Tables of parameters of symmetric configurations v_k*

Alexander A. Davydov
Institute for Information Transmission Problems (Kharkevich institute), Russian Academy of Sciences, Bol'shoi Karetnyi per. 19, GSP-4, Moscow, 127994, Russian Federation, e-mail: adav@iitp.ru

Giorgio Faina, Massimo Giulietti, Stefano Marcugini and Fernanda Pambianco
Dipartimento di Matematica e Informatica, Università degli Studi di Perugia, Via Vanvitelli 1, Perugia, 06123, Italy
e-mail: {faina, giuliet, gino, fernanda} @dmi.unipg.it

Abstract

Tables of the currently known parameters of symmetric configurations are given. Formulas for parameters of the known infinite families of symmetric configurations are presented as well. The results of the recent paper [18] are used. This work can be viewed as an appendix to [18], in the sense that the tables given here cover a much larger set of parameters.

Keywords: Configurations in Combinatorics, symmetric configurations, cyclic configurations, Golomb rulers, projective geometry, LDPC codes
Mathematics Subject Classification (2010). 05C10, 05B25, 94B05

1 Introduction

Configurations as combinatorial structures were defined in 1876. For an introduction to the problems connected with configurations, see [35–37] and the references therein.

Definition 1.1. [36]
(i) A configuration (v_r, b_k) is an incidence structure of v points and b lines such that each line contains k points, each point lies on r lines, and two distinct points are connected by *at most* one line.

*This research was supported by the Italian Ministry MIUR, Geometrie di Galois e strutture di incidenza, PRIN 2009–2010 and by the INdAM group G.N.S.A.G.A.

(ii) If $v = b$ and, hence, $r = k$, the configuration is *symmetric*, and it is referred to as a configuration v_k.

(iii) The *deficiency* d of a configuration (v_r, b_k) is the value $d = v - r(k-1) - 1$.

A symmetric configuration v_k is *cyclic* if there exists a permutation of the set of its points mappings blocks to blocks, and acting regularly on both points and blocks. Equivalently, v_k is cyclic if one of its incidence matrix is circulant.

Steiner systems are configurations with $d = 0$ [36]. The *deficiency* of a symmetric configuration v_k is $d = v - (k^2 - k + 1)$. The deficiency of v_k is zero if and only if v_k is a finite projective plane of order $k - 1$.

A configuration (v_r, b_k) can be viewed as a k-uniform r -regular linear hypergraph with v vertices and b hyperedges [34,36]. Connections of configurations (v_r, b_k) with numerical semigroups are noted in [14,60]. Some analogies between configurations (v_r, b_k), regular graphs, and molecule models of chemical elements are remarked in [32]. As an example of a practical application of configurations (both symmetric and non-symmetric), we mention also the problem of user privacy for using database; see [22,59] and the references therein.

Denote by $\mathbf{M}(v,k)$ an incidence matrix of a symmetric configuration v_k. Any matrix $\mathbf{M}(v,k)$ is a $v \times v$ 01-matrix with k ones in every row and column; moreover, the 2×2 matrix consisting of all ones is not a submatrix of $\mathbf{M}(v,k)$. Two incidence matrices of the same configuration may differ by a permutation on the rows and the columns.

A matrix $\mathbf{M}(v,k)$ can be considered as a biadjacency matrix of the *Levi graph* of the configuration v_k which is a k-regular bipartite graph without multiple edges [36, Sec. 7.2]. Clearly, the graph has girth at least six, i.e. it does not contain 4-cycles. Such graphs are useful for the construction of bipartite-graph codes that can be treated as *low-density parity-check* (LDPC) *codes*. If $\mathbf{M}(v,k)$ is *circulant*, then the corresponding LDPC code is *quasi-cyclic*; it can be encoded with the help of shift-registers with relatively small complexity; see [5,6,19,21,28,40,41,45] and the references therein.

Matrices $\mathbf{M}(v,k)$ consisting of square circulant submatrices have a number of useful properties, e.g. they are more suitable for LDPC codes implementation. We say that a 01-matrix $\mathbf{A}$ is *block double-circulant* (BDC for short) if $\mathbf{A}$ consists of square circulant blocks whose weights give rise to a circulant matrix (see Definition 3.1). A configuration v_k with a BDC incidence matrix $\mathbf{M}(v,k)$ is called a *BDC symmetric configuration.* Symmetric and non-symmetric configurations with incidence matrices consisting of square circulant blocks are considered, for example, in [3,4,19–21,40,41,52]. In [3,4], BDC symmetric configuration are considered in connection with $\mathbb{Z}_\mu$-schemes (see Remarks 3.2 and 3.3 in Section 3).

Cyclic configurations are considered, for instance, in [3,4,19–21,26,31,40,41,47,50]. A standard method to construct cyclic configurations (or, equivalently, circulant matrices $\mathbf{M}(v,k)$) is based on *Golomb rulers* [23,26,31,54–56].

Definition 1.2. [26,54]

(i) A *Golomb ruler* G_k of *order* k is an ordered set of k integers $(a_1, a_2, \ldots, a_k)$ such that $0 \le a_1 < a_2 < \ldots < a_k$ and all the differences $\{a_i - a_j \,|\, 1 \le j < i \le k\}$ are distinct.

The *length* $L_{\mathrm{G}}(k)$ of the ruler G_k is equal to $a_k - a_1$. Let $L_{\overline{\mathrm{G}}}(k)$ be the length of the *shortest known* Golomb ruler $\overline{\mathrm{G}}_k$.

(ii) A (v,k) *modular Golomb ruler* is an ordered set of k integers $(a_1, a_2, \ldots, a_k)$ such that $0 \le a_1 < a_2 < \ldots < a_k$ and all the differences $\{a_i - a_j \,|\, 1 \le i,j \le k;\ i \ne j\}$ are distinct and nonzero modulo v.

For any $\delta \ge 0$, Golomb rulers $(a_1, a_2, \ldots, a_k)$ and $(a_1 + \delta, a_2 + \delta, \ldots, a_k + \delta)$ have the same properties. Usually, $a_1 = 0$ is assumed. We say that a 0,1-*vector* $\mathbf{u} = (u_0, u_{1,}, \ldots, u_{v-1})$ *corresponds* to a (modular) Golomb ruler if the increasing sequence of integers $j \in \{0, 1, \ldots, v-1\}$ such that $u_j = 1$ form a (modular) Golomb ruler.

Recall that *weight* of a *circulant* 0, 1-*matrix* is the number of ones in each its row.

Theorem 1.3. [31, Sec. 4], [48]

(i) *Any Golomb ruler* G_k *of length* $L_{\mathrm{G}}(k)$ *is a* (v,k) *modular Golomb ruler for all* v *such that* $v \ge 2L_{\mathrm{G}}(k) + 1$.

(ii) *A circulant* $v \times v$ 0,1-*matrix of weight* k *is an incidence matrix* $\mathbf{M}(v,k)$ *of a cyclic symmetric configuration* v_k *if and only if the first row of the matrix corresponds to a* (v,k) *modular Golomb ruler.*

(iii) *For all* v *such that* $v \ge 2L_{\overline{\mathrm{G}}}(k)+1$, *there exists a cyclic symmetric configuration* v_k.

We call the value $G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1$ the *Golomb bound*. On the other hand, we call $P(k) = k^2 - k + 1$ the *projective plane bound*.

Let $v_\delta(k)$ be the smallest possible value of v for which a (v,k) modular Golomb ruler (or, equivalently, a cyclic symmetric configuration) exists.

In [18], two bounds are considered. The *existence bound* $E(k)$ is the least integer such that for any $v \ge E(k)$, there exists a symmetric configuration v_k. Similarly, the *cyclic existence bound* $E_c(k)$ is the least integer such that for any $v \ge E_c(k)$, there exists a cyclic v_k. Clearly, for a fixed k, we have

$$k^2 - k + 1 \;=\; P(k) \le E(k) \le E_c(k) \le G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1. \tag{1.1}$$

$$k^2 - k + 1 \;=\; P(k) \le v_\delta(k) \le E_c(k) \le G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1. \tag{1.2}$$

The aim of this work is to give tables of the currently known parameters of symmetric configurations v_k, including those arising from the recent work [18]. We consider the *spectrum* of possible parameters of v_k (with special attention to parameters of cyclic symmetric configurations) in the interval

$$k^2 - k + 1 = P(k) \le v < G(k) = 2L_{\overline{\mathrm{G}}}(k) + 1. \tag{1.3}$$

Also, we pay attention to parameters of circulant and block double-circulant incidence matrices $\mathbf{M}(v,k)$. Some upper bounds on $E(k)$ and $E_c(k)$ are pointed out.

From the stand point of applications, including Coding Theory, it is sometimes useful to have different matrices $\mathbf{M}(v,k)$ for the same v and k. This is why we remark situations when *different constructions* provide configurations with *the same parameters*.

The *Generalized Martinetti Construction* (Construction GM) proposed in [25] plays a key role for the investigation of the spectrum of possible parameters of symmetric configurations as it provides, for a fixed k, intervals of values of v for which a v_k exists. Construction GM has been considered also in [3, 5, 6][1] To be successfully applied, Construction GM needs a convenient starting incidence matrix. To this end, BDC matrices turn out to be particularly useful. In this work new starting matrices proposed in [18] are considered as well as those originally proposed in [3, 6].

We remark that new cyclic configurations provide new modular Golomb rulers, i.e. new deficient cyclic difference sets.

The work is organized as follows. In Section 2, we briefly summarize some constructions and parameters of configurations v_k. Preliminaries on BDC matrices are given in Section 3. In Section 4, parameters of block double-circulant incidence matrices $\mathbf{M}(v, k)$ are reported, according to some results from [18]. In Section 5, parameters of configurations v_k obtained by the Construction GM from the starting matrices proposed in [3,6,18] are given. In Sections 6 and 7, results on the spectra of parameters of cyclic and noncyclic configurations are reported. Finally, Section 8 contains the tables of parameters of symmetric configurations which are the main object of the paper. In particular, tables of parameters of BDC configurations v_k based on projective planes and punctured affine planes are given, as well as tables of values v for which a cyclic symmetric configuration v_k exists. Finally, aggregated tables on the existence of symmetric configurations are given. In the tables, the new parameters obtained from [18] are written in bold font.

2 Some known constructions and parameters of configurations v_k with $P(k) \le v < G(k)$

The aim of this section is to provide a list of pairs (v, k) for which a (cyclic) symmetric configuration v_k is known to exist, see Equations (2.1)–(2.16). Infinite families of configurations v_k given in this section are considered in [1–8, 10, 13, 17, 19–21, 25–27, 29, 31–37, 47, 50, 53, 54, 58]; see also the references therein.

Throughout the work, q is a prime power and p is a prime. Let F_q be Galois field of q elements. Let $F_q^* = F_q \setminus \{0\}$. Let $\mathbf{0}_u$ be the zero $u \times u$ matrix. Denote by $\mathbf{P}_u$ a permutation matrix of order u.

We recall that several pairs $(v, k - \delta)$ can be actually obtained from a given v_k; it is a basic result on symmetric configurations.

Theorem 2.1. [3, Sec. 2], [34, Sec. 5.2], [37, Sec. 2.5] [50] *If a (cyclic) configuration v_k exists, then for each δ with $0 \le \delta < k$ there exists a (cyclic) configuration $v_{k-\delta}$ as well.*

We note that from a cyclic configuration v_k, a cyclic configurations $v_{k-\delta}$ can be obtained by dismissing δ ones in the 1-st row of its incidence matrix. For the general case,

[1] The authors of the papers [5,6] (represented here by Davydov) regret that the paper [25] is not cited in [5,6]; the reason is that, unfortunately, the authors did not know the paper [25] during the preparation of [5,6].

Theorem 2.1 is based on the fact that an incidence matrix $\mathbf{M}(v,k)$ can be represented as a sum of k permutations $v \times v$ matrices (in different ways). This fact follows from the results of Steinits (1894) and König (1914), see e.g. [34, Sec. 5.2] and [37, Sec. 2.5].

The value δ appearing in Equations (2.1)–(2.16) is connected with Theorem 2.1. When a reference is given, it usually refers to the case $\delta = 0$.

The families giving rise to pairs (2.1)–(2.3) below are obtained from (v,k) modular Golomb rulers [23, Ch. 5], [24], [31, Sec. 5], [54, Sec. 19.3]; see Theorem 1.3(ii).

$$\text{cyclic } v_k \ : \ v = q^2+q+1, \ \ k = q+1-\delta, \ \ q+1 > \delta \geq 0; \tag{2.1}$$
$$\text{cyclic } v_k \ : \ v = q^2-1, \ \ k = q-\delta, \ \ q > \delta \geq 0; \tag{2.2}$$
$$\text{cyclic } v_k \ : \ v = p^2-p, \ \ k = p-1-\delta, \ \ p-1 > \delta \geq 0. \tag{2.3}$$

The configurations giving rise to (2.1) use the incidence matrix of the cyclic projective plane $PG(2,q)$ [23, Sec. 5.5], [24], [54, Th. 19.15], [58]. The family with parameters (2.2) is obtained from the *cyclic punctured affine plane* $AG(2,q)$ [13], [23, Sec. 5.6], [24,26], [54, Th. 19.17]; see also [21, Ex. 5] and [27] where the configurations are called *anti-flags*. We recall that the punctured plane $AG(2,q)$ is the affine plane without the origin and the lines through the origin. Punctured affine planes are also called elliptic (Desarguesian) semiplanes of type L. Finally, the configurations with parameters (2.3) follow from Ruzsa's construction [23, Sec. 5.4], [24,53], [54, Th. 19.19].

