrt{q}}{t} + (c - 1)\frac{q + 1 \pm \sqrt{q}}{t}, \quad (4.2)$$

$$c = 1, 2, \ldots, t, \ q = p^{2m}, \ t \text{ prime},$$

where $p \pmod{t}$ is a generator of the multiplicative group of $\mathbb{Z}_t$.

The needed for (4.2) hypothesis that $p \pmod{t}$ is a generator of the multiplicative group of $\mathbb{Z}_t$ holds, for example, in the following cases: $q = 3^4$, $t = 7$; $q = 2^8$, $t = 13$; $q = 5^4$, $t = 7$; $q = 2^{12}$, $t = 19$; $q = 3^8$, $t = 7$; $q = 2^{16}$, $t = 13$; $q = 17^4$, $t = 7$; $p \equiv 2 \pmod{t}$, $t = 3$.

In Table 4.1, parameters of configurations v'_n with BDC incidence matrices are given. We use both (ii) and (iii) of Section 3. The starting weights w_i^* are obtained by computer by considering orbits of subgroups of a Singer group of $PG(2, q)$. For $q = 81$ we use (4.2). The values k', v' are calculated by (3.4),(3.5). Only cases with $v' < G(k')$ are included in the tables. Then the smallest value $k^\#$ for which $v' < G(k^\#)$ is found. As a result, each row of the table provides configurations v'_n with $v' < G(n)$, $n = k^\#, k^\# + 1, \ldots, k'$, see (i) of Section 3 and (3.3). In column w_i^*, an entry s_j indicates that the weight s should be repeated j times.

4.2 BDC incidence matrices from punctured affine planes

In this subsection, the cyclic punctured affine plane is considered as a cyclic symmetric configuration $(q^2 - 1)_q$, see [8], [18, Sec. 5.6], [49, Th. 19.17], as well as [16, Ex. 5, Sec. 6]. The affine Singer group of $AG(2, q)$ is used as the automorphism group S.

The following BDC matrices with $d \times d$ circulant submatrices and the corresponding BDC configurations v_k are given in [13, Sec. 4.2] on the base [13, Th. 4]:

$$\text{BDC } v_k: \ d = (\sqrt{q} - 1)(q + 1), \ v = 2fd, \ k = (2f - 1)\sqrt{q}, \quad (4.3)$$

$$f = 1, 2, \ldots, \frac{\sqrt{q} + 1}{2}, \ q \text{ odd square.}$$

$$\text{BDC } v_k: \ d = 2(\sqrt{q} - 1)(q + 1), \ v = cd, \ k = (2c - 1)\sqrt{q},$$

$$c = 1, 2, \ldots, \frac{\sqrt{q} + 1}{2}, \ q \text{ odd square}, \ \sqrt{q} \equiv 1 \pmod{4}.$$

$$\text{BDC } v_k: \ d = 2(\sqrt{q} - 1)(q + 1), \ v = 2fd, \ k = (4f - 1)\sqrt{q},$$

$$f = 1, 2, \ldots, \frac{\sqrt{q} + 1}{4}, \ q \text{ odd square}, \ \sqrt{q} \equiv 3 \pmod{4}.$$

$$\text{BDC } v_k: \ d = 4(\sqrt{q} - 1)(q + 1), \ v = cd, \ k = (4c - 1)\sqrt{q},$$

$$c = 1, 2, \ldots, \frac{\sqrt{q} + 1}{4}, \ q \text{ odd square}, \ \sqrt{q} \equiv 3 \pmod{4}, \ \frac{\sqrt{q} + 1}{4} \text{ is odd.}$$

$$\text{BDC } v_k: \ d = 4(\sqrt{q} - 1)(q + 1), \ v = 2fd, \ k = (8f - 1)\sqrt{q},$$

$$f = 1, 2, \ldots, \frac{\sqrt{q} + 1}{8}, \ q \text{ odd square}, \ \sqrt{q} \equiv 3 \pmod{4}, \ \frac{\sqrt{q} + 1}{4} \text{ is even.}$$

In Table 4.2 parameters of configurations v'_n with BDC incidence matrices are given. We use both (ii) and (iii) of Section 3. The starting weights w_i^* are obtained by computer

through the constructions of the orbits of subgroups of the affine Singer group. For notations k' and $k^{\#}$ see Table 4.1.

5 Parameters of symmetric configurations v_k admitting an extension by [13]

The following infinite family of symmetric configuration v_k is obtained in [4] by Construction GM, starting from representation of the affine plane $AG(2, q)$ by a set of permutation matrices $\mathbf{P}_q$:

$$v_k : v = q^2 - qs + \theta, \ k = q - s - \Delta, \ q > s \geq 0, \ q - s > \Delta \geq 0, \ \theta = 0, 1, \ldots, q - s + 1. \ (5.1)$$

Theorem 5.1. [13, Corollary 1] *Let* $v = td$, $t \geq k$, $d \geq k - 1$ *, and let* v_k *be a symmetric configuration. Assume that an incidence matrix* $\mathbf{A}$ *of* v_k *is a BDC matrix as in* (3.1) *with weight vector* $\overline{\mathbf{W}}(\mathbf{A})$. *If* $\overline{\mathbf{W}}(\mathbf{A}) = (0, 1, 1, \ldots, 1)$ *or* $\overline{\mathbf{W}}(\mathbf{A}) = (1, 1, \ldots, 1)$ *then one can obtain a family of symmetric configurations* v_k *with parameters* (5.2) *or* (5.3)*, respectively*

$$v_k \ : \ v = cd + \theta, \ k = c - 1 - \delta, \ c = 2, 3, \ldots, t, \ \theta = 0, 1, \ldots, c + 1, \ \delta \geq 0. \quad (5.2)$$
$$v_k \ : \ v = cd + \theta, \ k = c - \delta, \ c = 2, 3, \ldots, t, \ \theta = 0, 1, \ldots, c + 1, \ \delta \geq 0. \qquad (5.3)$$

The families of configurations v_k (5.4) – (5.6) are obtained in [13, Sec. 5] by Construction GM, starting from some new starting matrices proposed in [13].

$$v_k : v = c(q - 1) + \theta, \ k = c - 1 - \delta, \ c = 2, 3, \ldots, q + 1, \ \theta = 0, 1, \ldots, c + 1, \ \delta \geq 0. \ (5.4)$$

$$v_k : v = cp + \theta, \ k = c - \delta, \ c = 2, 3, \ldots, p - 1, \ \theta = 0, 1, \ldots, c + 1, \ \delta \geq 0, \ p \text{ prime. } (5.5)$$

$$v_k : v = c(p - 1) + \theta, \ k = c - 1 - \delta, \ c = 2, 3, \ldots, p, \ \theta = 0, 1, \ldots, c + 1, \ \delta \geq 0, \ p \text{ prime. } (5.6)$$

6 The spectrum of parameters of cyclic symmetric configurations

Current data on the existence of cyclic configurations v_k, $k \leq 51$, are given in Table 6.1. New parameters obtained in [13] are written in bold font.

In Table 6.1, the values of v for which cyclic symmetric configurations v_k exist (resp. do not exist) are written in normal (resp. in italic) font. Moreover, $\overline{v}$ means that no configuration v_k exists, whereas $\overline{v^c}$ indicates that no cyclic configuration v_k exists. Data from [21, 26, 28, 31, 37, 42, 51] are used in the 4-th column of the table. We take into account that an entry of the form "$t+$" in the row "n" of [51, Tab. 1] means the existence of cyclic symmetric configurations v_n with $v \geq t$. An absence of a value "v" in the row "n" of [51, Tab. 1] means the non-existence of a cyclic symmetric configuration v_n. Also, we use the following *non-existence* results: $\overline{32_6}$ [26, Th. 4.8]; $\overline{33_6}$ [37]; $\overline{34_6^c}$, $\overline{59_8^c}$-$\overline{62_8^c}$ [42]; $\overline{75_9^c}$-$\overline{79_9^c}$, $\overline{81_9^c}$-$\overline{84_9^c}$ [21].

The values of k for which the spectrum of parameters of cyclic symmetric configurations v_k is completely known are indicated by a dot ".\"; the corresponding values of $E_c(k)$ are sharp and they are noted by the dot "." too.

An entry **v-w** indicates an interval of sizes from **v** to **w** without gaps. If an already known value lies within an interval **v-w** obtained in work [13], then it is written immediately before the interval.

Entries v_a, v_b, and v_c (here and in all tables) mean, respectively, that relations (2.1), (2.2), and (2.3) are applied.

The value $v_\delta(k)$ is defined in Introduction. In the second column, the exact values of $v_\delta(k)$ are marked by the dot ".". For $k \leq 16$, the exact values of $v_\delta(k)$ are taken from [25, Tab. IV], [33, Tab. 2], [51, Tab. 1a], [56]. Also, if $k - 1$ is a prime power then $v_\delta(k) = P(k) = k^2 - k + 1$. The remaining entries in the second column are lower bounds of $v_\delta(k)$. By the Bruck-Ryser Theorem, planes $PG(2, k-1)$ with $k - 1 = 6, 14, 21, 22, 30, 33, 38, 42, 46, 54, 57, 62$ do not exist. It is well-known that $P(k) \leq v_\delta(k)$, and that a *cyclic* symmetric configuration $(k^2 - k + 1)_k$ exists if and only if a *cyclic* projective plane of order $k - 1$ exists. By [6], no cyclic projective planes exist with non-prime power orders $\leq 2 \cdot 10^9$. Therefore cyclic projective planes $PG(2, k-1)$ with $k - 1 = 18, 20, 24, 26, 28, 34, 35, 36, 39, 40, 44, 45, 48, 50$ do not exist. Also we use Theorem 6.1 of [28]. The mentioned non-existence cases of cyclic configurations are marked in Table 6.1 by subscripts r, s, t.

For $k \leq 22$ the *filling of the interval* $P(k) - G(k)$ is expressed as a percentage in the last column of Table 6.1.

Theorem 6.1. [28, Th. 2.4] *There is no symmetric configuration $(k^2 - k + 2)_k$ if $5 \leq k \leq 10$ or if neither k or $k - 2$ is a square.*

In order to widen the ranges of parameter pairs (v, k) for which a cyclic symmetric configuration v_k exists, we consider a number of procedures that allow to define a new modular Golomb ruler from a known one. Some methods have already been introduced in the paper, see Theorem 2.1.

Here we first recall a result from [49], which describes a method to construct different rulers with the same parameters.

Theorem 6.2. [49] *If $(a_1, a_2, \ldots, a_k)$ is a (v, k) modular Golomb ruler and m and b are integers with $\gcd(m, v) = 1$ then $(ma_1 + b \pmod{v}, ma_2 + b \pmod{v}, \ldots, ma_k + b \pmod{v})$ is also a (v, k) modular Golomb ruler.*

It should be noted that a (v, k) modular Golomb ruler can be a $(v + \Delta, k)$ modular Golomb ruler for some integer Δ [26]. This property does not depend on parameters v and k only. This is why Theorem 6.2 can be useful for our purposes.