The non-cyclic families with parameters (2.4) and (2.5) are given in [1, Constructions (i),(ii), p. 126] and [29, Constructions 3.2,3.3, Rem. 3.5]; see also the references therein and [5], [6, Sec. 3], [21, Sec. 7.3], [27,31,50].

$$v_k \ : \ v = q^2-qs, \ \ k = q-s-\delta, \ \ q > s \geq 0, \ \ q-s > \delta \geq 0; \tag{2.4}$$
$$v_k \ : \ v = q^2-(q-1)s-1, \ \ k = q-s-\delta, \ \ q > s \geq 0, \ \ q-s > \delta \geq 0. \tag{2.5}$$

For q a *square*, in [1, Conjec. 4.4, Rem. 4.5, Ex. 4.6], [3, Th. 6.4], and [29, Construction 3.7, Th. 3.8], families of non-cyclic configuration v_k with parameters (2.6) are provided; see also [21, Ex. 8]. Taking $c = q - \sqrt{q}$, we obtain (2.7), see also [19, Ex. 2(ii)], [27].

$$v_k \ : \ v = c(q+\sqrt{q}+1), k = \sqrt{q}+c-\delta, \ c = 2,3,\ldots,q-\sqrt{q}, \ \delta \geq 0; \tag{2.6}$$
$$v_k \ : \ v = q^2-\sqrt{q}, \ \ k = q-\delta, \ \ q > \delta \geq 0. \tag{2.7}$$

In [27, Th. 1.1], a non-cyclic family with parameters

$$v_k : v = 2p^2, \ \ k = p+s-\delta, \ \ 0 < s \leq q+1, \ \ q^2+q+1 \leq p, \ \ p+s > \delta \geq 0 \tag{2.8}$$

is given. In [21, Sec. 6], a construction of non-cyclic configuration based on the cyclic punctured affine plane, is provided with parameters

$$v_k \ : \ v = c(q-1), \ \ k = c-\delta, \ \ c = 2,3,\ldots,b, \ \ b = q \text{ if } \delta \geq 1, \tag{2.9}$$
$$b \ = \ \left\lceil \frac{q}{2} \right\rceil \text{ if } \delta = 0, \ \ c > \delta \geq 0.$$

In [19, Sec. 2], [21, Sec. 3], the following geometrical construction which uses point orbits under the action of a collineation group is described.

Construction A. Take any point orbit $\mathcal{P}$ under the action of a collineation group in an affine or projective space of order q. Choose an integer $k \leq q+1$ such that the set $\mathcal{L}(\mathcal{P}, k)$ of lines meeting $\mathcal{P}$ in precisely k points is not empty. Define the following incidence structure: the points are the points of $\mathcal{P}$, the lines are the lines of $\mathcal{L}(\mathcal{P}, k)$, the incidence is that of the ambient space.

Theorem 2.2. *In Construction A the number of lines of* $\mathcal{L}(\mathcal{P}, k)$ *through a point of* $\mathcal{P}$ *is a constant* r_k. *The incidence structure is a configuration* (v_{r_k}, b_k) *with* $v_{r_k} = |\mathcal{P}|$, $b_k = |\mathcal{L}(\mathcal{P}, k)|$.

By Definition 1.1, if $r_k = k$ then Construction A produces a symmetric configuration v_k. It is noted in [19, 21] that Construction A works for any 2-$(v, k, 1)$ design D and for any group of automorphism of D. The size of any block in D plays the role of $q + 1$.

Families of non-cyclic configuration v_k obtained by Construction A with the following parameters are given in [21, Exs 2, 3].

$$v_k \ : \ v = \frac{q(q-1)}{2}, \ k = \frac{q+1}{2} - \delta, \ \frac{q+1}{2} > \delta \geq 0, \ q \text{ odd}. \tag{2.10}$$

$$v_k \ : \ v = \frac{q(q+1)}{2}, \ k = \frac{q-1}{2} - \delta, \ \frac{q-1}{2} > \delta \geq 0, \ q \text{ odd}. \tag{2.11}$$

$$v_k \ : \ v = q^2 + q - q\sqrt{q}, \ k = q - \sqrt{q}, \ q - \sqrt{q} > \delta \geq 0, \ q \text{ square}. \tag{2.12}$$

In [7, 8, 10], non-cyclic families with parameters (2.13)–(2.16) are described in connection with graph theory; see also [4] for another construction of (2.14).

$$v_k \ : \ v = q^2 - rq - 1, \ k = q - r - \delta, \ q - r > \delta \geq 0, \ q - 3 \geq r \geq 0. \tag{2.13}$$

$$v_k \ : \ v = q^2 - q - 2, \ \ k = q - 1 - \delta, \ q - 1 > \delta \geq 0. \tag{2.14}$$

$$v_k \ : \ v = tq - 1, \qquad k = t - \delta, \qquad t > \delta \geq 0, \quad q > t \geq 3. \tag{2.15}$$

$$v_k \ : \ v = tq - 2, \qquad k = t - \delta, \qquad t > \delta \geq 0, \quad q > t \geq 3. \tag{2.16}$$

A classical construction by V. Martinetti for configurations v_3, going back to 1887 [49], is described in detail, e.g. in [3, 6, 12, 15, 25, 32], [37, Sec. 2.4, Fig. 2.4.1]. In [25] a *Generalized Martinetti Construction* (Construction GM) for configurations v_k, $k \geq 3$, is proposed. Use of Construction GM to obtain a wide spectrum of symmetric configurations parameters is considered in [3, 5, 6].

Construction GM can be presented from different points of view, see [3, 25]. Here we focus on an approach based on incidence matrices, which will be used for obtaining new values of v, k.

Definition 2.3. Let $\mathbf{M}(v, k)$ be an incidence matrix of a symmetric configuration v_k. In $\mathbf{M}(v, k)$, we consider an aggregate $\mathcal{A}$ of $k - 1$ rows corresponding to pairwise disjoint lines of v_k and $k - 1$ columns corresponding to pairwise non-collinear points of v_k. If a

$(k-1)\times(k-1)$ submatrix $\mathbf{C}(\mathcal{A})$ formed by the intersection of the rows and columns of $\mathcal{A}$ is a permutation matrix $\mathbf{P}_{k-1}$ then $\mathcal{A}$ is called an *extending aggregate* (or *E-aggregate*). The matrix $\mathbf{M}(v,k)$ *admits an extension* if it contains at least one E-aggregate. The matrix $\mathbf{M}(v,k)$ *admits θ extensions* if it contains θ E-aggregates that do not intersect each other.

Procedure E (*Extension Procedure*). Let $\mathbf{M}(v,k)=[m_{ij}]$ be an incidence matrix of a symmetric configuration $v_k=(\mathcal{P},\mathcal{L})$. Assume that $\mathbf{M}(v,k)$ admits an extension. A matrix $\mathbf{M}(v+1,k)$ can be obtained by two steps.

1. To the matrix $\mathbf{M}(v,k)$, add a new row from below and a new column to the right. Denote the new $(v+1)\times(v+1)$ matrix by $\mathbf{B}=[b_{ij}]$, and let $b_{v+1,v+1}=1$ whereas $b_{v+1,1}=\ldots=b_{v+1,v}=0$, $b_{1,v+1}=\ldots=b_{v,v+1}=0$.

2. One of E-aggregates of $\mathbf{M}(v,k)$, say $\mathcal{A}$, is chosen. In the matrix $\mathbf{B}$, we "clone" all $k-1$ ones of the submatrix $\mathbf{C}(\mathcal{A})$ writing their "projections" to the new row and column. Finally, the ones cloned are changed by zeroes. In other words, let the aggregate $\mathcal{A}$ consist of rows with indexes i_u, $u=1,2,\ldots,k-1$, and columns with indexes j_d, $d=1,2,\ldots,k-1$. Then the ones of $\mathbf{C}(\mathcal{A})$ are as follows: $m_{i_u j_{\pi(u)}}=1$, $u=1,2,\ldots,k-1$, for some permutation π of the indexes $1,\ldots,k-1$. Then $\mathbf{B}$ arising from Step 1 is changed as follows: $b_{i_u,v+1}=1$, $b_{v+1,j_d}=1$, $b_{i_u j_{\pi(u)}}=0$, $u=1,2,\ldots,k-1$, $d=1,2,\ldots,k-1$.

As the Golomb bound $2L_{\overline{\mathrm{G}}}(k)+1$ is important for studying parameters v,k, we note that for sufficiently large orders k, relatively short Golomb rulers are constructed and are available online, see [23,55,56] and the references therein. For $k\le 150$, the order of magnitude of the lengths $L_{\overline{\mathrm{G}}}(k)$ of the shortest known Golomb rulers is ck^2 with $c\in[0.7,0.9]$, see [23,26,31,36,54,55]. Moreover, $L_{\overline{\mathrm{G}}}(k)<k^2$ for $k<65000$, see [23]. Constructions of Golomb rulers for large k can be found in [24]. Remind also that Sidon sets are equivalent to Golomb rulers, see [23,51] and the references therein.

3 Preliminaries on block double-circulant incidence matrices

Results of this section are taken from [18], see also the references therein, in particular, discussions of [18, Rem. 1,2].

Recall that the weight of a circulant binary is the number of 1's in each its rows.

Definition 3.1. Let $v=td$. A binary $v\times v$ matrix $\mathbf{A}$ is said to be a *block double-circulant matrix* (*BDC matrix* for short) if

$$\mathbf{A}=\begin{bmatrix}\mathbf{C}_{0,0} & \mathbf{C}_{0,1} & \ldots & \mathbf{C}_{0,t-1}\\ \mathbf{C}_{1,0} & \mathbf{C}_{1,1} & \ldots & \mathbf{C}_{1,t-1}\\ \vdots & \vdots & \vdots & \vdots\\ \mathbf{C}_{t-1,0} & \mathbf{C}_{t-1,1} & \ldots & \mathbf{C}_{t-1,t-1}\end{bmatrix}, \tag{3.1}$$

where $\mathbf{C}_{i,j}$ is a *circulant* $d\times d$ binary matrix for all i,j, and submatrices $\mathbf{C}_{i,j}$ and $\mathbf{C}_{l,m}$

with $j - i \equiv m - l \pmod{t}$ have the same weight. The matrix

$$\mathbf{W}(\mathbf{A}) = \begin{bmatrix} w_0 & w_1 & w_2 & w_3 & \dots & w_{t-2} & w_{t-1} \\ w_{t-1} & w_0 & w_1 & w_2 & \dots & w_{t-3} & w_{t-2} \\ w_{t-2} & w_{t-1} & w_0 & w_1 & \dots & w_{t-4} & w_{t-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_1 & w_2 & w_3 & w_4 & \dots & w_{t-1} & w_0 \end{bmatrix} \tag{3.2}$$

is a *circulant* $t \times t$ matrix whose entry in position i, j is the *weight* of $\mathbf{C}_{i,j}$. $\mathbf{W}(\mathbf{A})$ is called the *weight matrix* of $\mathbf{A}$. The vector $\overline{\mathbf{W}}(\mathbf{A}) = (w_0, w_1, \dots, w_{t-1})$ is called the *weight vector* of $\mathbf{A}$.

Remark 3.2. If in Definition 1 the matrices $C_{i,j}$ were assumed to be right-circulant and not left-circulant, then they would have been the sum of some right-circulant permutation matrices. A right-circulant $d \times d$ permutation matrix is always associated to a permutation of the set $\{1, 2, \dots, d\}$ in the subgroup generated by the cycle $(1\,2\,3\,\dots\,d)$. Then the notion of a BDC matrix is substantially equivalent to that of a $\mathbb{Z}_d$-scheme, as defined in [3].

Let $\mathbf{A}$ be as in Definition 3.1. In addition, assume that $\mathbf{A}$ is the incidence matrix of a symmetric configuration v_k with $k = \sum_{i=0}^{t} w_i$. From $\mathbf{A}$ one can obtain BDC incidence $v' \times v'$ matrices $\mathbf{A}'$ of symmetric configurations $v'_{k'}$ by the following way.

(i) For each $h \in \{0, 1, \dots, t-1\}$, in each row of every submatrix $\mathbf{C}_{i,j}$ with $j - i \equiv h \pmod{t}$ replace $\delta_h \geq 0$ values of 1 with zeros, in such a way that the obtained submatrix is still circulant. As a result, a BDC incidence matrix of a configuration $v'_{k'}$ with

$$v' = v,\ k' = k - \sum_{h=0}^{t-1} \delta_h,\ 0 \leq \delta_h \leq w_h,\ w'_h = w_h - \delta_h, \overline{\mathbf{W}}(\mathbf{A}') = (w'_0, \dots, w'_{t-1}) \tag{3.3}$$

is obtained.

(ii) Fix some non-negative integer $j \leq t-1$. Let m be such that $w_m \leq w_h$ for all $h \neq j$. Cyclically shift all block rows of $\mathbf{A}$ to the left by j block positions. A matrix $\mathbf{A}^*$ with $\overline{\mathbf{W}}(\mathbf{A}^*) = (w^*_0 = w_j, \dots, w^*_u = w_{u+j \pmod{t}}, \dots, w^*_{t-1} = w_{j-1})$ is obtained. By applying (i), construct a matrix $\mathbf{A}^{**}$ with $w^{**}_0 = w^*_0 = w_j$, $w^{**}_h = w_m$, $h \geq 1$. Now remove from $\mathbf{A}^{**}$ $t - c$ block rows and columns from the bottom and the right. In this way an incidence $cd \times cd$ BDC matrix $\mathbf{A}'$ of a configuration $v'_{k'}$ is obtained with

$$v' = cd,\ k' = w_j + (c-1)w_m,\ c = 1, 2, \dots, t,\ \overline{\mathbf{W}}(\mathbf{A}') = (w_j, w_m, \dots, w_m). \tag{3.4}$$

(iii) Let t be even. Let $\mathbf{A}^*$ be as in (ii). Let $w_{\mathrm{O}}, w_{\mathrm{E}}$ be weights such that $w_{\mathrm{O}} \leq w^*_h$ for odd h and $w_{\mathrm{E}} \leq w^*_h$ for even h. By applying (i), construct a matrix $\mathbf{A}^{**}$ with $w^{**}_0 = w^*_0 = w_j$, $w^{**}_h = w_{\mathrm{O}}$ for odd h, $w^{**}_h = w_{\mathrm{E}}$ for even $h \geq 2$. Let $f = 1, 2, \dots, t/2$. From $\mathbf{A}^{**}$ remove $t - 2f$ block rows and columns from the bottom and the right. An incidence $2fd \times 2fd$ BDC matrix $\mathbf{A}'$ of a configuration $v'_{k'}$ is obtained with

$$v' = 2fd, k' = w_j + w_{\mathrm{O}} + (f-1)(w_{\mathrm{E}} + w_{\mathrm{O}}\ ), \overline{\mathbf{W}}(\mathbf{A}') = (w_j, w_{\mathrm{O}}, \underbrace{w_{\mathrm{E}}, w_{\mathrm{O}}, \dots, w_{\mathrm{E}}, w_{\mathrm{O}}}_{f-1 \text{ pairs}}). \tag{3.5}$$

Remark 3.3. Construction (i) in this section essentially follows from Theorem 2.1; in [3] it is referred to as 1-factor deletion. Construction (ii) is essentialy a different formulation of Proposition 4.3 in [3], which is stated in terms of $\mathcal{S}_\mu$-schemes. Apart from terminology, the only difference is that here the case $m > 1$ is considered. Other methods for obtaining families of symmetric configurations from $\mathbf{A}$ can be found in [21, Sec. 4].

4 Constructions and parameters of block double-circulant incidence matrices from [18]

All the results of this section are taken from [18], apart from Tables 3.1 and 3.2 which essentially present more examples than the corresponding [18, Tab. 1]. In Subsections 4.1 and 4.2 we give some results based on a general method connected with the action of the automorphism group S of a configuration. The method was originally proposed in [19,21] and then developed in [18].

4.1 BDC incidence matrices from projective planes

In this subsection, the projective plane $PG(2, q)$ is considered as a cyclic symmetric configuration $(q^2 + q + 1)_{q+1}$ [21, Sec. 5], [23, Sec. 5.5], [54, Th. 19.15], [58]. The Singer group of $PG(2, q)$ is used as the automorphism group S.

The following BDC matrices with $d \times d$ circulant submatrices and the corresponding BDC configurations v_k are given in [18, Sec. 4.1]:

$$\begin{aligned} &\text{BDC } v_k: \ d = \frac{q^2+q+1}{3},\ v = 2d,\ k = \frac{2q+\sqrt{q}+2}{3},\ q = p^{4m+2},\ p \equiv 2 \pmod 3; \\ &\text{BDC } v_k: \ d = \frac{q^2+q+1}{3},\ v = 2d,\ k = \frac{2q-\sqrt{q}+2}{3},\ q = p^{4m},\ p \equiv 2 \pmod 3. \end{aligned} \tag{4.1}$$

$$\text{BDC } v_k: \ d = \frac{q^2+q+1}{t},\ v = cd,\ k = \frac{q+1 \pm (1-t)\sqrt{q}}{t} + (c-1)\frac{q+1\pm\sqrt{q}}{t}, \quad c = 1, 2, \dots, t,\ q = p^{2m},\ t \text{ prime}, \tag{4.2}$$

where $p \pmod t$ is a generator of the multiplicative group of $\mathbb{Z}_t$.

The needed for (4.2) hypothesis that $p \pmod t$ is a generator of the multiplicative group of $\mathbb{Z}_t$ holds, for example, in the following cases: $q = 3^4$, $t = 7$; $q = 2^8$, $t = 13$; $q = 5^4$, $t = 7$; $q = 2^{12}$, $t = 19$; $q = 3^8$, $t = 7$; $q = 2^{16}$, $t = 13$; $q = 17^4$, $t = 7$; $p \equiv 2 \pmod t$, $t = 3$.

In Table 4.1, parameters of configurations v'_n with BDC incidence matrices are given. We use both (ii) and (iii) of Section 3. The starting weights w_i^* are obtained by computer by considering orbits of subgroups of a Singer group of $PG(2, q)$. For $q = 81$ we use (4.2). The values k', v' are calculated by (3.4),(3.5). Only cases with $v' < G(k')$ are included in the tables. Then the smallest value $k^\#$ for which $v' < G(k^\#)$ is found. As a result, each row of the table provides configurations v'_n with $v' < G(n)$, $n = k^\#, k^\# + 1, \ldots, k'$, see (i) of Section 3 and (3.3). In column w^*_i, an entry s_j indicates that the weight s should be repeated j times.

4.2 BDC incidence matrices from punctured affine planes

In this subsection, the cyclic punctured affine plane is considered as a cyclic symmetric configuration $(q^2-1)_q$, see [13], [23, Sec. 5.6], [54, Th. 19.17], as well as [21, Ex. 5, Sec. 6]. The affine Singer group of $AG(2,q)$ is used as the automorphism group S.

The following BDC matrices with $d \times d$ circulant submatrices and the corresponding BDC configurations v_k are given in [18, Sec. 4.2] on the base [18, Th. 4]:

$$\begin{aligned}
&\text{BDC } v_k:\ d = (\sqrt{q}-1)(q+1),\ v = 2fd,\ k = (2f-1)\sqrt{q}, \qquad (4.3)\\
&\quad f = 1,2,\ldots,\frac{\sqrt{q}+1}{2},\ q \text{ odd square}.\\
&\text{BDC } v_k:\ d = 2(\sqrt{q}-1)(q+1),\ v = cd,\ k = (2c-1)\sqrt{q},\\
&\quad c = 1,2,\ldots,\frac{\sqrt{q}+1}{2},\ q \text{ odd square},\ \sqrt{q} \equiv 1 \pmod 4.\\
&\text{BDC } v_k:\ d = 2(\sqrt{q}-1)(q+1),\ v = 2fd,\ k = (4f-1)\sqrt{q},\\
&\quad f = 1,2,\ldots,\frac{\sqrt{q}+1}{4},\ q \text{ odd square},\ \sqrt{q} \equiv 3 \pmod 4.\\
&\text{BDC } v_k:\ d = 4(\sqrt{q}-1)(q+1),\ v = cd,\ k = (4c-1)\sqrt{q},\\
&\quad c = 1,2,\ldots,\frac{\sqrt{q}+1}{4},\ q \text{ odd square},\ \sqrt{q} \equiv 3 \pmod 4,\ \frac{\sqrt{q}+1}{4} \text{ is odd}.\\
&\text{BDC } v_k:\ d = 4(\sqrt{q}-1)(q+1),\ v = 2fd,\ k = (8f-1)\sqrt{q},\\
&\quad f = 1,2,\ldots,\frac{\sqrt{q}+1}{8},\ q \text{ odd square},\ \sqrt{q} \equiv 3 \pmod 4,\ \frac{\sqrt{q}+1}{4} \text{ is even}.
\end{aligned}$$

In Table 4.2 parameters of configurations v'_n with BDC incidence matrices are given. We use both (ii) and (iii) of Section 3. The starting weights w^*_i are obtained by computer through the constructions of the orbits of subgroups of the affine Singer group. For notations k' and $k^\#$ see Table 4.1.

5 Parameters of symmetric configurations v_k admitting an extension

The following infinite family of symmetric configuration v_k is obtained in [3, Th. 6.2], [6, Th. 1(i)] with the help of Construction GM:

$$v_k : v = q^2 - qs + \theta,\ k = q - s - \Delta,\ q > s \geq 0,\ q - s > \Delta \geq 0,\ \theta = 0, 1, \ldots, q - s + 1. \quad (5.1)$$

Theorem 5.1. [18, Corollary 1] *Let $v = td$, $t \geq k$, $d \geq k - 1$, and let v_k be a symmetric configuration. Assume that an incidence matrix* $\mathbf{A}$ *of v_k is a BDC matrix as in* (3.1) *with weight vector* $\overline{\mathbf{W}}(\mathbf{A})$. *If* $\overline{\mathbf{W}}(\mathbf{A}) = (0, 1, 1, \ldots, 1)$ *or* $\overline{\mathbf{W}}(\mathbf{A}) = (1, 1, \ldots, 1)$ *then one can obtain a family of symmetric configurations v_k with parameters* (5.2) *or* (5.3)*, respectively*

$$v_k \;:\; v = cd + \theta,\ k = c - 1 - \delta,\ c = 2, 3, \ldots, t,\ \theta = 0, 1, \ldots, c + 1,\ \delta \geq 0. \quad (5.2)$$

$$v_k \;:\; v = cd + \theta,\ k = c - \delta,\ c = 2, 3, \ldots, t,\ \theta = 0, 1, \ldots, c + 1,\ \delta \geq 0. \quad (5.3)$$

Configurations with parameters (5.4) were first obtained in [3, Th. 6.3] from the punctured affine plane by using Construction GM; see also [20, Eqn. (8)] and [18, Ex. 6(i)].

$$v_k : v = c(q - 1) + \theta,\ k = c - 1 - \delta,\ c = 2, 3, \ldots, q + 1,\ \theta = 0, 1, \ldots, c + 1,\ \delta \geq 0. \quad (5.4)$$

The families of configurations v_k (5.5) and (5.6) are obtained in [18, Sec. 5] by Construction GM, starting from some new starting matrices proposed in [18].

$$v_k : v = cp + \theta,\ k = c - \delta,\ c = 2, 3, \ldots, p - 1,\ \theta = 0, 1, \ldots, c + 1,\ \delta \geq 0,\ p \text{ prime}. \quad (5.5)$$

$$v_k : v = c(p-1)+\theta,\ k = c-1-\delta,\ c = 2, 3, \ldots, p,\ \theta = 0, 1, \ldots, c+1,\ \delta \geq 0,\ p \text{ prime}. \quad (5.6)$$

The family of configurations v_k (5.7) is obtained in [18].

$$\begin{aligned} v_k \;:\; & v = c(q + \sqrt{q} + 1) + \theta,\ k = c - \delta,\ c = 2, 3, \ldots, q - \sqrt{q} + 1, \\ & \theta = 0, 1, \ldots, c + 1,\ \delta \geq 0,\ q \text{ square}. \end{aligned} \quad (5.7)$$

6 The spectrum of parameters of cyclic symmetric configurations

Current data on the existence of cyclic configurations v_k, $k \leq 51$, are given in Table 6.1. New parameters obtained in [18] are written in bold font.

In Table 6.1, the values of v for which cyclic symmetric configurations v_k exist (resp. do not exist) are written in normal (resp. in italic) font. Moreover, $\overline{v}$ means that no configuration v_k exists, whereas $\overline{v^c}$ indicates that no cyclic configuration v_k exists. Data from [26, 31, 33, 36, 42, 47, 56] are used in the 4-th column of the table. We take into account that an entry of the form "$t+$" in the row "n" of [56, Tab. 1] means the existence of cyclic symmetric configurations v_n with $v \geq t$. An absence of a value "v" in the row "n" of [56, Tab. 1] means the non-existence of a cyclic symmetric configuration v_n. Also, we use the following *non-existence* results: $\overline{\mathit{32}_6}$ [31, Th. 4.8]; $\overline{\mathit{33}_6}$ [42]; $\overline{\mathit{34}^c_6}$, $\overline{\mathit{59}^c_8}$-$\overline{\mathit{62}^c_8}$ [47]; $\overline{\mathit{75}^c_9}$-$\overline{\mathit{79}^c_9}$, $\overline{\mathit{81}^c_9}$- $\overline{\mathit{84}^c_9}$ [26]; $\overline{\mathit{93}^c_{10}}$ – $\overline{\mathit{106}^c_{10}}$, $\overline{\mathit{121}^c_{11}}$ – $\overline{\mathit{132}^c_{11}}$, $\overline{\mathit{134}^c_{11}}$, $\overline{\mathit{135}^c_{12}}$ – $\overline{\mathit{155}^c_{12}}$, $\overline{\mathit{157}^c_{12}}$, $\overline{\mathit{160}^c_{12}}$, $\overline{\mathit{169}^c_{13}}$ – $\overline{\mathit{182}^c_{13}}$, $\overline{\mathit{184}^c_{13}}$ – $\overline{\mathit{192}^c_{13}}$, $\overline{\mathit{185}^c_{14}}$ – $\overline{\mathit{224}^c_{14}}$, $\overline{\mathit{256}^c_{15}}$ – $\overline{\mathit{260}^c_{15}}$, $\overline{\mathit{263}^c_{15}}$ [56, Tab. 1]; see also Theorem 6.1.

The values of k for which the spectrum of parameters of cyclic symmetric configurations v_k is completely known are indicated by a dot "$\centerdot$"; the corresponding values of $E_c(k)$ are sharp and they are noted by the dot "$\centerdot$" too.

An entry **v-w** indicates an interval of sizes from **v** to **w** without gaps. If an already known value lies within an interval **v-w** obtained in work [18], then it is written immediately before the interval.

Entries v_a, v_b, and v_c (here and in all tables) mean, respectively, that relations (2.1), (2.2), and (2.3) are applied.