Example 6.3. We consider the $(31, 6)$ modular Golomb ruler

$$(a_1, \ldots, a_6) = (0, 1, 4, 10, 12, 17)$$

obtained from $PG(2,5)$, see [50]. We can apply Theorem 6.2 for $m = 19$, $b = 0$. The $(31, 6)$ modular Golomb ruler $(ma_1 \pmod{31}, \ldots, ma_6 \pmod{31})$ is

$$(a'_1, \ldots, a'_6) = (0, 4, 11, 13, 14, 19).$$

Now we take $\Delta = 4$ and calculate the set of differences $\{a'_i - a'_j \pmod{35} \mid 1 \le i, j \le 6; i \ne j\}$, that is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$. As the all differences are distinct and nonzero, the starting $(31, 6)$ modular Golomb ruler is also a $(35, 6)$ modular Golomb ruler.

For $k \le 81$, we performed a computer search starting from the (v, k) modular Golomb rulers corresponding to (2.1)–(2.3). For projective and affine planes, we got a concrete description of the ruler from [50].

For Ruzsa's construction, we use the following known relations. Let p be a prime. Let g be a primitive element of F_p. The following Ruzsa's sequence [48], [18, Sec. 5.4], [49, Th. 19.19] forms a $(p^2 - p, p - 1)$ modular Golomb ruler:

$$e_u = pu + (p-1)g^u \pmod{p^2 - p}, \ u = 1, 2, \ldots, p-1, \ v = p^2 - p. \qquad (6.1)$$

For every starting (v, k) modular ruler we first considered all possible m with

$$\gcd(m, v) = 1,$$

and applied Theorem 6.2 for $b = 0$ to get new rulers with the same parameters v and k. Then, we checked whether this ruler was also a $(v + \Delta, k)$ for some Δ.

In Table 6.2, for $52 \le k \le 83$, the upper bounds on the cyclic existence bound $E_c(k)$ obtained in [13] are listed in bold font. An entry, say $A(u)$, in the column $E_c(k)$ on the row "u", means that in [13] *all* cyclic symmetric configurations v_u in the region $A(u), A(u) + 1, \ldots, G(u) - 1$ are obtained. These configurations are new. We obtained also many other new cyclic symmetric configurations v_k for $52 \le k \le 83$. However, we do not give here their sizes v here in order to save space.

The upper bounds on $E_c(k)$ in Tables 6.1 and 6.2, obtained in [13], are written in bold font.

7 The spectrum of parameters of symmetric (not necessarily cyclic) configurations

The known results regarding to parameters of symmetric configurations can be found in [1–5, 7, 8, 10, 14–16, 21, 22, 24–33, 37, 40, 42, 44, 45, 48, 49, 51]; see also the references therein.

The known families of configurations are described in Section 2, see also (5.1). New families obtained in the work [13] are given in Sections 4 and 5. In Table 7.1, for $k \le 51$, $P(k) \le v < G(k)$, values of v for which a symmetric configuration v_k from one of the families of Sections 2–5 exists are given. The new parameters obtained in the paper [13] are written in bold font.

An entry of type $v_{\text{subscript}}$ indicates that one of the following is used: $(2.i)$, $(4.j)$, $(5.k)$, Table 4.1, Table 6.1, the Bruck-Ryser Theorem, Theorem 6.1. More precisely v_a indicates that v is obtained from (2.1), and similarly $v_b \to (2.2)$, $v_c \to (2.3)$, $v_f \to (2.5)$, $v_g \to (2.6)$, $v_h \to (2.7)$, $v_j \to (2.8)$, $v_k \to (2.9)$, $v_m \to (5.1)$, $v_P \to (4.3)$, $v_R \to (5.4)$, $v_S \to (5.5)$, $v_T \to (5.6)$, $v_W \to$ Table 4.1, $v_y \to$ Table 6.1 with $k \leq 15$, $v_Z \to$ Table 6.1 with $k > 15$, $v_r \to$ the Bruck-Ryser Theorem, $v_t \to$ Theorem 6.1. Here capital letters in subscripts remark new results and constructions while lower case letters indicate the known ones.

An entry $v_{\text{subscript}_1 \cdot \text{subscript}_2 \cdots}$ with more than one subscript means that the same value can be obtained from different constructions. An entry of type $v_{\text{subscript}_1 \cdot \text{subscript}_2 \cdots} - v'_{\text{subscript}_1 \cdot \text{subscript}_2 \cdots}$ indicates that a whole interval of values from v to v' can be obtained from the constructions corresponding to the subscripts. We use the following known results on the existence of sporadic symmetric configurations: 45_7 [5]; 82_9 [22, Tab. 1]; 135_{12}, see [28] with reference to Mathon's talk at the British Combinatorial Conference 1987; 34_6 [40]. The non-existence of configuration 112_{11} is proven in [38]. The non-existence of the plane $PG(2, 10)$ implies the non-existence of configuration 111_{11}. The values of k for which the spectrum of parameters of symmetric configurations v_k is completely known are indicated by a dot "."; the corresponding values of $E(k)$ are exact and they are indicated by a dot as well.

To save space, in Table 7.1 for given v, k, we do not write all the constructions providing a configuration v_k, but often we describe some of them as a matter of illustration.

For the convenience of the reader we give also Table 7.3 where constructions are not indicated and for $k \leq 64$, $P(k) \leq v < G(k)$, values of v for which a symmetric configuration v_k exists are written. Also, in Table 7.2 we give new parameters obtained in [13] for $52 \leq k \leq 64$.

The filling of the interval $P(k) - G(k)$ is expressed as a percentage in the last column of Tables 7.1-7.3. It is interesting to note that such a percentage is quite high and that most gaps occur for v close to $k^2 - k + 1$.

We note that a number of parameters obtained in the work [13] are new, see bold font in Tables 7.1, 7.2. For $k \leq 51$, these new parameters are as follows: 322_{16}, $382_{17} - 390_{17}$, 401_{18}, $405_{18} - 407_{18}$, $409_{18} - 413_{18}$, 458_{19}, 459_{19}, 482_{20}, 574_{22}, 674_{24}, 782_{26}, $1066_{27} - 1072_{27}$, $1104_{27} - 1106_{27}$, $1066_{28} - 1072_{28}$, $1104_{28} - 1109_{28}$, $1142_{28} - 1146_{28}$, $1104_{29} - 1109_{29}$, $1142_{29} - 1146_{29}$, $1180_{29} - 1183_{29}$, 1220_{29}, $1142_{30}\text{-}1146_{30}$, $1180_{30}\text{-}1183_{30}$, $1218_{30} - 1220_{30}$, $1180_{31} - 1183_{31}$, 1218_{31}, 1219_{31}, 1220_{31}, 1256_{31}, 1257_{31}, $1218_{32} - 1220_{32}$, $1256_{32} - 1257_{32}$, 1294_{32}, 1256_{33}, 1257_{33}, 1294_{33}, 1294_{34}, $1430_{34} - 1434_{34}$, $1472_{35} - 1475_{35}$, $1514_{36} - 1516_{36}$, 1556_{37}, 1557_{37}, 1598_{38}, 1762_{38}, 1762_{39}, 1762_{40}, $1922_{40} - 1926_{40}$, $1970_{41} - 1973_{41}$, $2018_{42} - 2020_{42}$, 2066_{43}, 2067_{43}, $2486_{43} - 2490_{43}$, $2594_{43} - 2595_{43}$, 2114_{44}, 2302_{44}, $2486_{44} - 2490_{44}$, $2540_{44} - 2543_{44}$, 2594_{44}, 2595_{44}, 2648_{44}, 2649_{44}, 2302_{45}, $2486_{45} - 2490_{45}$, $2540_{45} - 2543_{45}$, $2594_{45} - 2596_{45}$, 2648_{45}, 2649_{45}, 2702_{45}, 2302_{46}, $2486_{46} - 2490_{46}$, $2540_{46} - 2543_{46}$, $2594_{46} - 2596_{46}$, 2648_{46}, 2649_{46}, 2702_{46}, $2540_{47} - 2543_{47}$, $2594_{47} - 2596_{47}$, 2648_{47}, 2649_{47}, 2702_{47}, $2594_{48} - 2596_{48}$, 2648_{48}, 2649_{48}, 2702_{48}, $2882_{48} - 2890_{48}$, 2648_{49}, 2649_{49}, 2702_{49}, $2942_{49} - 2949_{49}$, 2702_{50}, $3002_{50} - 3008_{50}$, $3062_{51} - 3067_{51}$; the gaps in the known range of parameters are filled by these new values.

Note that parameters of some new families are too big to be included in Tables 7.1–7.3.

Recall also (see Introduction) that from the stand point of applications, including Coding Theory, it is useful to have different matrices $\mathbf{M}(v, k)$ for the same v and k.

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8 Tables of parameters

8.1 Tables for Section 4

Table 4.1. Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^\#, k^\# + 1, \ldots, k'$, by (ii) of Section 3 from the cyclic projective plane $PG(2, q)$

q	t	d	w_i^*	c	k'	v'	$G(k')$	$k^\#$	$G(k^\#)$
25	3	217	$12, 7, 7$	2	19	434	493	19	493
25	7	93	$8, 3, 3, 3, 3, 3, 3$	4	17	372	399	17	399
25	7	93	$8, 3, 3, 3, 3, 3, 3$	5	20	465	567	19	493
25	7	93	$8, 3, 3, 3, 3, 3, 3$	6	23	558	745	20	567
32	7	151	$0, 5, 5, 6, 5, 6, 6$	6	25	906	961	25	961
37	3	469	$16, 9, 13$	2	25	938	961	25	961
43	3	631	$19, 13, 12$	2	31	1262	1495	30	1361
49	3	817	$21, 16, 13$	2	34	1634	1877	33	1719
61	3	1261	$25, 21, 16$	2	41	2522	2611	40	2565
64	3	1387	$27, 19, 19$	2	46	2774	3407	42	2795
64	19	219	$11, 3_{18}$	7	32	1533	1569	32	1569
64	19	219	$11, 3_{18}$	8	35	1752	1975	34	1877
64	19	219	$11, 3_{18}$	9	38	1971	2293	35	1975
64	19	219	$11, 3_{18}$	10	41	2190	2611	37	2199
64	19	219	$11, 3_{18}$	17	62	3723	6431	48	3775
64	19	219	$11, 3_{18}$	18	65	3942	7187	50	4189
67	3	1519	$28, 19, 21$	2	47	3038	3609	44	3193
73	3	1801	$28, 27, 19$	2	47	3602	3609	47	3609
79	3	2107	$31, 21, 28$	2	52	4214	4541	51	4381
81	7	949	$4, 13, 13, 13, 13, 13, 13$	6	69	5694	8291	58	5703
81	7	949	$4, 13, 13, 13, 13, 13, 13$	5	56	4745	5451	54	4747
97	3	3169	$39, 28, 31$	2	67	6338	7639	62	6431
103	3	3571	$39, 28, 37$	2	67	7142	7639	65	7187
107	7	1651	$24, 15, 15, 13, 15, 13, 13$	6	89	9906	13557	75	9965
107	7	1651	$24, 15, 15, 13, 15, 13, 13$	5	76	8255	10179	69	8291
109	3	3997	$43, 36, 31$	2	74	7994	9507	69	8291
109	7	1713	$8, 15, 15, 19, 15, 19, 19$	6	83	10278	12041	77	10409
121	37	399	$14, 3_{36}$	25	89	9975	13557	76	10179
127	3	5419	$49, 43, 36$	2	85	10838	12821	80	11127
128	7	2359	$24, 21, 21, 14, 21, 14, 14$	6	94	14154	15769	91	15085
137	7	2701	$24, 15, 15, 23, 15, 23, 23$	6	99	16206	17081	96	16243

Table 4.1 (continue). Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^\#, k^\# + 1, \ldots, k'$, by (ii) of Section 3 from the cyclic projective plane $PG(2, q)$

q	t	d	w_i^*	c	k'	v'	$G(k')$	$k^\#$	$G(k^\#)$
139	3	6487	52, 39, 49	2	91	12974	15085	86	13075
149	7	3193	12, 25, 25, 21, 25, 21, 21	6	117	19158	25035	104	19163
149	7	3193	12, 25, 25, 21, 25, 21, 21	5	96	15965	16243	96	16243
121	3	4921	48, 37, 37	2	85	9842	12821	75	9965
121	7	2109	21, 20, 13, 13, 21, 13, 21	6	86	12654	13075	85	12821
121	37	399	$14, 3_{36}$	20	71	7980	8661	69	8291
151	3	7651	57, 43, 52	2	100	15302	17663	93	15453
151	7	3279	32, 19, 19, 21, 19, 21, 21	6	127	19674	28921	105	19769
151	7	3279	32, 19, 19, 21, 19, 21, 21	5	108	16395	20831	97	16715
151	7	3279	32, 19, 19, 21, 19, 21, 21	4	89	13116	13557	87	13417
157	3	8269	61, 48, 49	2	109	16538	21167	97	16715
163	3	8911	63, 49, 52	2	112	17822	27043	102	18437
163	7	3819	32, 25, 25, 19, 25, 19, 19	6	127	22914	28921	114	23529
163	7	3819	32, 25, 25, 19, 25, 19, 19	5	108	19095	20831	104	19163
169	3	9577	64, 57, 49	2	113	19154	22847	104	19163
179	7	4603	24, 21, 21, 31, 21, 31, 31	6	129	27618	30151	124	27897
181	3	10981	67, 63, 52	2	119	21962	25823	111	22217

Table 4.2. Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^\#, k^\# + 1, \ldots, k'$, by (ii) and (iii) of Section 3 from the cyclic punctured affine plane $AG(2, q)$

q	t	d	w_i^*	c	f	k'	v'	$G(k')$	$k^\#$	$G(k^\#)$
16	3	85	$8, 4, 4$	2		12	170	171	12	171
31	3	320	$14, 9, 8$	2		22	640	713	21	667
37	3	456	$16, 9, 12$	2		25	912	961	25	961
49	4	600	$16, 12, 9, 12$	3		34	1800	1877	34	1877
49	6	400	$4, 9, 12, 8, 8, 8$	5		36	2000	2011	36	2011
53	4	702	$17, 12, 10, 14$	3		37	2106	2199	37	2199
61	3	1240	$25, 16, 20$	2		41	2480	2611	39	2505
61	4	930	$18, 18, 12, 13$	3		42	2790	2795	42	2795
61	5	744	$18, 9, 10, 12, 12$	4		45	2976	3375	43	3015
61	6	620	$15, 8, 8, 10, 8, 12$	5		47	3100	3609	44	3193
64	9	455	$0, 8, 8, 8, 8, 8, 8, 8, 8$	8		56	3640	5451	48	3775
64	9	455	$0, 8, 8, 8, 8, 8, 8, 8, 8$	7		48	3185	3775	44	3193
67	3	1496	$26, 24, 17$	2		43	2992	3015	43	3015
71	5	1008	$8, 14, 15, 16, 18$	4		50	4032	4189	50	4189
73	3	1776	$30, 21, 22$	2		51	3552	4381	47	3609
79	3	2080	$32, 25, 22$	2		54	4160	4747	50	4189
79	6	1040	$8, 14, 13, 14, 18, 12$	5		56	5200	5451	56	5451
79	6	1040	$18, 12, 8, 14, 13, 14$		2	50	4160	4189	50	4189
81	4	1640	$25, 20, 16, 20$	3		57	4920	5547	55	5197
81	4	1640	$25, 20, 16, 20$		1	45	3280	3375	45	3375
81	8	820	$16, 8, 8, 8, 9, 12, 8, 12$	7		64	5740	7055	59	5823
81	8	820	$16, 8, 8, 8, 9, 12, 8, 12$	6		56	4920	5451	55	5187
83	8	861	$6, 10, 10, 10, 15, 11, 10, 11$	7		66	6027	7515	60	6039
83	8	861	$6, 10, 10, 10, 15, 11, 10, 11$	6		56	5166	5451	55	5187
89	4	1980	$26, 25, 18, 20$	3		62	5940	6431	60	6039
89	8	990	$17, 8, 10, 12, 8, 10, 10, 14$	7		65	6930	7187	64	7055
97	3	3136	$37, 34, 26$	2		63	6272	6783	62	6431
97	4	2352	$29, 26, 20, 22$	3		69	7056	8291	65	7187
97	6	1586	$21, 20, 14, 16, 14, 12$	5		69	7930	8291	69	8291
101	4	2550	$30, 25, 20, 26$	3		70	7650	8435	68	7913
101	4	2550	$30, 25, 20, 26$		1	55	5100	5197	55	5197
103	3	3536	$41, 32, 30$	2		71	7072	8661	65	7187
103	6	1768	$24, 18, 14, 17, 14, 16$	5		80	8840	11127	72	8947
103	6	1768	$24, 18, 14, 17, 14, 16$	4		66	7072	7515	65	7187
103	6	1768	$24, 18, 14, 17, 14, 16$		2	70	7072	8435	65	7187
107	8	1431	$17, 15, 10, 13, 10, 12, 17, 13$	7		77	10017	10409	76	10179
107	8	1431	$17, 15, 10, 13, 10, 12, 17, 13$		3	73	8586	9027	71	8661

Table 4.2 (continue). Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^{\#}, k^{\#} + 1, \ldots, k'$, by (ii) and (iii) of Section 3 from the cyclic punctured affine plane $AG(2,q)$

q	t	d	w_i^*	c	f	k'	v'	$G(k')$	$k^{\#}$	$G(k^{\#})$
109	3	3960	42, 30, 37	2		72	7920	8947	69	8291
109	4	2970	32, 26, 22, 29	3		76	8910	10179	72	8947
109	6	1980	12, 19, 24, 18, 18, 18	5		84	9900	12319	75	9965
109	9	1320	8, 14, 20, 10, 13, 10, 12, 10, 12	8		78	10560	10599	78	10599
113	4	3192	32, 32, 24, 25	3		80	9576	11127	75	9965
113	7	1824	10, 20, 14, 17, 22, 16, 14	6		80	10944	11127	80	11127
121	4	3660	36, 30, 25, 30	3		86	10980	13075	80	11127
121	4	3660	36, 30, 25, 30		1	66	7320	7515	66	7515
121	5	2928	16, 24, 28, 25, 28	4		88	11712	13491	83	12041
125	4	3906	37, 32, 26, 30	3		89	11718	13557	83	12041
125	8	1953	24, 16, 14, 15, 13, 16, 12, 15	7		96	13671	16243	90	13935
125	8	1953	24, 16, 14, 15, 13, 16, 12, 15		3	93	11718	15453	83	12041
127	3	5376	49, 36, 42	2		85	10752	12821	79	10817
127	6	2688	28, 18, 18, 21, 18, 24	5		100	13440	17663	88	13491
127	6	2688	28, 18, 18, 21, 18, 24	4		82	10752	11629	79	10817
131	5	3432	19, 24, 32, 26, 30	4		91	13728	15085	90	13935
137	4	4692	40, 32, 29, 36	3		98	14076	16925	91	15085
139	3	6440	54, 41, 44	2		95	12880	15935	86	13075
149	4	5550	42, 41, 32, 34	3		106	16650	20271	97	16715
151	3	7600	58, 44, 49	2		102	15200	18437	92	15235
151	6	3800	18, 24, 32, 26, 25, 26		2	93	15200	15453	93	15453

8.2 Tables for Section 6

Table 6.1. Values of v for which a cyclic symmetric configuration v_k exists, $5 \leq k \leq 51$, $P(k) \leq v \leq G(k) - 1$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \leq v \leq G(k) - 1$	$E_c(k)$	$G(k)$	filling
5.	21	21.	$21_a, \overline{22}_t$	23.	23	100%
6.	31	31.	$31_a, \overline{32}_t, \overline{33}, \overline{34^c}$	35.	35	100%
7.	43	48.	$48_b, 49, 50$	48.	51	100%
8.	57	57.	$57_a, \overline{58}_t, \overline{59^c} - \overline{62^c}, 63_b, 64 - 68$	63.	69	100%
9.	73	73.	$73_a, \overline{74}_t, \overline{75^c} - \overline{79^c}, 80_b, \overline{81^c} - \overline{84^c}, 85 - 88$	85.	89	100%
10.	91	91.	$91_a, \overline{92}_t, \overline{93^c} - \overline{106^c}, 107 - 109, 110_c$	107.	111	100%
11.	111	120.	$120_b, \overline{121^c} - \overline{132^c}, 133_a, \overline{134^c}, 135 - 144$	135.	145	100%
12.	133	133.	$133_a, \overline{134}_t, \overline{135^c} - \overline{155^c}, 156_c, \overline{157^c}, 158,$ $159, \overline{160^c}, 168_b, 161 - 170$	161.	171	100%
13.	157	168.	$168_b, \overline{169^c} - \overline{182^c}, 183_a, \overline{184^c} - \overline{192^c},$ $193 - 212$	193.	213	100%
14.	183	183.	$183_a, \overline{184}_t, \overline{185^c} - \overline{224^c}, 225 - 254$	225.	255	100%
15	211	255.	$255_b, \overline{256^c} - \overline{260^c}, \overline{263^c}, 272_c, 273_a, 288_b,$ $267 - 302$	267	303	95%
16	241	255.	$255_b, 272_c, 273_a, 288_b, 307_a, \mathbf{318, 320 - 329,}$ $\mathbf{331 - 354}$	**331**	355	35%
17	273	273.	$273_a, \overline{274}_t, 288_b, 307_a, 342_c, \mathbf{343, 353}, 360_b,$ $\mathbf{357 - 363}, 381_a, \mathbf{365 - 398}$	**365**	399	40%
18	307	307.	$307_a, 342_c, 360_b, 381_a, \mathbf{401, 403, 405 - 407,}$ $\mathbf{409 - 432}$	**409**	433	26%
19	343	$\geq 345_{s,t}$	$360_b, 381_a, \mathbf{455, 457, 461, 464, 467, 468,}$ $\mathbf{470 - 492}$	**470**	493	21%
20	381	381.	$381_a, \overline{382}_t, \mathbf{498, 503}, 506_c, \mathbf{508, 513, 516,}$ $\mathbf{518 - 520, 522, 524, 525}, 528_b, \mathbf{527 - 530,}$ $553_a, \mathbf{532 - 566}$	**532**	567	28%
21	421	$\geq 423_{s,t}$	$506_c, 528_b, 553_a, \mathbf{592, 594, 597, 601, 602,}$ $\mathbf{604, 606 - 609, 611, 612}, 624_b, \mathbf{614 - 666}$	**614**	667	28%
22	463	$\geq 465_{r,t}$	$506_c, 528_b, 553_a, 624_b, \mathbf{633, 640, 644, 645,}$ $\mathbf{647}, 651_a, \mathbf{649 - 712}$	**649**	713	29%

Key to Table 6.1: $a \rightarrow$ (2.1), $b \rightarrow$ (2.2), $c \rightarrow$ (2.3), $r \rightarrow$ Bruck-Ryser Theorem, $s \rightarrow$ [6], $t \rightarrow$ Theorem 6.1