The value $v_\delta(k)$ is defined in Introduction. In the second column, the exact values of $v_\delta(k)$ are marked by the dot "**.**". For $k \leq 16$, the exact values of $v_\delta(k)$ are taken from [30, Tab. IV], [38, Tab. 2], [56, Tab. 1a], [61]. Also, if $k-1$ is a prime power then $v_\delta(k) = P(k) = k^2 - k + 1$. The remaining entries in the second column are lower bounds of $v_\delta(k)$. By the Bruck-Ryser Theorem, planes $PG(2, k-1)$ with $k-1 = 6, 14, 21, 22, 30, 33, 38, 42, 46, 54, 57, 62$ do not exist. It is well-known that $P(k) \leq v_\delta(k)$, and that a *cyclic* symmetric configuration $(k^2 - k + 1)_k$ exists if and only if a *cyclic* projective plane of order $k-1$ exists. By [11], no cyclic projective planes exist with non-prime power orders $\leq 2 \cdot 10^9$. Therefore cyclic projective planes $PG(2, k-1)$ with $k - 1 = 18, 20, 24, 26, 28, 34, 35, 36, 39, 40, 44, 45, 48, 50$ do not exist. Also we use Theorem 6.1 taken from [33]. The mentioned non-existence cases of cyclic configurations are marked in Table 6.1 by subscripts br (Bruck-Ryser Theorem), s ([11]), and t (Theorem 6.1).

For $k \leq 22$ the *filling of the interval* $P(k) - G(k)$ is expressed as a percentage in the last column of Table 6.1.

Theorem 6.1. [33, Th. 2.4] *There is no symmetric configuration* $(k^2 - k + 2)_k$ *if* $5 \leq k \leq 10$ *or if neither* k *or* $k-2$ *is a square.*

In order to widen the ranges of parameter pairs (v, k) for which a cyclic symmetric configuration v_k exists, we consider a number of procedures that allow to define a new modular Golomb ruler from a known one. Some methods have already been introduced in the paper, see Theorem 2.1.

Here we first recall a result from [54], which describes a method to construct different rulers with the same parameters.

Theorem 6.2. [54] *If* $(a_1, a_2, \ldots, a_k)$ *is a* (v, k) *modular Golomb ruler and* m *and* b *are integers with* $\gcd(m, v) = 1$ *then* $(ma_1 + b \pmod v, ma_2 + b \pmod v, \ldots, ma_k + b \pmod v)$ *is also a* (v, k) *modular Golomb ruler.*

It should be noted that a (v, k) modular Golomb ruler can be a $(v + \Delta, k)$ modular Golomb ruler for some integer Δ [31]. This property does not depend on parameters v and k only. This is why Theorem 6.2 can be useful for our purposes.

Example 6.3. We consider the $(31, 6)$ modular Golomb ruler

$$(a_1, \ldots, a_6) = (0, 1, 4, 10, 12, 17)$$

obtained from $PG(2, 5)$, see [55]. We can apply Theorem 6.2 for $m = 19$, $b = 0$. The $(31, 6)$ modular Golomb ruler $(ma_1 \pmod{31}, \ldots, ma_6 \pmod{31})$ is

$$(a'_1, \ldots, a'_6) = (0, 4, 11, 13, 14, 19).$$

Now we take $\Delta = 4$ and calculate the set of differences $\{a'_i - a'_j \pmod{35} \,|\, 1 \le i, j \le 6; i \ne j\}$, that is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$. As the all differences are distinct and nonzero, the starting $(31, 6)$ modular Golomb ruler is also a $(35, 6)$ modular Golomb ruler.

For $k \le 81$, we performed a computer search starting from the (v, k) modular Golomb rulers corresponding to (2.1)–(2.3). For projective and affine planes, we got a concrete description of the ruler from [55].

For Ruzsa's construction, we use the following known relations. Let p be a prime. Let g be a primitive element of F_p. The following Ruzsa's sequence [53], [23, Sec. 5.4], [54, Th. 19.19] forms a $(p^2 - p, p - 1)$ modular Golomb ruler:

$$e_u = pu + (p-1)g^u \pmod{p^2 - p},\ u = 1, 2, \dots, p-1,\ v = p^2 - p. \tag{6.1}$$

For every starting (v, k) modular ruler we first considered all possible m with

$$\gcd(m, v) = 1,$$

and applied Theorem 6.2 for all $b < v$ to get new rulers with the same parameters v and k. Then, we checked whether this ruler was also a $(v + \Delta, k)$ for some Δ.

In Table 6.2, for $52 \le k \le 83$, the upper bounds on the cyclic existence bound $E_c(k)$ obtained in [18] are listed in bold font. An entry, say $A(u)$, in the column $E_c(k)$ on the row "u", means that in [18] *all* cyclic symmetric configurations v_u in the region $A(u), A(u) + 1, \dots, G(u) - 1$ are obtained. These configurations are new. We obtained also many other new cyclic symmetric configurations v_k for $52 \le k \le 83$. However, we do not give here their sizes v here in order to save space.

The upper bounds on $E_c(k)$ in Tables 6.1 and 6.2, obtained in [18], are written in bold font.

Some new cyclic symmetric configurations important for Table 7.1 of Section 7 are given in Table 6.3 where we write the first rows of their incidence matrices; these rows may be the same for distinct v.

7 The spectrum of parameters of symmetric (not necessarily cyclic) configurations

The known results regarding to parameters of symmetric configurations can be found in [1–10, 12, 13, 15, 17–21, 25–27, 29–38, 42, 44, 47, 49, 50, 53, 54, 56]; see also the references therein.

The known families of configurations are described in Section 2, see also (5.1),(5.4). New families obtained in the work [18] are given in Sections 4 and 5. In Table 7.1, for $k \le 51$, $P(k) \le v < G(k)$, values of v for which a symmetric configuration v_k from one of the families of Sections 2–5 exists are given. The new parameters obtained in the paper [18] are written in bold font.

An entry of type $v_{\text{subscript}}$ indicates that one of the following is used: $(2.i)$, $(4.j)$, $(5.k)$, Table 4.1,Table 6.1, the Bruck-Ryser Theorem, Theorem 6.1. More precisely v_a indicates that v is obtained from (2.1), and similarly $v_b \to (2.2)$, $v_c \to (2.3)$, $v_f \to (2.5)$, $v_g \to (2.6)$, $v_h \to (2.7)$, $v_j \to (2.8)$, $v_k \to (2.9)$, $v_\lambda \to (2.13) - (2.16)$, $v_m \to (5.1)$, $v_P \to (4.3)$, $v_r \to (5.4)$, $v_S \to (5.5)$, $v_T \to (5.6)$, $v_W \to$ Table 4.1, $v_y \to$ Table 6.1 with $k \leq 15$, $v_Z \to$ Table 6.1 with $k > 15$, $v_{br} \to$ the Bruck-Ryser Theorem, $v_t \to$Theorem 6.1. Here capital letters in subscripts remark new results and constructions of [18] while lower case letters indicate the known ones.

An entry $v_{\text{subscript}_1 \cdot \text{subscript}_2 \cdot \ldots}$ with more than one subscript means that the same value can be obtained from different constructions. An entry of type $v_{\text{subscript}_1 \cdot \text{subscript}_2 \cdot \ldots} - v'_{\text{subscript}_1 \cdot \text{ subscript}_2 \cdot \ldots}$ indicates that a whole interval of values from v to v' can be obtained from the constructions corresponding to the subscripts. We use the following known results on the existence of sporadic symmetric configurations: 45_7 [9]; 82_9 [27, Tab. 1]; 135_{12}, see [33] with reference to Mathon's talk at the British Combinatorial Conference 1987; 34_6 [44], see also [4]. The non-existence of configuration 112_{11} is proven in [43]. The non-existence of the plane $PG(2, 10)$ implies the non-existence of configuration 111_{11}. The values of k for which the spectrum of parameters of symmetric configurations v_k is completely known are indicated by a dot "**.**"; the corresponding values of $E(k)$ are exact and they are indicated by a dot as well.

To save space, in Table 7.1 for given v, k, we do not write all the constructions providing a configuration v_k, but often we describe some of them as a matter of illustration.

For the convenience of the reader we give also Table 7.2 where constructions are not indicated and for $k \leq 64$, $P(k) \leq v < G(k)$, values of v for which a symmetric configuration v_k exists are written.

The filling of the interval $P(k) - G(k)$ is expressed as a percentage in the last column of Tables 7.1 and 7.2. It is interesting to note that such a percentage is quite high and that most gaps occur for v close to $k^2 - k + 1$.

We note that a number of parameters obtained in the work [18] are new, see bold font in Table 7.1. Note that parameters of some new families are too big to be included in Tables 7.1 and 7.2. Recall also (see Introduction) that from the stand point of applications, including Coding Theory, it is useful to have different matrices $\mathbf{M}(v, k)$ for the same v and k.

References

[1] M. Abreu, M. Funk, D. Labbate, and V. Napolitano, On (minimal) regular graphs of girth 6, *Australas. J. Combin.* **35**, 119–132 (2006).

[2] M. Abreu, M. Funk, D. Labbate, and V. Napolitano, Configuration graphs of neighbourhood geometries, *Contrib. Discr. Math.* **3**, 109–122 (2008).

[3] Abreu, M., Funk, M., Labbate, D., Napolitano, V.: Deletions, extension, and reductions of elliptic semiplanes, Innov. Incidence Geom. **11**, 139–155 (2010).

[4] Abreu, M., Funk, M., Labbate, D., Napolitano, V.: On the ubiquity and utility of cyclic schemes, Australas. J. Combin. **55**, 95–120 (2013).

[5] V. B. Afanassiev, A. A. Davydov, and V. V. Zyablov, Low density concatenated codes with Reed-Solomon component codes. In: *Proc. XI Int. Symp. on Problems of Redundancy in Information and Control Syst.*, St.-Petersburg, Russia, pp. 47–51 (2007). http://k36.org/redundancy2007

[6] V. B. Afanassiev, A. A. Davydov, and V. V. Zyablov, Low density parity check codes on bipartite graphs with Reed-Solomon constituent codes, *Information Processes (Electronic Journal)* **9**, no. 4, 301–331 (2009). http://www.jip.ru/2009/301-331-2009.pdf

[7] Araujo-Pardo, G., Balbuena, C.: Constructions of small regular bipartite graphs of girth 6, Networks **57**, 121–127 (2011).

[8] Araujo-Pardo, G., Balbuena, C., Héger, T.: Finding small regular graphs of girths 6, 8 and 12 as subgraphs of cages, Discrete Math. **310**, 1301–1306 (2010).

[9] R. D. Baker, An elliptic semiplane, *J. Combin. Theory, ser. A* **25**, 193–195 (1978).

[10] Balbuena, C.: Incidence matrices of projective planes and of some regular bipartite graphs of girth 6 with few vertices, SIAM J. Discrete Math. **22**, 1351-1363 (2008).

[11] L. D. Baumert and D. M. Gordon, On the existence of cyclic difference sets with small parameters, In: *High Primes and Misdemeanours: Lectures in Honour of the 60th Birthday of Hugh Cowie Williams* , van der Poorten, A., Stein, A., eds., Amer. Math. Soc., Fields Institute Communications, **41**, pp. 61–68, Providence (2004).

[12] M. Boben, Irreducible ($v3$) configurations and graphs, *Discrete Math.* **307**, 331–344, (2007).

[13] R. C. Bose, An affine analogue of Singer's theorem, *J. Ind. Math. Society* **6**, 1–15 (1942).

[14] M. Bras-Amorós and K. Stokes, The semigroup of combinatorial configurations, *Semigroup forum* **84**, 91-96 (2012).

[15] H. G. Carstens, T. Dinski, and E. Steffen, Reduction of symmetric configurations n_3, *Discrete Applied Math.* **99**, 401–411 (2000).

[16] J. Coykendall and J. Dover, Sets with few intersection numbers from Singer subgroup orbits, *Europ. J. Combin.* **22**, 455–464 (2001).

[17] A. A. Davydov, G. Faina, M. Giulietti, S. Marcugini, and F. Pambianco, On constructions and parameters of symmetric configurations v_k. (2012) http://arxiv.org/abs/arXiv:1203.0709v1

[18] A. A. Davydov, G. Faina, M. Giulietti, S. Marcugini, and F. Pambianco, On constructions and parameters of symmetric configurations v_k, submitted.

[19] A. A. Davydov, M. Giulietti, S. Marcugini, and F. Pambianco, Symmetric configurations for bipartite-graph codes. In: *Proc. XI Int. Workshop Algebraic Comb. Coding Theory, ACCT2008,* Pamporovo, Bulgaria, pp. 63–69 (2008). http://www.moi.math.bas.bg/acct2008/b11.pdf

[20] A. A. Davydov, M. Giulietti, S. Marcugini, and F. Pambianco, On the spectrum of possible parameters of symmetric configurations. In: *Proc. XII Int. Symp. on Problems of Redundancy in Information and Control Systems*, Saint-Petersburg, Russia, pp. 59–64 (2009). http://k36.org/redundancy2009

[21] A. A. Davydov, M. Giulietti, S. Marcugini, and F. Pambianco, Some combinatorial aspects of constructing bipartite-graph codes, *Graphs Combinatorics* **29**, 187–212 (2013).

[22] J. Domingo-Ferrer, M. Bras-Amorós, Q. Wu, and J. Manjón, User-private information retrieval based on a peer-to-peer community, *Data Knowl. Eng.* **68,** 1237–1252 (2009).

[23] A. Dimitromanolakis, *Analysis of the Golomb Ruler and the Sidon Set Problems, and Determination of Large, Near-Optimal Golomb Rulers*, Depart. Electronic Comput. Eng. Techn. University of Crete, 2002.
http://www.cs.toronto.edu/∼apostol/golomb/main.pdf

[24] K. Drakakis, A review of the available construction methods for Golomb rulers, *Advanc. Math. Commun.* **3**, 235–250 (2009).

[25] M. Funk, On configurations of type n_k with constant degree of irreducibility, *J. Combin. Theory, ser. A,* **65**, 173–201 (1993).

[26] M. Funk, Cyclic difference sets of positive deficiency, *Bull. Inst. Combin. Appl.* **53**, 47–56 (2008).

[27] M. Funk, D. Labbate, and V. Napolitano, Tactical (de-)compositions of symmetric configurations, *Discrete Math.* **309**, 741–747 (2009).

[28] E. Gabidulin, A. Moinian, and B. Honary, Generalized construction of quasi-cyclic regular LDPC codes based on permutation matrices. In: *Proc. Int. Symp. Inf. Theory 2006, ISIT 2006*, Seattle, USA, pp. 679–683 (2006).

[29] A. Gács and T. Héger, On geometric constructions of (k, g)-graphs, *Contrib. Discr. Math.* **3**, 63–80 (2008).

[30] R. L. Graham and N. J. A. Sloane, On additive bases and Harmonious Graphs, *Siam J. Algeb. Discrete Methods* **1**, 382–404 (1980).

[31] H. Gropp, On the existence and non-existence of configurations n_k, *J. Combin. Inform. System Sci.* **15**, 34–48 (1990).

[32] H. Gropp, Configurations, regular graphs and chemical compounds, *J. Mathematical Chemistry* **11**, 145-153 (1992).

[33] H. Gropp, Non-symmetric configurations with deficiencies 1 and 2. In: Barlotti, A., Bichara, A., Ceccherini, P.V., Tallini, G. (eds.) *Combinatorics '90: Recent Trends and Applications. Ann. Discrete Math.* vol. 52, pp. 227–239. Elsevier, Amsterdam (1992).

[34] H. Gropp, Configurations and graps – II, *Discrete Math.* **164**, 155–163 (1997).

[35] H. Gropp, Configurations between geometry and combinatorics, *Discr. Appl. Math.* **138**, 79–88 (2004).

[36] H. Gropp, Configurations. In: Colbourn, C.J., Dinitz, J. (eds.) *The CRC Handbook of Combinatorial Designs,* 2-nd edition, Chapter VI.7, pp. 353–355. CRC Press, Boca Raton (2007).

[37] B. Grünbaum, *Configurations of points and line* . Gradute studies in mathematics vol. 103, American Mathematical Society, Providence (2009).

[38] H. Haanpää, A. Huima, P. R. J. Östergård, Sets in Z_n with distinct sums of pairs, *Discrete Appl. Math.* **138**, 99–106 (2004).

[39] J. W. P. Hirschfeld, *Projective Geometries over Finite Fields.* Second edition, Oxford University Press, Oxford (1998).

[40] Q. Huang, Q. Diao, S. Lin, Circulant decomposition: Cyclic, quasi-cyclic and LDPC codes, In: *Proc. 2010 International Symposium on Information Theory and its Applications (ISITA)*, pp.383–388.

[41] Q. Huang, Q. Diao, S. Lin, K. Abdel-Ghaffar, Cyclic and quasi-cyclic LDPC Codes: new developments, In: *Proc. Information Theory and Applications Workshop (ITA)*, 2011, pp. 1–10.

[42] P. Kaski, P. R. J. Östergård, There exists no symmetric configuration with 33 points and line size 6, *Australasian Journal of Combin.* **38**, 273–277 (2007).

[43] P. Kaski and P. R. J. Östergård, There are exactly five biplanes with k = 11, *J. Combin. Des.* **16**, 117-127 (2007).

[44] V. Krčadinac, *Construction and classification of finite structures by computer.* PhD thesis, University of Zagreb, in Croatian. (2004).

[45] Z.-W. Li, L. Chen, L. Zeng, S. Lin, and W. H. Fong, Efficient encoding of quasi-cyclic low-density parity-check codes, *IEEE Trans. Commun.* **54**, 71–81 (2006).

[46] A. C. H. Ling, Difference triangle sets from affine planes, *IEEE Trans. Inform. Theory* **48**, 2399–2401 (2002).

[47] M. J. Lipman, The existence of small tactical configurations. In: *Graphs and Combinatorics, Springer Lecture Notes in Mathematics* **406**, Springer-Verlag, pp. 319–324 (1974).

[48] J. Q. Longyear, Tactical constructions, *J. Combin. Theory, ser. A*, **19**, 200–207 (1975).

[49] V. Martinetti, Sulle configurazioni piane μ_3. *Annali di matematica pura ed applicata* (2) **15** 1–26 (1887-88).

[50] N. S. Mendelsohn, R. Padmanabhan, and B. Wolk, Planar projective configurations I, *Note di Matematica* **7**, 91–112 (1987). http://siba2.unile.it/ese/issues/1/13/Notematv7n1p91.pdf

[51] K. O'Bryant, A complete annotated bibliography of work related to Sidon sequences, *Electronic J. Combin.* **11**, #39 (2004).

[52] V. Pepe, LDPC codes from the Hermitian curve, *Des. Codes Crypt.* **42**, 303–315 (2007).

[53] I. Z. Ruzsa, Solving a linear equation in a set of integers I, *Acta Arithmetica* **65**, 259–282 (1993).

[54] J. Shearer, Difference triangle sets. In: Colbourn, C.J., Dinitz, J. (eds.) *The CRC Handbook of Combinatorial Designs,* 2-nd edition, Chapter VI.19, pp. 436–440. CRC Press, Boca Raton (2007).

[55] J. Shearer, Table of lengths of shortest known Golomb rulers, http://www.research.ibm.com/people/s/shearer/grtab.html

[56] J. Shearer, Modular Golomb rulers, http://www.research.ibm.com/people/s/shearer/ mgrule.html http://www.research.ibm.com/people/s/shearer/grule.html

[57] J. B. Shearer, *Difference triangle sets constructions,* IBM Research report, RC24623(W0808-045) (2008).

[58] J. Singer, A theorem in finite projective geometry and some applications to number theory, *Trans. American Math. Soc.* **43**, 377–385 (1938).

[59] K. Stokes and M. Bras-Amorós, Optimal configurations for peer-to-peer user-private information retrieval. *Comput Math Applications* **59** (2010), 1568–1577.

[60] K. Stokes and M. Bras-Amorós, Linear, non-homogeneous, symmetric patterns and prime power generators in numerical semigroups associated to combinatorial configurations, *Semigroup Forum* **88** (2014), 11–20.

[61] C. N. Swanson, Planar cyclic difference packings, *J. Combin. Des.* **8**, 426–434 (2000).

8 Tables of parameters

8.1 Tables for Section 4

Table 4.1. Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^{\#}, k^{\#}+1, \dots, k'$, by (ii) of Section 3 from the cyclic projective plane $PG(2, q)$

q	t	d	w_i^*	c	k'	v'	$G(k')$	$k^{\#}$	$G(k^{\#})$
25	3	217	$12, 7, 7$	2	19	434	493	19	493
25	7	93	$8, 3, 3, 3, 3, 3, 3$	4	17	372	399	17	399
25	7	93	$8, 3, 3, 3, 3, 3, 3$	5	20	465	567	19	493
25	7	93	$8, 3, 3, 3, 3, 3, 3$	6	23	558	745	20	567
32	7	151	$0, 5, 5, 6, 5, 6, 6$	6	25	906	961	25	961
37	3	469	$16, 9, 13$	2	25	938	961	25	961
43	3	631	$19, 13, 12$	2	31	1262	1495	30	1361
49	3	817	$21, 16, 13$	2	34	1634	1877	33	1719
61	3	1261	$25, 21, 16$	2	41	2522	2611	40	2565
64	3	1387	$27, 19, 19$	2	46	2774	3407	42	2795
64	19	219	$11, 3_{18}$	7	32	1533	1569	32	1569
64	19	219	$11, 3_{18}$	8	35	1752	1975	34	1877
64	19	219	$11, 3_{18}$	9	38	1971	2293	35	1975
64	19	219	$11, 3_{18}$	10	41	2190	2611	37	2199
64	19	219	$11, 3_{18}$	17	62	3723	6431	48	3775
64	19	219	$11, 3_{18}$	18	65	3942	7187	50	4189
67	3	1519	$28, 19, 21$	2	47	3038	3609	44	3193
73	3	1801	$28, 27, 19$	2	47	3602	3609	47	3609
79	3	2107	$31, 21, 28$	2	52	4214	4541	51	4381
81	7	949	$4, 13, 13, 13, 13, 13, 13$	6	69	5694	8291	58	5703
81	7	949	$4, 13, 13, 13, 13, 13, 13$	5	56	4745	5451	54	4747
97	3	3169	$39, 28, 31$	2	67	6338	7639	62	6431
103	3	3571	$39, 28, 37$	2	67	7142	7639	65	7187
107	7	1651	$24, 15, 15, 13, 15, 13, 13$	6	89	9906	13557	75	9965
107	7	1651	$24, 15, 15, 13, 15, 13, 13$	5	76	8255	10179	69	8291
109	3	3997	$43, 36, 31$	2	74	7994	9507	69	8291
109	7	1713	$8, 15, 15, 19, 15, 19, 19$	6	83	10278	12041	77	10409
121	37	399	$14, 3_{36}$	25	89	9975	13557	76	10179
127	3	5419	$49, 43, 36$	2	85	10838	12821	80	11127
128	7	2359	$24, 21, 21, 14, 21, 14, 14$	6	94	14154	15769	91	15085
137	7	2701	$24, 15, 15, 23, 15, 23, 23$	6	99	16206	17081	96	16243

Table 4.1 (continue). Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^{\#}, k^{\#}+1, \ldots, k'$, by (ii) of Section 3 from the cyclic projective plane $PG(2,q)$

q	t	d	w_i^*	c	k'	v'	$G(k')$	$k^{\#}$	$G(k^{\#})$
139	3	6487	52, 39, 49	2	91	12974	15085	86	13075
149	7	3193	12, 25, 25, 21, 25, 21, 21	6	117	19158	25035	104	19163
149	7	3193	12, 25, 25, 21, 25, 21, 21	5	96	15965	16243	96	16243
121	3	4921	48, 37, 37	2	85	9842	12821	75	9965
121	7	2109	21, 20, 13, 13, 21, 13, 21	6	86	12654	13075	85	12821
121	37	399	$14, 3_{36}$	20	71	7980	8661	69	8291
151	3	7651	57, 43, 52	2	100	15302	17663	93	15453
151	7	3279	32, 19, 19, 21, 19, 21, 21	6	127	19674	28921	105	19769
151	7	3279	32, 19, 19, 21, 19, 21, 21	5	108	16395	20831	97	16715
151	7	3279	32, 19, 19, 21, 19, 21, 21	4	89	13116	13557	87	13417
157	3	8269	61, 48, 49	2	109	16538	21167	97	16715
163	3	8911	63, 49, 52	2	112	17822	27043	102	18437
163	7	3819	32, 25, 25, 19, 25, 19, 19	6	127	22914	28921	114	23529
163	7	3819	32, 25, 25, 19, 25, 19, 19	5	108	19095	20831	104	19163
169	3	9577	64, 57, 49	2	113	19154	22847	104	19163
179	7	4603	24, 21, 21, 31, 21, 31, 31	6	129	27618	30151	124	27897
181	3	10981	67, 63, 52	2	119	21962	25823	111	22217

Table 4.2. Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^{\#}, k^{\#}+1, \ldots, k'$, by (ii) and (iii) of Section 3 from the cyclic punctured affine plane $AG(2,q)$

q	t	d	w_i^*	c	f	k'	v'	$G(k')$	$k^{\#}$	$G(k^{\#})$
16	3	85	8, 4, 4	2		12	170	171	12	171
31	3	320	14, 9, 8	2		22	640	713	21	667
37	3	456	16, 9, 12	2		25	912	961	25	961
49	4	600	16, 12, 9, 12	3		34	1800	1877	34	1877
49	6	400	4, 9, 12, 8, 8, 8	5		36	2000	2011	36	2011
53	4	702	17, 12, 10, 14	3		37	2106	2199	37	2199
61	3	1240	25, 16, 20	2		41	2480	2611	39	2505
61	4	930	18, 18, 12, 13	3		42	2790	2795	42	2795
61	5	744	18, 9, 10, 12, 12	4		45	2976	3375	43	3015
61	6	620	15, 8, 8, 10, 8, 12	5		47	3100	3609	44	3193
64	9	455	0, 8, 8, 8, 8, 8, 8, 8, 8	8		56	3640	5451	48	3775
64	9	455	0, 8, 8, 8, 8, 8, 8, 8, 8	7		48	3185	3775	44	3193
67	3	1496	26, 24, 17	2		43	2992	3015	43	3015
71	5	1008	8, 14, 15, 16, 18	4		50	4032	4189	50	4189
73	3	1776	30, 21, 22	2		51	3552	4381	47	3609
79	3	2080	32, 25, 22	2		54	4160	4747	50	4189
79	6	1040	8, 14, 13, 14, 18, 12	5		56	5200	5451	56	5451
79	6	1040	18, 12, 8, 14, 13, 14		2	50	4160	4189	50	4189
81	4	1640	25, 20, 16, 20	3		57	4920	5547	55	5197
81	4	1640	25, 20, 16, 20		1	45	3280	3375	45	3375
81	8	820	16, 8, 8, 8, 9, 12, 8, 12	7		64	5740	7055	59	5823
81	8	820	16, 8, 8, 8, 9, 12, 8, 12	6		56	4920	5451	55	5187
83	8	861	6, 10, 10, 10, 15, 11, 10, 11	7		66	6027	7515	60	6039
83	8	861	6, 10, 10, 10, 15, 11, 10, 11	6		56	5166	5451	55	5187
89	4	1980	26, 25, 18, 20	3		62	5940	6431	60	6039
89	8	990	17, 8, 10, 12, 8, 10, 10, 14	7		65	6930	7187	64	7055
97	3	3136	37, 34, 26	2		63	6272	6783	62	6431
97	4	2352	29, 26, 20, 22	3		69	7056	8291	65	7187
97	6	1586	21, 20, 14, 16, 14, 12	5		69	7930	8291	69	8291
101	4	2550	30, 25, 20, 26	3		70	7650	8435	68	7913
101	4	2550	30, 25, 20, 26		1	55	5100	5197	55	5197
103	3	3536	41, 32, 30	2		71	7072	8661	65	7187
103	6	1768	24, 18, 14, 17, 14, 16	5		80	8840	11127	72	8947
103	6	1768	24, 18, 14, 17, 14, 16	4		66	7072	7515	65	7187
103	6	1768	24, 18, 14, 17, 14, 16		2	70	7072	8435	65	7187
107	8	1431	17, 15, 10, 13, 10, 12, 17, 13	7		77	10017	10409	76	10179
107	8	1431	17, 15, 10, 13, 10, 12, 17, 13		3	73	8586	9027	71	8661