Table 6.1 (continue 1) Values of v for which a cyclic symmetric configuration v_k exists, $5 \leq k \leq 51$, $P(k) \leq v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \leq v \leq G(k) - 1$	$E_c(k)$	$G(k)$
23	507	$\geq 509_{r,t}$	$528_b, 553_a, 624_b, 651_a, \mathbf{683}, \mathbf{687}, \mathbf{696}, \mathbf{698},$ $\mathbf{699}, \mathbf{702}, \mathbf{703}, \mathbf{705 - 711}, 728_b, \mathbf{713 - 744}$	**713**	745
24	553	$553\textbf{.}$	$553_a, \overline{554}_t, 624_b, 651_a, 728_b, \ \mathbf{739}, \mathbf{747}, \mathbf{748},$ $\mathbf{752}, 757_a, \mathbf{759 - 761}, \mathbf{763 - 770}, \mathbf{772},$ $812_c, 840_b, \mathbf{774 - 850}$	**774**	851
25	601	$\geq 602_s$	$624_b, 651_a, 728_b, 757_a, 812_c, \mathbf{837}, 840_b,$ $\mathbf{839 - 844}, \mathbf{846 - 863}, 871_a, 930_c,$ $960_b, \mathbf{865 - 960}$	**865**	961
26	651	$651\textbf{.}$	$651_a, \overline{652}_t, 728_b, 757_a, 812_c, 840_b, 871_a,$ $\mathbf{900}, \mathbf{905 - 907}, \mathbf{910}, \mathbf{912}, \mathbf{913}, \mathbf{916}, \mathbf{917},$ $\mathbf{919 - 921}, \mathbf{924}, \mathbf{925}, \mathbf{927}, \mathbf{929}, 930_c,$ $\mathbf{932 - 941}, 960_b, \mathbf{943 - 984}$	**943**	985
27	703	$\geq 704_s$	$728_b, 757_a, 812_c, 840_b, 871_a, 930_c, 960_b,$ $\mathbf{970}, \mathbf{971}, \mathbf{975}, \mathbf{977}, \mathbf{978}, 985, \mathbf{987}, \mathbf{988},$ $\mathbf{991}, 993_a, \mathbf{994}, \mathbf{996}, \mathbf{997}, \mathbf{1000}, \mathbf{1001},$ $\mathbf{1003 - 1006}, \mathbf{1008 - 1015}, \mathbf{1017 - 1019},$ $1023_b, 1057_a, \mathbf{1021 - 1106}$	**1021**	1107
28	757	$757\textbf{.}$	$757_a, \overline{758}_t, 812_c, 840_b, 871_a, 930_c, 960_b,$ $993_a, 1023_b, \mathbf{1045}, 1057_a, \mathbf{1063}, \mathbf{1067}, \mathbf{1070},$ $\mathbf{1071}, \mathbf{1074}, \mathbf{1075}, \mathbf{1077}, \mathbf{1079 - 1082},$ $\mathbf{1085 - 1170}$	**1085**	1171
29	813	$\geq 815_{s,t}$	$840_b, 871_a, 930_c, 960_b, 993_a, 1023_b, 1057_a,$ $\mathbf{1145}, \mathbf{1146}, \mathbf{1151}, \mathbf{1152}, \mathbf{1155 - 1158},$ $\mathbf{1162 - 1167}, \mathbf{1169}, \mathbf{1172 - 1175},$ $\mathbf{1177}, \mathbf{1179 - 1246}$	**1179**	1247
30	871	$871\textbf{.}$	$871_a, \overline{872}_t, 930_c, 960_b, 993_a, 1023_b, 1057_a, \mathbf{1198},$ $\mathbf{1199}, \mathbf{1219}, \mathbf{1220}, \mathbf{1223}, \mathbf{1224}, \mathbf{1229}, \mathbf{1235},$ $\mathbf{1236}, \mathbf{1238}, \mathbf{1240}, \mathbf{1241}, \mathbf{1243}, \mathbf{1248}, \mathbf{1249},$ $\mathbf{1251}, \mathbf{1253 - 1255}, \mathbf{1258}, \mathbf{1259}, \mathbf{1261},$ $\mathbf{1262}, \mathbf{1264 - 1267}, 1332_c, \mathbf{1269 - 1360}$	**1269**	1361
31	931	$\geq 933_{r,t}$	$960_b, 993_a, 1023_b, 1057_a, \mathbf{1324}, \mathbf{1325},$ $1332_c, \mathbf{1341}, \mathbf{1343 - 1346}, \mathbf{1348},$ $\mathbf{1349}, 1368_b, \mathbf{1351\text{-}1494}$	**1351**	1495
32	993	$993\textbf{.}$	$993_a, \overline{994}_t, 1023_b, 1057_a, 1332_c, 1368_b,$ $\mathbf{1383}, \mathbf{1393}, \mathbf{1401}, 1407_a, \mathbf{1409}, \mathbf{1411}, \mathbf{1412},$ $\mathbf{1414}, \mathbf{1421}, \mathbf{1424}, \mathbf{1425}, \mathbf{1428 - 1430}, \mathbf{1432},$ $\mathbf{1434}, \mathbf{1438 - 1441}, \mathbf{1443 - 1445},$ $\mathbf{1447 - 1457}, \mathbf{1459 - 1568}$	**1459**	1569

Key to Table 6.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $r \to$ Bruck-Ryser Theorem, $s \to [6]$, $t \to$ Theorem 6.1

Table 6.1 (continue 2) Values of v for which a cyclic symmetric configuration v_k exists, $5 \leq k \leq 51$, $P(k) \leq v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \leq v \leq G(k)-1$	$E_c(k)$	$G(k)$
33	1057	1057.	$1057_a, \overline{1058}_t, 1332_c, 1368_b, 1407_a,$ **1492, 1506,** **1507, 1518, 1521, 1528, 1529, 1533, 1535,** **1540, 1542, 1543, 1545, 1547 − 1553, 1555,** **1557 − 1559, 1561 − 1563, 1565 − 1567,** **1569 − 1578,**$1640_c, 1680_b,$ **1580 − 1718**	**1580**	1719
34	1123	$\geq 1125_{r,t}$	$1332_c, 1368_b, 1407_a, 1640_c,$ **1664, 1665, 1670** **1676,**$1680_b,$ **1686, 1693, 1698, 1699, 1702,** **1705, 1708 − 1712, 1714, 1717, 1721,** $1723_a,$ **1723 − 1726, 1728, 1730 − 1742,** **1744 − 1752,**$1806_c, 1848_b,$ **1754 − 1876**	**1754**	1877
35	1191	$\geq 1193_{s,t}$	$1332_c, 1368_b, 1407_a, 1640_c, 1680_b, 1723_a,$ **1777, 1781, 1783, 1788, 1792, 1793, 1795,** **1798, 1800 − 1803,**$1806_c,$ **1805 − 1807,** **1810, 1812 − 1814, 1817, 1818,** $1848_b, 1893_a,$ **1820 − 1974**	**1820**	1975
36	1261	$\geq 1262_s$	$1332_c, 1368_b, 1407_a, 1640_c, 1680_b, 1723_a, 1806_c,$ $1848_b,$ **1853, 1855, 1860, 1868, 1883,**$1886,$ $1893_a,$ **1890 − 1897, 1900 − 1903, 1905,** **1907 − 1939, 1941 − 2010**	**1941**	2011
37	1333	$\geq 1335_{s,t}$	$1368_b, 1407_a, 1640_c, 1680_b, 1723_a, 1806_c,$ $1848_b, 1893_a,$ **1910, 1944, 1957, 1959,** **1967, 1973, 1974, 1976, 1978, 1986,** **1989, 1992, 1995 − 1999, 2001 − 2004,** **2006 − 2008, 2010, 2012, 2015 − 2019,** **2021, 2023 − 2031, 2033,**$2162_c,$ **2035 − 2198**	**2035**	2199
38	1407	1407.	$1407_a, 1640_c, 1680_b, 1723_a, 1806_c, 1848_b,$ $1893_a,$ **2059, 2061, 2073, 2088, 2089,** **2092, 2094, 2096, 2097, 2099 − 2101,** **2103, 2105 − 2108, 2110, 2111,** **2114 − 2116, 2118, 2123, 2124,** **2126 − 2130, 2135 − 2153, 2155- 2157,** $2162_c,$ **2159 − 2164, 2166 − 2170,** $2208_b, 2257_a,$ **2172 − 2292**	**2172**	2293

Key to Table 6.1: $a \rightarrow$ (2.1), $b \rightarrow$ (2.2), $c \rightarrow$ (2.3), $r \rightarrow$ Bruck-Ryser Theorem, $s \rightarrow$ [6], $t \rightarrow$ Theorem 6.1

Table 6.1 (continue 3) Values of v for which a cyclic symmetric configuration v_k exists, $5 \leq k \leq 51$, $P(k) \leq v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \leq v \leq G(k) - 1$	$E_c(k)$	$G(k)$
39	1483	$\geq 1485_{r,t}$	$1640_c, 1680_b, 1723_a, 1806_c, 1848_b, 1893_a,$ $2162_c, \mathbf{2187}, \mathbf{2195}, 2208_b, \mathbf{2240}, \mathbf{2241},$ $\mathbf{2243}, \mathbf{2247}, \mathbf{2248}, \mathbf{2251}, \mathbf{2252}, \mathbf{2254},$ $\mathbf{2255}, 2257_a, \mathbf{2258-2260}, \mathbf{2263-2265},$ $\mathbf{2269}, \mathbf{2270}, \mathbf{2277-2279}, \mathbf{2281}, \mathbf{2283},$ $\mathbf{2287}, \mathbf{2289-2309}, \mathbf{2311}, \mathbf{2313-2328},$ $2400_b, 2451_a, \mathbf{2330-2504}$	**2330**	2505
40	1561	$\geq 1563_{s,t}$	$1640_c, 1680_b, 1723_a, 1806_c, 1848_b, 1893_a,$ $2162_c, 2208_b, 2257_a, \mathbf{2326}, \mathbf{2338}, \mathbf{2345},$ $\mathbf{2349}, \mathbf{2353}, \mathbf{2355}, \mathbf{2357}, \mathbf{2360}, \mathbf{2361},$ $\mathbf{2363}, \mathbf{2364}, \mathbf{2366-2368}, \mathbf{2370}, \mathbf{2372},$ $\mathbf{2374-2377}, \mathbf{2379-2381}, \mathbf{2387-2389},$ $\mathbf{2393}, \mathbf{2395-2397}, \mathbf{2399}, 2400_b, \mathbf{2401},$ $\mathbf{2403}, \mathbf{2404}, \mathbf{2406}, \mathbf{2407}, \mathbf{2409},$ $\mathbf{2411-2418}, 2451_a, \mathbf{2420-2436},$ $\mathbf{2438-2564}$	**2438**	2565
41	1641	$\geq 1643_{s,t}$	$1680_b, 1723_a, 1806_c, 1848_b, 1893_a, 2162_c,$ $2208_b, 2257_a, \mathbf{2345}, \mathbf{2399}, 2400_b, \mathbf{2436},$ $2449, 2451_a, \mathbf{2459}, \mathbf{2460}, \mathbf{2465}, \mathbf{2471},$ $\mathbf{2472}, \mathbf{2479}, \mathbf{2480}, \mathbf{2483}, \mathbf{2485}, \mathbf{2491},$ $\mathbf{2493}, \mathbf{2494}, \mathbf{2496-2500}, \mathbf{2502}, \mathbf{2503},$ $\mathbf{2505}, \mathbf{2507-2513}, \mathbf{2515-2525},$ $\mathbf{2528-2540}, \mathbf{2542}, \mathbf{2544-2610}$	**2544**	2611
42	1723	1723**.**	$1723_a, \overline{1724_t}, 1806_c, 1848_b, 1893_a, 2162_c,$ $2208_b, 2257_a, 2400_b, 2451_a, \mathbf{2510}, \mathbf{2522},$ $\mathbf{2539}, \mathbf{2541}, \mathbf{2557-2559}, \mathbf{2562}, \mathbf{2564},$ $\mathbf{2566-2568}, \mathbf{2570}, \mathbf{2573}, \mathbf{2577}, \mathbf{2578},$ $\mathbf{2580-2584}, \mathbf{2586-2590}, \mathbf{2593}, \mathbf{2595},$ $\mathbf{2597-2601}, \mathbf{2603-2610}, \mathbf{2612},$ $\mathbf{2613}, \mathbf{2615-2626}, 2756_c, \mathbf{2628-2794}$	**2628**	2795