Table 4.2 (continue). Parameters of configurations v'_n with BDC incidence matrices, $v' < G(n)$, $n = k^{\#}, k^{\#}+1, \ldots, k'$, by (ii) and (iii) of Section 3 from the cyclic punctured affine plane $AG(2,q)$

q	t	d	w_i^*	c	f	k'	v'	$G(k')$	$k^{\#}$	$G(k^{\#})$
109	3	3960	42, 30, 37	2		72	7920	8947	69	8291
109	4	2970	32, 26, 22, 29	3		76	8910	10179	72	8947
109	6	1980	12, 19, 24, 18, 18, 18	5		84	9900	12319	75	9965
109	9	1320	8, 14, 20, 10, 13, 10, 12, 10, 12	8		78	10560	10599	78	10599
113	4	3192	32, 32, 24, 25	3		80	9576	11127	75	9965
113	7	1824	10, 20, 14, 17, 22, 16, 14	6		80	10944	11127	80	11127
121	4	3660	36, 30, 25, 30	3		86	10980	13075	80	11127
121	4	3660	36, 30, 25, 30		1	66	7320	7515	66	7515
121	5	2928	16, 24, 28, 25, 28	4		88	11712	13491	83	12041
125	4	3906	37, 32, 26, 30	3		89	11718	13557	83	12041
125	8	1953	24, 16, 14, 15, 13, 16, 12, 15	7		96	13671	16243	90	13935
125	8	1953	24, 16, 14, 15, 13, 16, 12, 15		3	93	11718	15453	83	12041
127	3	5376	49, 36, 42	2		85	10752	12821	79	10817
127	6	2688	28, 18, 18, 21, 18, 24	5		100	13440	17663	88	13491
127	6	2688	28, 18, 18, 21, 18, 24	4		82	10752	11629	79	10817
131	5	3432	19, 24, 32, 26, 30	4		91	13728	15085	90	13935
137	4	4692	40, 32, 29, 36	3		98	14076	16925	91	15085
139	3	6440	54, 41, 44	2		95	12880	15935	86	13075
149	4	5550	42, 41, 32, 34	3		106	16650	20271	97	16715
151	3	7600	58, 44, 49	2		102	15200	18437	92	15235
151	6	3800	18, 24, 32, 26, 25, 26		2	93	15200	15453	93	15453

8.2 Tables for Section 6

Table 6.1. Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51$, $P(k) \le v \le G(k) - 1$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k) - 1$	$E_c(k)$	$G(k)$	filling
5.	21	21**.**	$21_a, \overline{22_t}$	23**.**	23	100%
6.	31	31**.**	$31_a, \overline{32_t}, \overline{33}, \overline{34^c}$	35**.**	35	100%
7.	43	48**.**	$48_b, 49, 50$	48**.**	51	100%
8.	57	57**.**	$57_a, \overline{58_t}, \overline{59^c} - \overline{62^c}, 63_b, 64 - 68$	63**.**	69	100%
9.	73	73**.**	$73_a, \overline{74_t}, \overline{75^c} - \overline{79^c}, 80_b, \overline{81^c} - \overline{84^c}, 85 - 88$	85**.**	89	100%
10.	91	91**.**	$91_a, \overline{92_t}, \overline{93^c} - \overline{106^c}, 107 - 109, 110_c$	107**.**	111	100%
11.	111	120**.**	$120_b, \overline{121^c} - \overline{132^c}, 133_a, \overline{134^c}, 135 - 144$	135**.**	145	100%
12.	133	133**.**	$133_a, \overline{134_t}, \overline{135^c} - \overline{155^c}, 156_c, \overline{157^c}, 158, 159, \overline{160^c}, 168_b, 161 - 170$	161**.**	171	100%
13.	157	168**.**	$168_b, \overline{169^c} - \overline{182^c}, 183_a, \overline{184^c} - \overline{192^c}, 193 - 212$	193**.**	213	100%
14.	183	183**.**	$183_a, \overline{184_t}, \overline{185^c} - \overline{224^c}, 225 - 254$	225**.**	255	100%
15	211	255**.**	$255_b, \overline{256^c} - \overline{260^c}, \overline{263^c}, 272_c, 273_a, 288_b, 267 - 302$	267	303	95%
16	241	255**.**	$255_b, 272_c, 273_a, 288_b, 307_a, \mathbf{313}, \mathbf{317}, \mathbf{318}, \mathbf{320} - \mathbf{354}$	**320**	355	43%
17	273	273**.**	$273_a, \overline{274_t}, 288_b, 307_a, 342_c, \mathbf{343}, \mathbf{349}, \mathbf{353}, 360_b, 381_a, \mathbf{356} - \mathbf{398}$	**356**	399	42%
18	307	307**.**	$307_a, 342_c, 360_b, 381_a, \mathbf{389}, \mathbf{391}, \mathbf{395} - \mathbf{398}, \mathbf{401}, \mathbf{403} - \mathbf{432}$	**403**	433	32%
19	343	$\ge 345_{s,t}$	$360_b, 381_a, \mathbf{445}, \mathbf{450}, \mathbf{453}, \mathbf{455} - \mathbf{458}, \mathbf{460} - \mathbf{492}$	**460**	493	29%
20	381	381**.**	$381_a, \overline{382}, \mathbf{482}, \mathbf{497}, \mathbf{498}, \mathbf{501} - \mathbf{503}, 506_c, \mathbf{505} - \mathbf{509}, 528_b, 553_a, \mathbf{511} - \mathbf{566}$	**511**	567	37%
21	421	$\ge 423_{s,t}$	$506_c, 528_b, 553_a, \mathbf{586}, \mathbf{589}, \mathbf{591}, \mathbf{592}, \mathbf{594}, \mathbf{595}, \mathbf{597}, \mathbf{598}, 624_b, \mathbf{600} - \mathbf{666}$	**600**	667	32%
22	463	$\ge 465_{br,t}$	$506_c, 528_b, 553_a, 624_b, \mathbf{633}, \mathbf{637}, \mathbf{640} - \mathbf{642}, 651_a, \mathbf{644} - \mathbf{712}$	**644**	713	33%

Key to Table 6.1: $a \to$ (2.1), $b \to$ (2.2), $c \to$ (2.3), $br \to$Bruck-Ryser Theorem, $s \to$ [11], $t \to$Theorem 6.1

Table 6.1 (continue 1) Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51$, $P(k) \le v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k)-1$	$E_c(k)$	$G(k)$
23	507	$\ge 509_{br,t}$	$528_b, 553_a, 624_b, 651_a, \mathbf{683}, \mathbf{686-688}, \mathbf{692}, \mathbf{695-700}, 728_b, \mathbf{702-744}$	$\mathbf{702}$	745
24	553	$553_\bullet$	$553_a, \overline{554}_t, 624_b, 651_a, 728_b, \mathbf{738}, \mathbf{739}, \mathbf{742}, \mathbf{747-749}, \mathbf{752}, \mathbf{753}, \mathbf{755}, 757_a, 812_c, 840_b, \mathbf{757-850}$	$\mathbf{757}$	851
25	601	$\ge 602_s$	$624_b, 651_a, 728_b, 757_a, 812_c, \mathbf{830}, 840_b, 871_a, 930_c, 960_b, \mathbf{837-960}$	$\mathbf{837}$	961
26	651	$651_\bullet$	$651_a, \overline{652}_t, 728_b, 757_a, 812_c, 840_b, 871_a, \mathbf{885}, \mathbf{888}, \mathbf{895}, \mathbf{900}, \mathbf{903}, \mathbf{905-907}, \mathbf{910-913}, \mathbf{915-917}, \mathbf{919-925}, \mathbf{927}, 930_c, 960_b, \mathbf{929-984}$	$\mathbf{929}$	985
27	703	$\ge 704_s$	$728_b, 757_a, 812_c, 840_b, 871_a, 930_c, 960_b, \mathbf{970}, \mathbf{971}, \mathbf{972}, \mathbf{975}, \mathbf{977}, \mathbf{978}, \mathbf{985}, \mathbf{987}, \mathbf{988}, \mathbf{991}, 993_a, \mathbf{993-997}, \mathbf{1000}, \mathbf{1001}, \mathbf{1003-1015}, 1023_b, 1057_a, \mathbf{1017-1106}$	$\mathbf{1017}$	1107
28	757	$757_\bullet$	$757_a, \overline{758}_t, 812_c, 840_b, 871_a, 930_c, 960_b, 993_a, \mathbf{1006}, 1023_b, \mathbf{1045}, \mathbf{1051}, \mathbf{1053}, 1057_a, \mathbf{1063-1067}, \mathbf{1070-1972}, \mathbf{1074}, \mathbf{1075}, \mathbf{1077}, \mathbf{1079-1170}$	$\mathbf{1079}$	1171
29	813	$\ge 815_{s,t}$	$840_b, 871_a, 930_c, 960_b, 993_a, 1023_b, 1057_a, \mathbf{1091}, \mathbf{1127}, \mathbf{1135}, \mathbf{1137}, \mathbf{1141}, \mathbf{1143}, \mathbf{1145}, \mathbf{1146}, \mathbf{1151-1246}$	$\mathbf{1151}$	1247
30	871	$871_\bullet$	$871_a, \overline{872}_t, 930_c, 960_b, 993_a, 1023_b, 1057_a, \mathbf{1196}, \mathbf{1198-1201}, \mathbf{1206}, \mathbf{1207}, \mathbf{1216}, \mathbf{1217}, \mathbf{1219-1224}, 1332_c, \mathbf{1226-1360}$	$\mathbf{1226}$	1361
31	931	$\ge 933_{br,t}$	$960_b, 993_a, 1023_b, 1057_a, \mathbf{1298}, \mathbf{1309}, \mathbf{1314}, \mathbf{1315}, \mathbf{1320}, \mathbf{1321}, \mathbf{1324}, \mathbf{1325}, 1332_c, \mathbf{1330-1335}, \mathbf{1339-1346}, 1368_b, \mathbf{1348-1494}$	$\mathbf{1348}$	1495
32	993	$993_\bullet$	$993_a, \overline{994}_t, 1023_b, 1057_a, 1332_c, \mathbf{1366}, 1368_b, \mathbf{1383}, \mathbf{1388}, \mathbf{1391-1395}, \mathbf{1397}, \mathbf{1398}, \mathbf{1400}, \mathbf{1401}, \mathbf{1403}, 1407_a, \mathbf{1406-1409}, \mathbf{1411-1414}, \mathbf{1416}, \mathbf{1420}, \mathbf{1421}, \mathbf{1424-1434}, \mathbf{1436-1568}$	$\mathbf{1436}$	1569

Key to Table 6.1: $a \to$ (2.1), $b \to$ (2.2), $c \to$ (2.3), $br \to$Bruck-Ryser Theorem, $s \to$ [11], $t \to$Theorem 6.1

Table 6.1 (continue 2) Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51$, $P(k) \le v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k) - 1$	$E_c(k)$	$G(k)$
33	1057	1057**.**	1057_a, $\overline{\mathit{1058}}_t$, 1332_c, 1368_b, 1407_a, **1492, 1506, 1507, 1515, 1518, 1520, 1521, 1528, 1529, 1533, 1535, 1537, 1540, 1542, 1543, 1545, 1547 − 1553, 1555 − 1559**, 1640_c, 1680_b, **1561 − 1718**	**1561**	1719
34	1123	$\ge 1125_{br,t}$	1332_c, 1368_b, 1407_a, 1640_c, **1664, 1665, 1670, 1676**, 1680_b, **1686, 1693, 1698, 1699, 1702, 1705, 1708 − 1712, 1714, 1717, 1721**, 1723_a, **1723 − 1726, 1728, 1730 − 1742, 1744 − 1752**, 1806_c, 1848_b, **1754 − 1876**	**1754**	1877
35	1191	$\ge 1193_{s,t}$	1332_c, 1368_b, 1407_a, 1640_c, 1680_b, 1723_a, **1777, 1781, 1783, 1788, 1792, 1793, 1795, 1798, 1800 − 1803**, 1806_c, **1805 − 1807, 1810, 1812 − 1815**, 1848_b, 1893_a, **1817 − 1974**	**1817**	1975
36	1261	$\ge 1262_s$	1332_c, 1368_b, 1407_a, 1640_c, 1680_b, 1723_a, 1806_c, 1848_b, **1853, 1855, 1860, 1867 − 1870, 1872 − 1876, 1878, 1882 − 1884**, 1893_a, **1886 − 2010**	**1886**	2011
37	1333	$\ge 1335_{s,t}$	1368_b, 1407_a, 1640_c, 1680_b, 1723_a, 1806_c, 1848_b, **1892**, 1893_a, **1910, 1922, 1930, 1934, 1938, 1943, 1944, 1947 − 1953, 1957, 1959, 1960, 1962, 1963, 1965 − 1967, 1969**, 2162_c, **1972 − 2198**	**1972**	2199
38	1407	1407**.**	1407_a, 1640_c, 1680_b, 1723_a, 1806_c, 1848_b, 1893_a, **2059, 2061, 2073, 2088, 2089, 2092, 2094, 2096, 2097, 2099 − 2101, 2103, 2105 − 2108, 2110, 2111, 2114 − 2116, 2118, 2123, 2124, 2126 − 2130, 2135 − 2153, 2155- 2157**, 2162_c, **2159 − 2164, 2166 − 2170**, 2208_b, 2257_a, **2172 − 2292**	**2172**	2293