Key to Table 6.1: $a \rightarrow$ (2.1), $b \rightarrow$ (2.2), $c \rightarrow$ (2.3), $r \rightarrow$ Bruck-Ryser Theorem, $s \rightarrow$ [6], $t \rightarrow$ Theorem 6.1

Table 6.1 (continue 4) Values of v for which a cyclic symmetric configuration v_k exists, $5 \leq k \leq 51$, $P(k) \leq v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \leq v \leq G(k) - 1$	$E_c(k)$	$G(k)$
43	1807	$\geq 1809_{r,t}$	$1848_b, 1893_a, 2162_c, 2208_b, 2257_a, 2400_b,$ $2451_a, \mathbf{2684}, \mathbf{2686}, \mathbf{2688}, \mathbf{2715}, \mathbf{2725},$ $\mathbf{2728}, \mathbf{2734}, \mathbf{2737}, \mathbf{2739}, \mathbf{2744}, \mathbf{2752},$ $2756_c, \mathbf{2757}, \mathbf{2759}, \mathbf{2762}, \mathbf{2763},$ $\mathbf{2766 - 2768}, \mathbf{2771}, \mathbf{2772}, \mathbf{2776}, \mathbf{2777},$ $\mathbf{2783}, \mathbf{2786 - 2789}, \mathbf{2791}, \mathbf{2792},$ $\mathbf{2794 - 2798}, \mathbf{2800}, \mathbf{2801}, 2808_b,$ $\mathbf{2803 - 2811}, \mathbf{2813 - 2815}, \mathbf{2817 - 2858},$ $2863_a, \mathbf{2860 - 3014}$	**2860**	3015
44	1893	$1893.$	$1893_a, \overline{1894}_t, 2162_c, 2208_b, 2257_a, 2400_b,$ $2451_a, 2756_c, 2808_b, \mathbf{2811}, \mathbf{2821}, \mathbf{2826},$ $\mathbf{2834}, \mathbf{2836}, \mathbf{2844}, \mathbf{2848}, \mathbf{2849}, \mathbf{2861},$ $\mathbf{2862}, 2863_a, \mathbf{2865 - 2867}, \mathbf{2870}, \mathbf{2871},$ $\mathbf{2873 - 2875}, \mathbf{2879 - 2881}, \mathbf{2884}, \mathbf{2887},$ $\mathbf{2890 - 2895}, \mathbf{2898}, \mathbf{2899}, \mathbf{2901 - 2912},$ $\mathbf{2914}, \mathbf{2916 - 3192}$	**2916**	3193
45	1981	$\geq 1983_{s,t}$	$2162_c, 2208_b, 2257_a, 2400_b, 2451_a, 2756_c,$ $2808_b, 2863_a, \mathbf{2994}, \mathbf{3013}, \mathbf{3014}, \mathbf{3019},$ $\mathbf{3038}, \mathbf{3052}, \mathbf{3054}, \mathbf{3056}, \mathbf{3066}, \mathbf{3069},$ $\mathbf{3082 - 3085}, \mathbf{3087}, \mathbf{3088}, \mathbf{3090}, \mathbf{3091},$ $\mathbf{3093}, \mathbf{3095 - 3098}, \mathbf{3101 - 3103},$ $\mathbf{3105 - 3112}, \mathbf{3114}, \mathbf{3116 - 3120},$ $\mathbf{3122 - 3163}, \mathbf{3165 - 3374}$	**3165**	3375
46	2071	$\geq 2073_{s,t}$	$2162_c, 2208_b, 2257_a, 2400_b, 2451_a, 2756_c,$ $2808_b, 2863_a, \mathbf{3124}, \mathbf{3171}, \mathbf{3188}, \mathbf{3191},$ $\mathbf{3194}, \mathbf{3196}, \mathbf{3197}, \mathbf{3198}, \mathbf{3206}, \mathbf{3216},$ $\mathbf{3218}, \mathbf{3219}, \mathbf{3221}, \mathbf{3227}, \mathbf{3228}, \mathbf{3231},$ $\mathbf{3233}, \mathbf{3234}, \mathbf{3238 - 3241}, \mathbf{3244 - 3247},$ $\mathbf{3249}, \mathbf{3250}, \mathbf{3252 - 3265}, \mathbf{3267 - 3271},$ $\mathbf{3273 - 3278}, \mathbf{3280 - 3406}$	**3280**	3407

Key to Table 6.1: $a \rightarrow$ (2.1), $b \rightarrow$ (2.2), $c \rightarrow$ (2.3), $r \rightarrow$ Bruck-Ryser Theorem, $s \rightarrow$ [6], $t \rightarrow$ Theorem 6.1

Table 6.1 (continue 5) Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51$, $P(k) \le v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k)-1$	$E_c(k)$	$G(k)$
47	2163	$2165_{r,t}$	$2208_b, 2257_a, 2400_b, 2451_a, 2756_c, 2808_b,$ $2863_a, \mathbf{3255}, \mathbf{3261}, \mathbf{3271}, \mathbf{3280}, \mathbf{3285},$ $\mathbf{3292}, \mathbf{3301}, \mathbf{3312}, \mathbf{3327}, \mathbf{3331}, \mathbf{3342},$ $\mathbf{3343}, \mathbf{3346 - 3348}, \mathbf{3351}, \mathbf{3353},$ $\mathbf{3355 - 3357}, \mathbf{3360 - 3371}, \mathbf{3375 - 3379},$ $\mathbf{3381 - 3384}, \mathbf{3387}, \mathbf{3389}, \mathbf{3390},$ $\mathbf{3392 - 3402}, \mathbf{3404 - 3408}, \mathbf{3412}, \mathbf{3413},$ $3422_c, \mathbf{3415 - 3427}, 3480_b, 3541_a,$ $\mathbf{3429 - 3608}$	**3429**	3609
48	2257	$2257.$	$2257_a, \overline{2258_t}, 2400_b, 2451_a, 2756_c, 2808_b,$ $2863_a, \mathbf{3418}, 3422_c, \mathbf{3431}, \mathbf{3445}, \mathbf{3459},$ $3480_b, \mathbf{3487}, \mathbf{3491}, \mathbf{3492}, \mathbf{3495}, \mathbf{3499},$ $\mathbf{3509}, \mathbf{3512}, \mathbf{3515}, \mathbf{3518}, \mathbf{3519}, \mathbf{3522},$ $\mathbf{3523}, \mathbf{3526 - 3528}, \mathbf{3535}, \mathbf{3540},$ $3541_a, \mathbf{3545 - 3547}, \mathbf{3549 - 3552},$ $\mathbf{3554 - 3563}, \mathbf{3565}, \mathbf{3567}, \mathbf{3568}, \mathbf{3570},$ $\mathbf{3572 - 3578}, \mathbf{3580 - 3583}, \mathbf{3586}, \mathbf{3587},$ $\mathbf{3589 - 3595}, \mathbf{3597 - 3605}, \mathbf{3607 - 3618},$ $\mathbf{3620 - 3630}, \mathbf{3632 - 3640}, 3660_c,$ $\mathbf{3642 - 3774}$	**3642**	3775
49	2353	$\ge 2354_s$	$2400_b, 2451_a, 2756_c, 2808_b, 2863_a, 3422_c,$ $3480_b, 3541_a, \mathbf{3608}, \mathbf{3627}, \mathbf{3637}, \mathbf{3640},$ $\mathbf{3642}, \mathbf{3644}, \mathbf{3646}, \mathbf{3647}, \mathbf{3649}, \mathbf{3652},$ $\mathbf{3653}, \mathbf{3655}, 3660_c, \mathbf{3665}, \mathbf{3669}, \mathbf{3671},$ $\mathbf{3675}, \mathbf{3677}, \mathbf{3678}, \mathbf{3680 - 3683}, \mathbf{3685},$ $\mathbf{3688 - 3692}, \mathbf{3694 - 3701}, \mathbf{3707}, \mathbf{3709},$ $3720_b, \mathbf{3711 - 3722}, \mathbf{3724 - 3730},$ $\mathbf{3732 - 3735}, \mathbf{3737 - 3740}, \mathbf{3742}, \mathbf{3743},$ $\mathbf{3745}, \mathbf{3747 - 3755}, 3783_a, \mathbf{3757 - 3825},$ $\mathbf{3827 - 3837}, \mathbf{3839 - 3916}$	**3839**	3917

Key to Table 6.1: $a \to$ (2.1), $b \to$ (2.2), $c \to$ (2.3), $r \to$ Bruck-Ryser Theorem, $s \to$ [6], $t \to$ Theorem 6.1

Table 6.1 (continue 6). Values of v for which a cyclic symmetric configuration v_k exists, $5 \leq k \leq 51$, $P(k) \leq v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \leq v \leq G(k)-1$	$E_c(k)$	$G(k)$
50	2451	2451.	$2451_a, \overline{2452}_t, 2756_c, 2808_b, 2863_a, 3422_c,$ $3480_b, 3541_a, 3660_c,$ **3685, 3688, 3692,** **3712, 3714, 3716,**3720_b,**3726, 3743,** **3745, 3749, 3750, 3752, 3753, 3758,** **3762, 3766, 3767, 3769, 3770, 3772,** **3775, 3779, 3780,**3783_a,**3782 − 3785,** **3788 − 3791, 3796, 3803, 3805, 3811,** **3813, 3817 − 3823, 3825,** **3828 − 3832, 3834 − 3840, 3842, 3843,** **3845, 3846, 3848 − 3869,**$4095_b, 4161_a,$ **3871 − 4188**	**3871**	4189
51	2551	$\geq 2552_s$	$2756_c, 2808_b, 2863_a, 3422_c, 3480_b, 3541_a,$ $3660_c, 3720_b, 3783_a,$ **3871, 3894, 3927,** **3938, 3986, 3998, 4004, 4018, 4032,** **4035, 4037, 4042, 4048 − 4050, 4053,** **4060, 4064 − 4066, 4068, 4072, 4076,** **4079 − 4083, 4085, 4087, 4090, 4091,** **4093,**4095_b,**4095 − 4100, 4102 − 4104,** **4107 − 4109, 4112 − 4120, 4124, 4125,** **4127 − 4131, 4133 − 4156,**$4161_a,$ **4158 − 4164, 4166 − 4196, 4199, 4200,** **4202 − 4206, 4208 − 4231, 4233 − 4380**	**4233**	4381