Key to Table 6.1: $a \to$ (2.1), $b \to$ (2.2), $c \to$ (2.3), $br \to$Bruck-Ryser Theorem, $s \to$ [11], $t \to$Theorem 6.1

Table 6.1 (continue 3) Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51,\ P(k) \le v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k) - 1$	$E_c(k)$	$G(k)$
39	1483	$\ge 1485_{br,t}$	$1640_c, 1680_b, 1723_a, 1806_c, 1848_b, 1893_a$, 2162_c, **2187**, **2195**, 2208_b, **2240**, **2241**, **2243**, **2247**, **2248**, **2251**, **2252**, **2254**, **2255**, 2257_a, **2258 − 2260**, **2263 − 2265**, **2269**, **2270**, **2277 − 2279**, **2281**, **2283**, **2287**, **2289 − 2309**, **2311**, **2313 − 2328**, 2400_b, 2451_a, **2330 − 2504**	**2330**	2505
40	1561	$\ge 1563_{s,t}$	$1640_c, 1680_b, 1723_a, 1806_c, 1848_b, 1893_a$, $2162_c, 2208_b, 2257_a$, **2326**, **2338**, **2345**, **2349**, **2353**, **2355**, **2357**, **2360**, **2361**, **2363**, **2364**, **2366 − 2368**, **2370**, **2372**, **2374 − 2377**, **2379 − 2381**, **2387 − 2389**, **2393**, **2395 − 2397**, **2399**, 2400_b, **2401**, **2403**, **2404**, **2406**, **2407**, **2409**, **2411 − 2418**, 2451_a, **2420 − 2436**, **2438 − 2564**	**2438**	2565
41	1641	$\ge 1643_{s,t}$	$1680_b, 1723_a, 1806_c, 1848_b, 1893_a, 2162_c$, $2208_b, 2257_a$, **2345**, **2399**, 2400_b, **2436**, **2449**, 2451_a, **2459**, **2460**, **2465**, **2471**, **2472**, **2479**, **2480**, **2483**, **2485**, **2491**, **2493**, **2494**, **2496 − 2500**, **2502**, **2503**, **2505**, **2507 − 2513**, **2515 − 2525**, **2528 − 2540**, **2542**, **2544 − 2610**	**2544**	2611
42	1723	1723.	1723_a, $\overline{\mathit{1724}}_t$, $1806_c, 1848_b, 1893_a, 2162_c$, $2208_b, 2257_a, 2400_b, 2451_a$, **2510**, **2522**, **2539**, **2541**, **2557 − 2559**, **2562**, **2564**, **2566 − 2568**, **2570**, **2573**, **2577**, **2578**, **2580 − 2584**, **2586 − 2590**, **2593**, **2595**, **2597 − 2601**, **2603 − 2610**, **2612**, **2613**, **2615 − 2626**, 2756_c, **2628 − 2794**	**2628**	2795

Key to Table 6.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, br →Bruck-Ryser Theorem, $s \to$ [11], t →Theorem 6.1

Table 6.1 (continue 4) Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51,\ P(k) \le v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k) - 1$	$E_c(k)$	$G(k)$
43	1807	$\ge 1809_{br,t}$	$1848_b, 1893_a, 2162_c, 2208_b, 2257_a, 2400_b, 2451_a$, **2684, 2686, 2688, 2715, 2725, 2728, 2734, 2737, 2739, 2744, 2752**, 2756_c, **2757, 2759, 2762, 2763, 2766 − 2768, 2771, 2772, 2776, 2777, 2783, 2786 − 2789, 2791, 2792, 2794 − 2798, 2800, 2801**,2808_b, **2803 − 2811, 2813 − 2815, 2817 − 2858**, 2863_a, **2860 − 3014**	**2860**	3015
44	1893	1893**.**	1893_a, $\overline{\mathit{1894}}_t$,$2162_c, 2208_b, 2257_a, 2400_b, 2451_a$,$2756_c, 2808_b$, **2811, 2821, 2826, 2834, 2836, 2844, 2848, 2849, 2861, 2862**,2863_a, **2865 − 2867, 2870, 2871, 2873 − 2875, 2879 − 2881, 2884, 2887, 2890 − 2895, 2898, 2899, 2901 − 2912, 2914, 2916 − 3192**	**2916**	3193
45	1981	$\ge 1983_{s,t}$	$2162_c, 2208_b, 2257_a, 2400_b, 2451_a, 2756_c, 2808_b, 2863_a$, **2994, 3013, 3014, 3019, 3038, 3052, 3054, 3056, 3066, 3069, 3082 − 3085, 3087, 3088, 3090, 3091, 3093, 3095 − 3098, 3101 − 3103, 3105 − 3112, 3114, 3116 − 3120, 3122 − 3163, 3165 − 3374**	**3165**	3375
46	2071	$\ge 2073_{s,t}$	$2162_c, 2208_b, 2257_a, 2400_b, 2451_a, 2756_c, 2808_b, 2863_a$, **3124, 3171, 3188, 3191, 3194, 3196, 3197, 3198, 3206, 3216, 3218, 3219, 3221, 3227, 3228, 3231, 3233, 3234, 3238 − 3241, 3244 − 3247, 3249, 3250, 3252 − 3265, 3267 − 3271, 3273 − 3278, 3280 − 3406**	**3280**	3407

Key to Table 6.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $br \to$Bruck-Ryser Theorem, $s \to [11]$, $t \to$Theorem 6.1

Table 6.1 (continue 5) Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51,\ P(k) \le v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k)-1$	$E_c(k)$	$G(k)$
47	2163	$2165_{br,t}$	$2208_b, 2257_a, 2400_b, 2451_a, 2756_c, 2808_b, 2863_a, \mathbf{3255}, \mathbf{3261}, \mathbf{3271}, \mathbf{3280}, \mathbf{3285}, \mathbf{3292}, \mathbf{3301}, \mathbf{3312}, \mathbf{3327}, \mathbf{3331}, \mathbf{3342}, \mathbf{3343}, \mathbf{3346-3348}, \mathbf{3351}, \mathbf{3353}, \mathbf{3355-3357}, \mathbf{3360-3371}, \mathbf{3375-3379}, \mathbf{3381-3384}, \mathbf{3387}, \mathbf{3389}, \mathbf{3390}, \mathbf{3392-3402}, \mathbf{3404-3408}, \mathbf{3412}, \mathbf{3413}, 3422_c, \mathbf{3415-3427}, 3480_b, 3541_a, \mathbf{3429-3608}$	**3429**	3609
48	2257	2257**.**	$2257_a, \overline{\mathit{2258}}_t, 2400_b, 2451_a, 2756_c, 2808_b, 2863_a, \mathbf{3418}, 3422_c, \mathbf{3431}, \mathbf{3445}, \mathbf{3459}, 3480_b, \mathbf{3487}, \mathbf{3491}, \mathbf{3492}, \mathbf{3495}, \mathbf{3499}, \mathbf{3509}, \mathbf{3512}, \mathbf{3515}, \mathbf{3518}, \mathbf{3519}, \mathbf{3522}, \mathbf{3523}, \mathbf{3526-3528}, \mathbf{3535}, \mathbf{3540}, 3541_a, \mathbf{3545-3547}, \mathbf{3549-3552}, \mathbf{3554-3563}, \mathbf{3565}, \mathbf{3567}, \mathbf{3568}, \mathbf{3570}, \mathbf{3572-3578}, \mathbf{3580-3583}, \mathbf{3586}, \mathbf{3587}, \mathbf{3589-3595}, \mathbf{3597-3605}, \mathbf{3607-3618}, \mathbf{3620-3630}, \mathbf{3632-3640}, 3660_c, \mathbf{3642-3774}$	**3642**	3775
49	2353	$\ge 2354_s$	$2400_b, 2451_a, 2756_c, 2808_b, 2863_a, 3422_c, 3480_b, 3541_a, \mathbf{3608}, \mathbf{3627}, \mathbf{3637}, \mathbf{3640}, \mathbf{3642}, \mathbf{3644}, \mathbf{3646}, \mathbf{3647}, \mathbf{3649}, \mathbf{3652}, \mathbf{3653}, \mathbf{3655}, 3660_c, \mathbf{3665}, \mathbf{3669}, \mathbf{3671}, \mathbf{3675}, \mathbf{3677}, \mathbf{3678}, \mathbf{3680-3683}, \mathbf{3685}, \mathbf{3688-3692}, \mathbf{3694-3701}, \mathbf{3707}, \mathbf{3709}, 3720_b, \mathbf{3711-3722}, \mathbf{3724-3730}, \mathbf{3732-3735}, \mathbf{3737-3740}, \mathbf{3742}, \mathbf{3743}, \mathbf{3745}, \mathbf{3747-3755}, 3783_a, \mathbf{3757-3825}, \mathbf{3827-3837}, \mathbf{3839-3916}$	**3839**	3917

Key to Table 6.1: $a \to$ (2.1), $b \to$ (2.2), $c \to$ (2.3), $br \to$Bruck-Ryser Theorem, $s \to$ [11], $t \to$Theorem 6.1

Table 6.1 (continue 6). Values of v for which a cyclic symmetric configuration v_k exists, $5 \le k \le 51$, $P(k) \le v < G(k)$

k	$P(k)$	$v_\delta(k)$	$v_\delta(k) \le v \le G(k) - 1$	$E_c(k)$	$G(k)$
50	2451	2451.	2451_a, $\overline{2452}_t$, 2756_c, 2808_b, 2863_a, 3422_c, 3480_b, 3541_a, 3660_c, **3685**, **3688**, **3692**, **3712**, **3714**, **3716**, 3720_b, **3726**, **3743**, **3745**, **3749**, **3750**, **3752**, **3753**, **3758**, **3762**, **3766**, **3767**, **3769**, **3770**, **3772**, **3775**, **3779**, **3780**, 3783_a, **3782 − 3785**, **3788 − 3791**, **3796**, **3803**, **3805**, **3811**, **3813**, **3817 − 3823**, **3825**, **3828 − 3832**, **3834 − 3840**, **3842**, **3843**, **3845**, **3846**, **3848 − 3869**, 4095_b, 4161_a, **3871 − 4188**	**3871**	4189
51	2551	$\ge 2552_s$	2756_c, 2808_b, 2863_a, 3422_c, 3480_b, 3541_a, 3660_c, 3720_b, 3783_a, **3871**, **3894**, **3927**, **3938**, **3986**, **3998**, **4004**, **4018**, **4032**, **4035**, **4037**, **4042**, **4048 − 4050**, **4053**, **4060**, **4064 − 4066**, **4068**, **4072**, **4076**, **4079 − 4083**, **4085**, **4087**, **4090**, **4091**, **4093**, 4095_b, **4095 − 4100**, **4102 − 4104**, **4107 − 4109**, **4112 − 4120**, **4124**, **4125**, **4127 − 4131**, **4133 − 4156**, 4161_a, **4158 − 4164**, **4166 − 4196**, **4199**, **4200**, **4202 − 4206**, **4208 − 4231**, **4233 − 4380**	**4233**	4381

Key to Table 6.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $br \to$Bruck-Ryser Theorem, $s \to$ [11], $t \to$Theorem 6.1

Table 6.2. Upper bounds on the cyclic existence bound $E_c(k)$, $52 \le k \le 83$

k	$E_c(k)$	$G(k)$	k	$E_c(k)$	$G(k)$	k	$E_c(k)$	$G(k)$	k	$E_c(k)$	$G(k)$
52	**4359**	4541	60	**5687**	6039	68	**7463**	7913	76	**10023**	10179
53	**4463**	4695	61	**5994**	6269	69	**8111**	8291	77	**10229**	10409
54	**4513**	4747	62	**6150**	6431	70	**8125**	8435	78	**10395**	10599
55	**5195**	5197	63	**6611**	6783	71	**8288**	8661	79	**10800**	10817
56	**5341**	5451	64	**6796**	7055	72	**8694**	8947	80	**10977**	11127
57	**5501**	5547	65	**6853**	7187	73	**8813**	9027	81	**11396**	11435
58	**5551**	5703	66	**7279**	7515	74	**8965**	9507	82	**11443**	11629
59	**5612**	5823	67	**7359**	7639	75	**9883**	9965	83	**11593**	12041