Key to Table 6.1: $a \rightarrow (2.1)$, $b \rightarrow (2.2)$, $c \rightarrow (2.3)$, $r \rightarrow$ Bruck-Ryser Theorem, $s \rightarrow [6]$, $t \rightarrow$ Theorem 6.1

Table 6.2. Upper bounds on the cyclic existence bound $E_c(k)$, $52 \leq k \leq 83$

k	$E_c(k)$	$G(k)$	k	$E_c(k)$	$G(k)$	k	$E_c(k)$	$G(k)$	k	$E_c(k)$	$G(k)$
52	**4359**	4541	60	**5687**	6039	68	**7463**	7913	76	**10023**	10179
53	**4463**	4695	61	**5994**	6269	69	**8111**	8291	77	**10229**	10409
54	**4513**	4747	62	**6150**	6431	70	**8125**	8435	78	**10395**	10599
55	**5195**	5197	63	**6611**	6783	71	**8288**	8661	79	**10800**	10817
56	**5341**	5451	64	**6796**	7055	72	**8694**	8947	80	**10977**	11127
57	**5501**	5547	65	**6853**	7187	73	**8813**	9027	81	**11396**	11435
58	**5551**	5703	66	**7279**	7515	74	**8965**	9507	82	**11443**	11629
59	**5612**	5823	67	**7359**	7639	75	**9883**	9965	83	**11593**	12041

8.3 Tables for Section 7

Table 7.1. The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k) - 1$	$E(k) \leq$	$G(k)$	filling
3.	7	7	7.	7	100%
4.	13	13	13.	13	100%
5.	21	$21_a, \overline{22}_t$	23.	23	100%
6.	31	$31_a, \overline{32}_t, \overline{33}, 34$	34.	35	100%
7	43	$\overline{43}_r, \overline{44}_t, 45, 48_{b \cdot f \cdot R}, 49_{m \cdot R}, 50_{m \cdot R}$	48	51	75%
8	57	$57_a, \overline{58}_t, 63_{b \cdot f \cdot R}, 64_{m \cdot R} - 68_{m \cdot R}$	63	69	67%
9	73	$73_a, \overline{74}_t, 78_{g \cdot h}, 80_{b \cdot f \cdot R}, 81_{m \cdot R} - 88_{m \cdot R}$	80	89	75%
10	91	$91_a, \overline{92}_t, 98_j, 107_y - 109_y, 110_{c \cdot f \cdot k \cdot m \cdot R \cdot S \cdot T}$	107	111	35%
11	111	$\overline{111}, \overline{112}, 120_{b \cdot f \cdot R}, 121_{m \cdot R} - 133_{m \cdot R}, 135_y - 142_y,$ $143_{m \cdot y \cdot S}, 144_{m \cdot y \cdot R \cdot S \cdot T}$	135	145	76%
12	133	$133_a, \overline{134}_t, 135, 156_{m \cdot R \cdot S \cdot T} - 169_{m \cdot R \cdot S \cdot T}, 170_{m \cdot R \cdot T \cdot X}$	156	171	47%
13	157	$\overline{158}_t, 168_{b \cdot f \cdot R}, 169_{m \cdot R} - 183_{m \cdot R}, 189_g, 193_y - 209_y,$ $210_{m \cdot y \cdot R} - 212_{m \cdot y \cdot R}$	193	213	68%
14	183	$183_a, \overline{184}_t, 210_g, 224_m, 225_{m \cdot y \cdot R} - 254_{m \cdot y \cdot R}$	224	255	47%
15	211	$\overline{211}_r, \overline{212}_t, 231_g, 240_{m \cdot R} - 266_{m \cdot R}, 267_{m \cdot y \cdot R} - 302_{m \cdot y \cdot R}$	240	303	72%
16	241	$252_{g \cdot h}, 255_{b \cdot f \cdot R}, 256_{m \cdot R} - 321_{m \cdot R}, \mathbf{322_{R \cdot T \cdot Z}},$ $323_{m \cdot R \cdot S} - 354_{m \cdot R \cdot S}$	**255**	355	89%
17	273	$273_a, \overline{274}_t, 288_{b \cdot f \cdot R}, 289_{m \cdot R} - 307_{m \cdot R}, 323_{m \cdot S}, 372_W,$ $324_{m \cdot R} - 381_{m \cdot R}, \mathbf{382_Z - 390_Z}, 391_{m \cdot S \cdot Z} - 398_{m \cdot S \cdot Z}$	**323**	399	78%
18	307	$307_a, 342_{m \cdot R} - 381_{m \cdot R}, \mathbf{401_Z}, 403_{g \cdot Z}, \mathbf{405_Z - 407_Z},$ $\mathbf{409_Z - 413_Z}, 414_{m \cdot S \cdot Z} - 432_{m \cdot S \cdot Z}$	**409**	433	55%
19	343	$\overline{344}_t, 360_{b \cdot f \cdot R}, 361_{m \cdot R} - 381_{m \cdot R}, 434_{g, W},$ $437_{m \cdot S} - 457_{m \cdot S}, \mathbf{458_{R \cdot T}}, \mathbf{459_{R \cdot T}},$ $465_W, 460_{m \cdot R \cdot T} - 492_{m \cdot R \cdot T}$	**437**	493	53%
20	381	$381_a, \overline{382}_t, 465_W, 460_{m \cdot S} - 481_{m \cdot S}, \mathbf{482_{R \cdot T}},$ $558_W, 483_{m \cdot R \cdot T} - 566_{m \cdot R \cdot T}$	**460**	567	59%
21	421	$\overline{422}_t, 483_{m \cdot S}, 558_W, 640_X, 484_{m \cdot R} - 666_{m \cdot R}$	483	667	75%

Key to Table 7.1: $a \to (2.1), b \to (2.2), c \to (2.3), f \to (2.5), g \to (2.6), h \to (2.7),$ $j \to (2.8), k \to (2.9), m \to (5.1), P \to (4.3), R \to (5.4), S \to (5.5), T \to (5.6), W \to$ Table 4.1, $X \to$ Table 4.2, $y \to$ Table 6.1 with $k \leq 15$, $Z \to$ Table 6.1 with $k > 15$, $r \to$ Bruck-Ryser Theorem, $t \to$ Theorem 6.1

Table 7.1 (continue 1). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k)-1$	$E(k) \leq$	$G(k)$	filling
22	463	$\overline{463}_r, \overline{464}_t, 558_W, 506_{m\cdot R} - 573_{m\cdot R}, \mathbf{574_R},$ $640_X, 575_{m\cdot R} - 712_{m\cdot R}$	**506**	713	84%
23	507	$\overline{507}_r, \overline{508}_t, 528_{b\cdot f\cdot R}, 529_{m\cdot R} - 553_{m\cdot R}, 558_{g\cdot W},$ $575_m, 576_{m\cdot R} - 744_{m\cdot R}$	575	745	84%
24	553	$553_a, \overline{554}_t, 589_g, 600_{m\cdot R} - 673_{m\cdot R}, \mathbf{674_R}, 675_{m\cdot R} - 850_{m\cdot R}$	**600**	851	85%
25	601	$620_{g\cdot h}, 624_{b\cdot f\cdot P\cdot R}, 625_{m\cdot R} - 651_{m\cdot R},$ $675_m, 906_W, 912_X, 938_W, 676_{m\cdot R} - 960_{m\cdot R}$	675	961	88%
26	651	$651_a, \overline{652}_t, 702_{m\cdot R} - 781_{m\cdot R}, \mathbf{782_R}, 783_{m\cdot R} - 984_{m\cdot R}$	**702**	985	85%
27	703	$728_{b\cdot f\cdot R}, 729_{m\cdot R} - 757_{m\cdot R}, 783_{m\cdot S},$ $784_{m\cdot R} - 1065_{m\cdot R}, \mathbf{1066_{R\cdot T\cdot Z} - 1072_{R\cdot T\cdot Z}},$ $1073_{m\cdot S\cdot Z} - 1103_{m\cdot S\cdot Z}, \mathbf{1104_{R\cdot T\cdot Z} - 1106_{R\cdot T\cdot Z}}$	**783**	1107	88%
28	757	$757_a, \overline{758}_t, 812_{m\cdot R} - 1065_{m\cdot R}, \mathbf{1066_T - 1072_T},$ $1073_{m\cdot S} - 1103_{m\cdot S}, \mathbf{1104_{R\cdot T\cdot Z} - 1109_{R\cdot T\cdot Z}},$ $1110_{m\cdot R\cdot S\cdot Z} - 1141_{m\cdot R\cdot S\cdot Z}, \mathbf{1142_{R\cdot T\cdot Z} - 1146_{R\cdot T\cdot Z}},$ $1147_{m\cdot R\cdot S\cdot Z} - 1170_{m\cdot R\cdot S\cdot Z}$	**812**	1171	87%
29	813	$\overline{814}_t, 840_{b\cdot f\cdot R}, 841_{m\cdot R} - 871_{m\cdot R}, 899_{m\cdot S},$ $900_{m\cdot R} - 1057_{m\cdot R}, 1073_{m\cdot S} - 1103_{m\cdot S},$ $\mathbf{1104_{R\cdot T} - 1109_{R\cdot T}}, 1110_{m\cdot S} - 1141_{m\cdot S},$ $\mathbf{1142_{R\cdot T} - 1146_{R\cdot T}}, 1147_{m\cdot S} - 1179_{m\cdot S},$ $\mathbf{1180_{R\cdot T\cdot Z} - 1183_{R\cdot T\cdot Z}}, 1184_{m\cdot S} - 1219_{m\cdot S},$ $\mathbf{1220_{R\cdot T\cdot Z}}, 1221_{m\cdot S} - 1246_{m\cdot S}$	**1073**	1247	84%
30	871	$871_a, \overline{872}_t, 930_{m\cdot R} - 1057_{m\cdot R}, 1110_{m\cdot S} - 1141_{m\cdot S},$ $\mathbf{1142_{R\cdot T} - 1146_{R\cdot T}}, 1147_{m\cdot S} - 1179_{m\cdot S},$ $\mathbf{1180_{R\cdot T} - 1183_{R\cdot T}}, 1184_{m\cdot S} - 1217_{m\cdot S},$ $\mathbf{1218_{R\cdot T} - 1220_{R\cdot T}}, 1262_W, 1221_{m\cdot S} - 1360_{m\cdot S}$	**1110**	1361	78%

Key to Table 7.1: $a \to$ (2.1), $b \to$ (2.2), $c \to$ (2.3), $f \to$ (2.5), $g \to$ (2.6), $h \to$ (2.7), $j \to$ (2.8), $k \to$ (2.9), $m \to$ (5.1), $P \to$ (4.3), $R \to$ (5.4), $S \to$ (5.5), $T \to$ (5.6), $W \to$ Table 4.1, $X \to$ Table 4.2, $y \to$ Table 6.1 with $k \leq 15$, $Z \to$ Table 6.1 with $k > 15$, $r \to$ Bruck-Ryser Theorem, $t \to$ Theorem 6.1

Table 7.1 (continue 2). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k) - 1$	$E(k) \leq$	$G(k)$	filling
31	931	$\overline{931}_r, \overline{932}_t, 960_{b \cdot f \cdot R}, 961_{m \cdot R} - 1057_{m \cdot R},$ $1147_{m \cdot S} - 1179_{m \cdot S}, \mathbf{1180_{R \cdot T}} - \mathbf{1183_{R \cdot T}},$ $1184_{m \cdot S} - 1217_{m \cdot S}, \mathbf{1218_{R \cdot T}} - \mathbf{1220_{R \cdot T}},$ $1221_{m \cdot S} - 1255_{m \cdot S}, \mathbf{1256_{R \cdot T}}, \mathbf{1257_{R \cdot T}},$ $1262_W, 1258_{m \cdot S \cdot R \cdot T} - 1494_{m \cdot S \cdot R \cdot T}$	**1147**	1495	68%
32	993	$993_a, \overline{994}_t, 1023_{b \cdot f \cdot R}, 1024_{m \cdot R} - 1057_{m \cdot R},$ $1184_{m \cdot S} - 1217_{m \cdot S}, \mathbf{1218_{R \cdot T}} - \mathbf{1220_{R \cdot T}},$ $1221_{m \cdot S} - 1255_{m \cdot S}, \mathbf{1256_{R \cdot T}}, \mathbf{1257_{R \cdot T}},$ $1258_{m \cdot S \cdot R \cdot T} - 1293_{m \cdot S \cdot R \cdot T}, \mathbf{1294_{R \cdot T}},$ $1533_W, 1295_{m \cdot S \cdot R \cdot T} - 1568_{m \cdot S \cdot R \cdot T}$	**1184**	1569	73%
33	1057	$1057_a, \overline{1058}_t, 1221_{m \cdot S} - 1255_{m \cdot S}, \mathbf{1256_{R \cdot T}},$ $\mathbf{1257_{R \cdot T}}, 1258_{m \cdot S \cdot R \cdot T} - 1293_{m \cdot S \cdot R \cdot T}, \mathbf{1294_{R \cdot T}},$ $1634_W, 1295_{m \cdot R} - 1718_{m \cdot R}$	**1221**	1719	76%
34	1123	$\overline{1123}_r, \overline{1124}_t, 1258_{m \cdot S} - 1293_{m \cdot S}, \mathbf{1294_{R \cdot T}},$ $1295_{m \cdot R} - 1429_{m \cdot R}, \mathbf{1430_{R \cdot T}} - \mathbf{1434_{R \cdot T}},$ $1634_W, 1800_X, 1435_{m \cdot S} - 1876_{m \cdot S}$	**1258**	1877	82%
35	1191	$\overline{1192}_t, 1295_{m \cdot S}, 1296_{m \cdot R} - 1407_{m \cdot R},$ $1435_{m \cdot S} - 1471_{m \cdot S}, \mathbf{1472_{R \cdot T}} - \mathbf{1475_{R \cdot T}},$ $1800_P, 1476_{m \cdot S} - 1974_{m \cdot S}$	**1435**	1975	83%
36	1261	$1332_{m \cdot R} - 1407_{m \cdot R}, 1476_{m \cdot S} - 1513_{m \cdot S},$ $\mathbf{1514_{R \cdot T}} - \mathbf{1516_{R \cdot T}}, 1517_{m \cdot S} - 1519_{m \cdot S},$ $2000_X, 1520_{m \cdot R \cdot T} - 2010_{m \cdot R \cdot T}$	**1476**	2011	81%
37	1333	$\overline{1334}_t, 1368_{b \cdot f \cdot R}, 1369_{m \cdot R} - 1407_{m \cdot R},$ $1517_{m \cdot S} - 1555_{m \cdot S}, \mathbf{1556_{R \cdot T}} - \mathbf{1557_{R \cdot T}},$ $1558_{m \cdot R} - 2198_{m \cdot R}$	**1517**	2199	84%
38	1407	$1407_a, 1558_{m \cdot S}, 1559_{m \cdot S}, 1560_{m \cdot R \cdot S \cdot T} - 1597_{m \cdot R \cdot S \cdot T},$ $\mathbf{1598_{R \cdot T}}, 1599_{m \cdot R \cdot T} - 1761_{m \cdot R \cdot T}, \mathbf{1762_{R \cdot T}},$ $1763_{m,R} - 2292_{m,R}$	**1558**	2293	83%

Key to Table 7.1: $a \to$ (2.1), $b \to$ (2.2), $c \to$ (2.3), $f \to$ (2.5), $g \to$ (2.6), $h \to$ (2.7), $j \to$ (2.8), $k \to$ (2.9), $m \to$ (5.1), $P \to$ (4.3), $R \to$ (5.4), $S \to$ (5.5), $T \to$ (5.6), $W \to$ Table 4.1, $X \to$ Table 4.2, $y \to$ Table 6.1 with $k \leq 15$, $Z \to$ Table 6.1 with $k > 15$, $r \to$ Bruck-Ryser Theorem, $t \to$ Theorem 6.1

Table 7.1 (continue 3). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k) - 1$	$E(k)$ $\leq$	$G(k)$	filling
39	1483	$\overline{1483}_r, \overline{1484}_t, 1599_{m\cdot S}, 1600_{m\cdot R\cdot T} - 1761_{m\cdot R\cdot T},$ $\mathbf{1762_{R\cdot T}}, 1763_{m\cdot R} - 2504_{m\cdot R}$	**1599**	2505	89%
40	1561	$\overline{1562}_t, 1640_{m\cdot R} - 1761_{m\cdot R}, \mathbf{1762 \ _{R\cdot T}},$ $1763_{m\cdot R} - 1921_{m\cdot R}, \mathbf{1922_{R\cdot T}} - \mathbf{1926_{R\cdot T}},$ $1927_m - 1931_m, 1932_{m\cdot R} - 2564_{m\cdot R}$	**1640**	2565	92%
41	1641	$\overline{1642}_t, 1680_{b\cdot R}, 1681_{m\cdot R} - 1723_{m\cdot R}, 1763_m,$ $1764_{m\cdot R} - 1893_{m\cdot R}, 1927_{m\cdot S} - 1969_{m\cdot S},$ $\mathbf{1970_{R\cdot T}} - \mathbf{1973_{R\cdot T}}, 1974_m - 2610_m$	**1927**	2611	89%
42	1723	$1723_a, \overline{1724}_t, 1806_{m\cdot R} - 1893_{m\cdot R}, 1974_{m\cdot S} - 2017_{m\cdot S},$ $\mathbf{2018_{R\cdot T}} - \mathbf{2020_{R\cdot T}}, 2021_m - 2794_m$	**1974**	2795	85%
43	1807	$\overline{1807}_r, \overline{1808}_t, 1848_{b\cdot R}, 1849_{m\cdot R} - 1893_{m\cdot R},$ $2021_{m\cdot S} - 2065_{m\cdot S}, \mathbf{2066_{R\cdot T}} - \mathbf{2067_{R\cdot T}},$ $2068_{m\cdot R} - 2485_{m\cdot R}, \mathbf{2486_{R\cdot T}} - \mathbf{2490_{R\cdot T}},$ $2491_{m\cdot S} - 2593_{m\cdot S}, \mathbf{2594_{R\cdot T}} - \mathbf{2595_{R\cdot T}},,$ $2596_{m\cdot S} - 3014_{m\cdot S}$	**2021**	3015	86%
44	1893	$1893_a, \overline{1894}_t, 2068_{m\cdot S} - 2113_{m\cdot S}, \ \mathbf{2114_{R\cdot T}},$ $2115 - 2301m, \mathbf{2302_R}, 2303 - 2485m,$ $\mathbf{2486_{R\cdot T}} - \mathbf{2490_{R\cdot T}}, 2491_{m\cdot S} - 2539_{m\cdot S},$ $\mathbf{2540_{R\cdot T}} - \mathbf{2543_{R\cdot T}}, 2544_{m\cdot S} - 2593_{m\cdot S},$ $\mathbf{2594_{R\cdot T}} - \mathbf{2595_{R\cdot T}}, 2596_{m\cdot S} - 2647_{m\cdot S},$ $\mathbf{2648_{R\cdot T}} - \mathbf{2649_{R\cdot T}}, 2650_{m\cdot S} - 3192_{m\cdot S}$	**2068**	3193	87%
45	1981	$\overline{1982}_t, 2115_{m\cdot S}, 2116_{m\cdot R} - 2301_{m\cdot R},$ $\mathbf{2302_R}, 2303_{m\cdot R} - 2485_{m\cdot R}, \ \mathbf{2486_{R\cdot T}} - \mathbf{2490_{R\cdot T}},$ $2491_{m\cdot S} - 2539_{m\cdot S}, \mathbf{2540_{R\cdot T}} - \mathbf{2543_{R\cdot T}},$ $2544_{m\cdot S} - 2593_{m\cdot S}, \mathbf{2594_{R\cdot T}} - \mathbf{2596_{R\cdot T}},$ $2597_{m\cdot S} - 2647_{m\cdot S}, \mathbf{2648_{R\cdot T}} - \mathbf{2649_{R\cdot T}},$ $2650_{m\cdot R\cdot S\cdot T} - 2701_{m\cdot R\cdot S\cdot T}, \mathbf{2702_{R\cdot T}},$ $2703_{m\cdot R\cdot S\cdot T} - 3374_{m\cdot R\cdot S\cdot T}$	**2115**	3375	91%

Key to Table 7.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $f \to (2.5)$, $g \to (2.6)$, $h \to (2.7)$, $j \to (2.8)$, $k \to (2.9)$, $m \to (5.1)$, $P \to (4.3)$, $R \to (5.4)$, $S \to (5.5)$, $T \to (5.6)$, $W \to$ Table 4.1, $X \to$ Table 4.2, $y \to$ Table 6.1 with $k \leq 15$, $Z \to$ Table 6.1 with $k > 15$, $r \to$ Bruck-Ryser Theorem, $t \to$ Theorem 6.1

Table 7.1 (continue 4). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k)-1$	$E(k) \leq$	$G(k)$	filling
46	2071	$\overline{2072}_t, 2162_{m\cdot R} - 2301_{m\cdot R}, \mathbf{2302_R}, 2303_{m\cdot R} - 2485_{m\cdot R},$ $\mathbf{2486_{R\cdot T}} - \mathbf{2490_{R\cdot T}}, 2491_{m\cdot S} - 2539_{m\cdot S},$ $\mathbf{2540_{R\cdot T}} - \mathbf{2543_{R\cdot T}}, 2544_{m\cdot S} - 2593_{m\cdot S},$ $\mathbf{2594_{R\cdot T}} - \mathbf{2596_{R\cdot T}}, 2597_{m\cdot S} - 2647_{m\cdot S},$ $\mathbf{2648_{R\cdot T}} - \mathbf{2649_{R\cdot T}}, 2650_{m\cdot R\cdot S\cdot T} - 2701_{m\cdot R\cdot S\cdot T},$ $\mathbf{2702_{R\cdot T}}, 2703_{m\cdot R\cdot S\cdot T} - 3446_{m\cdot R\cdot S\cdot T}$	**2162**	3407	93%
47	2163	$\overline{2163}_r, \overline{2164}_t, 2208_{b\cdot R}, 2209_{m\cdot R} - 2257_{m\cdot R},$ $2303_m, 2304_{m\cdot R} - 2451_{m\cdot R}, 2491_{m\cdot S} - 2539_{m\cdot S},$ $\mathbf{2540_{R\cdot T}} - \mathbf{2543_{R\cdot T}}, 2544_{m\cdot S} - 2593_{m\cdot S},$ $\mathbf{2594_{R\cdot T}} - \mathbf{2596_{R\cdot T}}, 2597_{m\cdot S} - 2647_{m\cdot S},$ $\mathbf{2648_{R\cdot T}} - \mathbf{2649_{R\cdot T}}, 2650_{m\cdot R\cdot S} - 2701_{m\cdot R\cdot S},$ $\mathbf{2702_{R\cdot T}}, 2703_{m\cdot R} - 3608_{m\cdot R}$	**2303**	3609	94%
48	2257	$2257_a, \overline{2258}_t, 2352_{m\cdot R} - 2451_{m\cdot R}, 2544_{m\cdot S} - 2593_{m\cdot S},$ $\mathbf{2594_{R\cdot T}} - \mathbf{2596_{R\cdot T}}, 2597_{m\cdot S} - 2647_{m\cdot S},$ $\mathbf{2648_{R\cdot T}} - \mathbf{2649_{R\cdot T}}, 2650_{m\cdot R\cdot S\cdot T} - 2701_{m\cdot R\cdot S\cdot T},$ $\mathbf{2702_{R\cdot T}}, 2703_{m\cdot R} - 2881_{m\cdot R}, \mathbf{2882_{R\cdot T}} - \mathbf{2890_{R\cdot T}},$ $2891_{m\cdot S} - 3774_{m\cdot S}$	**2544**	3775	88%
49	2353	$2400_{b\cdot R}, 2401_{m\cdot R} - 2451_{m\cdot R}, 2597_{m\cdot S} - 2647_{m\cdot S},$ $\mathbf{2648_{R\cdot T}} - \mathbf{2649_{R\cdot T}}, 2650_{m\cdot R\cdot S\cdot T} - 2701_{m\cdot R\cdot S\cdot T},$ $\mathbf{2702_{R\cdot T}}, 2703_{m\cdot R} - 2863_{m\cdot R},$ $2891_{m\cdot S} - 2941_{m\cdot S}, \mathbf{2942_{R\cdot T}} - \mathbf{2949_{R\cdot T}},$ $2950_{m\cdot S} - 3916_{m\cdot S}$	**2891**	3917	86%
50	2451	$2451_a, \overline{2452}_t, 2650_{m\cdot S} - 2701_{m\cdot S}, \mathbf{2702_{R\cdot T}},$ $2703_{m\cdot R} - 2863_{m\cdot R}, 2950_{m\cdot S} - 3001_{m\cdot S},$ $\mathbf{3002_{R\cdot T}} - \mathbf{3008_{R\cdot T}}, 3009_{m\cdot S} - 4188_{m\cdot S}$	**2950**	4189	84%
51	2551	$2703_{m\cdot S}, 2704_{m\cdot R} - 2863_{m\cdot R}, 3009_{m\cdot S} - 3061_{m\cdot S},$ $\mathbf{3062_{R\cdot T}} - \mathbf{3067_{R\cdot T}}, 3068_{m\cdot S} - 4380_{m\cdot S}$	**3009**	4381	84%

Key to Table 7.1: $a \rightarrow (2.1)$, $b \rightarrow (2.2)$, $c \rightarrow (2.3)$, $f \rightarrow (2.5)$, $g \rightarrow (2.6)$, $h \rightarrow (2.7)$, $j \rightarrow (2.8)$, $k \rightarrow (2.9)$, $m \rightarrow (5.1)$, $P \rightarrow (4.3)$, $R \rightarrow (5.4)$, $S \rightarrow (5.5)$, $T \rightarrow (5.6)$, $W \rightarrow$ Table 4.1, $X \rightarrow$ Table 4.2, $y \rightarrow$ Table 6.1 with $k \leq 15$, $Z \rightarrow$ Table 6.1 with $k > 15$, $r \rightarrow$ Bruck-Ryser Theorem, $t \rightarrow$ Theorem 6.1

Table 7.2. New parameters of symmetric configurations v_k obtained in [13], $52 \leq k \leq 64$

k	$P(k)$	$P(k) \leq v \leq G(k) - 1$	$E(k) \leq$	$G(k)$	filling
52	2653	$3122_{RT} - 3126_{RT}$	**3068**	4541	84%
53	2757	$3182_{RT} - 3185_{RT}$	**3127**	4695	84%
54	2863	$3242_{RT} - 3244_{RT}$	**3186**	4747	83%
55	2971	$3302_{RT} - 3303_{RT}$	**3245**	5197	88%
56	3081	3362_{RT}	**3304**	5451	91%
57	3193	3598_{RT}	**3363**	5547	93%
58	3307	$3598_{RT}, 3837_{R} - 3839_{R}$	**3422**	5703	95%
59	3423	$3837_{R} - 3839_{R}, 3902_{R} - 3903_{R}$	**3599**	5823	95%
60	3541	$3902_{R} - 3903_{R}, 3967_{R}, 4218_{RT} - 4220_{RT}$	**3840**	6039	93%
61	3661	$3967_{R}, 4218_{RT} - 4220_{RT}, 4286_{RT} - 4287_{RT}$	**3904**	6269	93%
62	3783	$4218_{RT} - 4220_{RT}, 4286_{RT} - 4287_{RT}, 4354_{RT}$	**3968**	6431	93%
63	3907	$4286_{RT} - 4287_{RT}, 4354_{RT}$	**4221**	6783	94%
64	4033	$4354_{RT}, 4610_{RT} - 4614_{RT}$	**4288**	7055	94%

Key to Table 7.2: $R \rightarrow (5.4), T \rightarrow (5.6)$

Table 7.3. The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic). Constructions are not remarked.

k	$P(k)$	$P(k) \le v \le G(k) - 1$	$E(k) \le$	$G(k)$	filling
3.	7	7	7.	7	100%
4.	13	13	13.	13	100%
5.	21	$21, \overline{22}$	23.	23	100%
6.	31	$31, \overline{32}, \overline{33}, 34$	34.	35	100%
7	43	$\overline{43}, \overline{44}, 45, 48, 49, 50$	48	51	75%
8	57	$57, \overline{58}, 63 - 68$	63	69	67%
9	73	$73, \overline{74}, 78, 80 - 88$	80	89	75%
10	91	$91, \overline{92}, 98, 107 - 110$	107	111	35%
11	111	$\overline{111}, \overline{112}, 120 - 133, 135 - 144$	135	145	76%
12	133	$133, \overline{134}, 135, 156 - 170$	156	171	47%
13	157	$\overline{158}, 168 - 183, 189, 193 - 212$	193	213	68%
14	183	$183, \overline{184}, 210, 224 - 254$	224	255	47%
15	211	$\overline{211}, \overline{212}, 231, 240 - 302$	240	303	72%
16	241	$252, 255 - 354$	**255**	355	89%
17	273	$273, \overline{274}, 288 - 307, 323 - 398$	**323**	399	78%
18	307	$307, 342 - 381, 401, 403, 405 - 407, 409 - 432$	**409**	433	55%
19	343	$\overline{344}, 360 - 381, 434, 437 - 492$	**437**	493	53%
20	381	$381, \overline{382}, 460 - 566$	**460**	567	59%
21	421	$\overline{422}, 483 - 666$	483	667	75%
22	463	$\overline{463}, \overline{464}, 506 - 712$	**506**	713	84%
23	507	$\overline{507}, \overline{508}, 528 - 553_{m \cdot R}, 558, 575 - 744$	575	745	84%
24	553	$553, \overline{554}, 589, 600 - 850$	**600**	851	85%
25	601	$620, 624 - 651, 675 - 960$	675	961	88%
26	651	$651, \overline{652}, 702 - 984$	**702**	985	85%
27	703	$728 - 757, 783 - 1106$	**783**	1107	88%
28	757	$757, \overline{758}, 812 - 1170$	**812**	1171	87%
29	813	$\overline{814}, 840 - 871, 899 - 1057, 1073 - 1246$	**1073**	1247	84%
30	871	$871, \overline{872}, 930 - 1057, 1110 - 1360$	**1110**	1361	78%
31	931	$\overline{931}, \overline{932}, 960 - 1057, 1147 - 1494$	**1147**	1495	68%
32	993	$993, \overline{994}, 1023 - 1057, 1184 - 1568$	**1184**	1569	73%

Table 7.3 (continue)**.** The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic). Constructions are not remarked.

k	$P(k)$	$P(k) \leq v \leq G(k)-1$	$E(k) \leq$	$G(k)$	filling
33	1057	$1057, \overline{1058}, 1221 - 1718$	**1221**	1719	76%
34	1123	$\overline{1123}, \overline{1124}, 1258 - 1876$	**1258**	1877	82%
35	1191	$\overline{1192}, 1295 - 1407, 1435 - 1974$	**1435**	1975	83%
36	1261	$1332 - 1407, 1476 - 2010$	**1476**	2011	81%
37	1333	$\overline{1334}, 1368 - 1407, 1517 - 2198$	**1517**	2199	84%
38	1407	$1407, 1558 - 2292$	**1558**	2293	83%
39	1483	$\overline{1483}, \overline{1484}, 1599 - 2504$	**1599**	2505	89%
40	1561	$\overline{1562}, 1640 - 2564$	**1640**	2565	92%
41	1641	$\overline{1642}, 1680 - 1723, 1763 - 1893, 1927 - 2610$	**1927**	2611	89%
42	1723	$1723, \overline{1724}, 1806 - 1893, 1974 - 2794$	**1974**	2795	85%
43	1807	$\overline{1807}, \overline{1808}, 1848 - 1893, 2021_{m \cdot S} - 3014$	**2021**	3015	86%
44	1893	$1893, \overline{1894}, 2068 - 3192$	**2068**	3193	87%
45	1981	$\overline{1982}, 2115 - 3374$	**2115**	3375	91%
46	2071	$\overline{2072}, 2162 - 3446$	**2162**	3407	93%
47	2163	$\overline{2163}, \overline{2164}, 2208 - 2257, 2303 - 3608$	**2303**	3609	94%
48	2257	$2257, \overline{2258}, 2352 - 2451, 2544 - 3774$	**2544**	3775	88%
49	2353	$2400 - 2451, 2597 - 2863, 2891 - 3916$	**2891**	3917	86%
50	2451	$2451, \overline{2452}, 2650 - 2863, 2950 - 4188$	**2950**	4189	84%
51	2551	$2703 - 2863, 3009 - 4380$	**3009**	4381	84%
52	2653	$\overline{2654}, 2756 - 2863, 3068 - 4540$	**3068**	4541	84%
53	2757	$\overline{2758}, 2808 - 2863, 3127 - 4694$	**3127**	4695	84%
54	2863	$2863, \overline{2864}, 3186 - 4746$	**3186**	4747	83%
55	2971	$\overline{2971}, \overline{2972}, 3245 - 5196$	**3245**	5197	88%
56	3081	$\overline{3082}, 3304 - 5450$	**3304**	5451	91%
57	3193	$\overline{3194}, 3363 - 5546$	**3363**	5547	93%
58	3307	$\overline{3307}, \overline{3308}, 3422 - 5702$	**3422**	5703	95%
59	3423	$\overline{3424}, 3480 - 3541, 3599 - 5822$	**3599**	5823	95%
60	3541	$3541, \overline{3542}, 3660 - 3783, 3840 - 6038$	**3840**	6039	93%
61	3661	$\overline{3662}, 3720 - 3783, 3904 - 6268$	**3904**	6269	93%
62	3783	$3783, 3968 - 6430$	**3968**	6431	93%
63	3907	$\overline{3907}, \overline{3908}, 4032 - 4161, 4221 - 6782$	**4221**	6783	94%
64	4033	$4095 - 4161, 4288 - 7054$	**4288**	7055	94%