Table 6.3. New cyclic symmetric configurations v_k

k	v	the 1-st row of incidence matrix
17	382	0,25,69,81,88,89,112,123,126,128,141,174,196,202,206,223,232
17	383 385 386 388	0,5,15,34,35,42,73,75,86,89,98,134,151,155,177,183,201
17	384	0,68,70,84,90,107,111,120,139,151,185,186,193,196,211,244,249
17	387	0,5,7,17,52,56,67,80,81,100,122,138,159,165,168,191,199
18	389	0,10,27,28,35,50,74,94,103,105,108,146,159,165,191,195,207,228
18	391	0, 1, 4,15,35,42,75,85,94,111,133,139,141,157,162,194,206,219
18	395	0,71,73,81,85,88 117 118 141 167 172 183 192 210 231 244 250 272
18	396	0,79,89,106,107,114,129,153,173,182,184,187,225,238,244,270,274,286
18	397	0,36,50,56,59,74,78,85,122,139,147,149,179,180,192,226,231,247
18	398	0,18,30,32,71,84,90,93,119,127,152,169,176,192,196,197,207,243
18	401	0,71,94,104,136,160,164,176,195,217,238,243,256,263,290,292,293,301
18	404	0, 2,10,14,17,46,47,70,96,101,112,121,139,160,173,179,201,236
18	405 407	0,2,10,22,53,56,82,83,89,98,130,148,153,167,188,192,205,216
18	406	0,49,59,62,97,99,117,141,173,180,184,192,201,206,207,237,253,278
18	408	0,14,15,27,50,53,60,81,97,115,137,139,145,156,188,208,213,217
18	409	0,91,193,195,203,215,246,249,275,276,282,291,323,341,346,360,381,385
18	410	0,36,68,103,110,111,121,124,130,161,176,200,202,225,230,247,259,263
18	411	0,93,195,197,205,217,248,251,277,278,284,293,325,343,348,362,383,387

8.3 Tables for Section 7

Table 7.1. The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \le v \le G(k)-1$	$E(k) \le$	$G(k)$	filling
3.	7	7	7.	7	100%
4.	13	13	13.	13	100%
5.	21	$21_a, \overline{22}_t$	23.	23	100%
6.	31	$31_a, \overline{32}_t, \overline{33}, 34$	34.	35	100%
7	43	$\overline{43}_{br}, \overline{44}_t, 45, 48_{b\cdot f\cdot r}, 49_{m\cdot r}, 50_{m\cdot r}$	48	51	75%
8	57	$57_a, \overline{58}_t, 63_{b\cdot f\cdot r}, 64_{m\cdot r} - 68_{m\cdot r}$	63	69	67%
9	73	$73_a, \overline{74}_t, 78_{g\cdot h}, 80_{b\cdot f\cdot r}, 81_{m\cdot r} - 88_{m\cdot r}$	80	89	75%
10	91	$91_a, \overline{92}_t, 98_j, 107_y - 109_y, 110_{c\cdot f\cdot k\cdot m\cdot r\cdot S\cdot T}$	107	111	35%
11	111	$\overline{111}, \overline{112}, 120_{b\cdot f\cdot r}, 121_{m\cdot r} - 133_{m\cdot r}, 135_y - 142_y, 143_{m\cdot y\cdot S}, 144_{m\cdot y\cdot r\cdot S\cdot T}$	135	145	76%
12	133	$133_a, \overline{134}_t, 135, 154_\lambda, 155_\lambda, 156_{m\cdot r\cdot S\cdot T} - 169_{m\cdot r\cdot S\cdot T}, 170_{m\cdot r\cdot T\cdot X}$	154	171	52%
13	157	$\overline{158}_t, 168_{b\cdot f\cdot r}, 169_{m\cdot r} - 183_{m\cdot r}, 189_g, 193_y - 209_y, 210_{m\cdot y\cdot r} - 212_{m\cdot y\cdot r}$	193	213	68%
14	183	$183_a, \overline{184}_t, 210_g, 222_\lambda, 223_\lambda, 224_m, 225_{m\cdot y\cdot r} - 254_{m\cdot y\cdot r}$	222	255	50%
15	211	$\overline{211}_{br}, \overline{212}_t, 231_g, 238_\lambda, 239_\lambda, 240_{m\cdot r} - 266_{m\cdot r}, 267_{m\cdot y\cdot r} - 302_{m\cdot y\cdot r}$	238	303	73%
16	241	$252_{g\cdot h}, 255_{b\cdot f\cdot r}, 256_{m\cdot r} - 321_{m\cdot r}, 322_{\lambda\cdot r\cdot T\cdot Z}, 323_{m\cdot r\cdot S} - 354_{m\cdot r\cdot S}$	255	355	89%
17	273	$273_a, \overline{274}_t, 288_{b\cdot f\cdot r}, 289_{m\cdot r} - 307_{m\cdot r}, 321_\lambda, 322_\lambda, 323_{m\cdot S}, 372_W, 324_{m\cdot r} - 381_{m\cdot r}, \mathbf{382_Z - 388_Z}, 389_{\lambda\cdot Z}, 390_{\lambda\cdot Z}, 391_{m\cdot S\cdot Z} - 398_{m\cdot S\cdot Z}$	**321**	399	79%
18	307	$307_a, 340_\lambda, 341_\lambda, 342_{m\cdot r} - 381_{m\cdot r}, \mathbf{389}_Z, \mathbf{391}_Z, \mathbf{395_Z - 398_Z}, \mathbf{401_Z}, 403_{g\cdot Z}, \mathbf{404_Z - 411_Z}, 412_{\lambda\cdot Z}, 413_{\lambda\cdot Z}, 414_{m\cdot S\cdot Z} - 432_{m\cdot S\cdot Z}$	**403**	433	63%

Key to Table 7.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $f \to (2.5)$, $g \to (2.6)$, $h \to (2.7)$, $j \to (2.8)$, $k \to (2.9)$, $\lambda \to (2.13) - (2.16)$, $m \to (5.1)$, $P \to (4.3)$, $r \to (5.4)$, $S \to (5.5)$, $T \to (5.6)$, $W \to$ Table 4.1, $X \to$ Table 4.2, $y \to$ Table 6.1 with $k \le 15$, $Z \to$ Table 6.1 with $k > 15$, $br \to$ Bruck-Ryser Theorem, $t \to$ Theorem 6.1

Table 7.1 (continue 1). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k)-1$	$E(k) \leq$	$G(k)$	filling
19	343	$\overline{344}_t, 360_{b\cdot f\cdot r}, 361_{m\cdot r}-381_{m\cdot r}, 434_{g\cdot W}, 435_\lambda, 436_\lambda,$ $437_{m\cdot S}-457_{m\cdot S}, 458_{\lambda\cdot r\cdot T}, 459_{\lambda\cdot r\cdot T},$ $465_W, 460_{m\cdot r\cdot T}-492_{m\cdot r\cdot T}$	434	493	54%
20	381	$381_a, \overline{382}_t, 458_\lambda, 459_\lambda, 465_W, 460_{m\cdot S}-481_{m\cdot S}, 482_{\lambda\cdot r\cdot T},$ $558_W, 483_{m\cdot r\cdot T}-566_{m\cdot r\cdot T}$	458	567	59%
21	421	$\overline{422}_t, 481_\lambda, 482_\lambda, 483_{m\cdot S}, 558_W, 640_X, 484_{m\cdot r}-666_{m\cdot r}$	481	667	76%
22	463	$\overline{463}_{br}, \overline{464}_t, 504_\lambda, 505_\lambda, 558_W, 506_{m\cdot r}-573_{m\cdot r}, 574_{\lambda\cdot r},$ $640_X, 575_{m\cdot r}-712_{m\cdot r}$	504	713	84%
23	507	$\overline{507}_{br}, \overline{508}_t, 528_{b\cdot f\cdot r}, 529_{m\cdot r}-553_{m\cdot r}, 558_{g\cdot W},$ $573_\lambda, 574_\lambda, 575_m, 576_{m\cdot r}-744_{m\cdot r}$	573	745	84%
24	553	$553_a, \overline{554}_t, 589_g, 598_\lambda, 599_\lambda, 600_{m\cdot r}-673_{m\cdot r},$ $674_{\lambda\cdot r}, 675_{m\cdot r}-850_{m\cdot r}$	598	851	85%
25	601	$620_{g\cdot h}, 624_{b\cdot f\cdot P\cdot r}, 625_{m\cdot r}-651_{m\cdot r}, 673_\lambda,$ $674_\lambda, 675_m, 906_W, 912_X, 938_W, 676_{m\cdot r}-960_{m\cdot r}$	673	961	88%
26	651	$651_a, \overline{652}_t, 700_\lambda, 701_\lambda, 702_{m\cdot r}-781_{m\cdot r}, 782_r,$ $783_{m\cdot r}-984_{m\cdot r}$	700	985	85%
27	703	$728_{b\cdot f\cdot r}, 729_{m\cdot r}-757_{m\cdot r}, 781_\lambda, 782_\lambda, 783_{m\cdot S},$ $784_{m\cdot r}-1065_{m\cdot r}, 1066_{r\cdot T\cdot Z}-1072_{r\cdot T\cdot Z},$ $1073_{m\cdot S\cdot Z}-1103_{m\cdot S\cdot Z}, 1104_{r\cdot T\cdot Z}, 1105_{\lambda\cdot r\cdot T\cdot Z},$ $1106_{\lambda\cdot r\cdot T\cdot Z}$	781	1107	88%
28	757	$757_a, \overline{758}_t, 810_\lambda, 811_\lambda, 812_{m\cdot r}-1065_{m\cdot r},$ $1066_{r\cdot T}-1072_{r\cdot T}, 1073_{m\cdot S}-1103_{m\cdot S},$ $1104_{r\cdot T\cdot Z}-1109_{r\cdot T\cdot Z}, 1110_{m\cdot r\cdot S\cdot Z}-1141_{m\cdot r\cdot S\cdot Z},$ $1142_{r\cdot T\cdot Z}-1145_{r\cdot T\cdot Z}, 1146_{\lambda\cdot r\cdot T\cdot Z},$ $1147_{m\cdot r\cdot S\cdot Z}-1170_{m\cdot r\cdot S\cdot Z}$	810	1171	87%

Key to Table 7.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $f \to (2.5)$, $g \to (2.6)$, $h \to (2.7)$, $j \to (2.8)$, $k \to (2.9)$, $\lambda \to (2.13)-(2.16)$, $m \to (5.1), P \to (4.3), r \to (5.4), S \to (5.5)$, $T \to (5.6), W \to$ Table 4.1, $X \to$Table 4.2, $y \to$ Table 6.1 with $k \leq 15$, $Z \to$ Table 6.1 with $k > 15$, $br \to$Bruck-Ryser Theorem, $t \to$Theorem 6.1

Table 7.1 (continue 2). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k)-1$	$E(k) \leq$	$G(k)$	filling
29	813	$\overline{814}_t, 840_{b\cdot f\cdot r}, 841_{m\cdot r}-871_{m\cdot r}, 897_\lambda, 898_\lambda, 899_{m\cdot S},$ $900_{m\cdot r}-1057_{m\cdot r}, 1071_\lambda, 1072_\lambda, 1073_{m\cdot S}-1103_{m\cdot S},$ $1104_{r\cdot T}-1109_{r\cdot T}, 1110_{m\cdot S}-1141_{m\cdot S},$ $1142_{r\cdot T}-1146_{r\cdot T}, 1147_{m\cdot S}-1179_{m\cdot S},$ $1180_{r\cdot T\cdot Z}-1183_{r\cdot T\cdot Z}, 1184_{m\cdot S}-1219_{m\cdot S},$ $1220_{r\cdot T\cdot Z}, 1221_{m\cdot S}-1246_{m\cdot S}$	1071	1247	85%
30	871	$871_a, \overline{872}_t, 928_\lambda, 929_\lambda, 930_{m\cdot r}-1057_{m\cdot r}, 1108_\lambda,$ $1109_\lambda, 1110_{m\cdot S}-1141_{m\cdot S}, 1142_{r\cdot T}-1146_{r\cdot T},$ $1147_{m\cdot S}-1179_{m\cdot S}, 1180_{r\cdot T}-1183_{r\cdot T},$ $1184_{m\cdot S}-1217_{m\cdot S}, 1218_{r\cdot T}-1220_{r\cdot T}, 1262_W,$ $1221_{m\cdot S}-1360_{m\cdot S}$	1108	1361	78%
31	931	$\overline{931}_{br}, \overline{932}_t, 960_{b\cdot f\cdot r}, 961_{m\cdot r}-1057_{m\cdot r}, 1145_\lambda,$ $1146_\lambda, 1147_{m\cdot S}-1179_{m\cdot S}, 1180_{r\cdot T}-1183_{r\cdot T},$ $1184_{m\cdot S}-1217_{m\cdot S}, 1218_{r\cdot T}-1220_{r\cdot T},$ $1221_{m\cdot S}-1255_{m\cdot S}, 1256_{r\cdot T}, 1257_{r\cdot T},$ $1262_W, 1258_{m\cdot S\cdot r\cdot T}-1494_{m\cdot S\cdot r\cdot T}$	1145	1495	79%
32	993	$993_a, \overline{994}_t, 1023_{b\cdot f\cdot r}, 1024_{m\cdot r}-1057_{m\cdot r}, 1182_\lambda,$ $1183_\lambda, 1184_{m\cdot S}-1217_{m\cdot S}, 1218_{r\cdot T}-1220_{r\cdot T},$ $1221_{m\cdot S}-1255_{m\cdot S}, 1256_{r\cdot T}, 1257_{r\cdot T},$ $1258_{m\cdot S\cdot r\cdot T}-1293_{m\cdot S\cdot r\cdot T}, 1294_{r\cdot T},$ $1533_W, 1295_{m\cdot S\cdot r\cdot T}-1568_{m\cdot S\cdot r\cdot T}$	1182	1569	73%
33	1057	$1057_a, \overline{1058}_t, 1219_\lambda, 1220_\lambda, 1221_{m\cdot S}-1255_{m\cdot S},$ $1256_{r\cdot T}, 1257_{r\cdot T}, 1258_{m\cdot S\cdot r\cdot T}-1293_{m\cdot S\cdot r\cdot T},$ $1294_{r\cdot T}, 1634_W, 1295_{m\cdot r}-1718_{m\cdot r}$	1219	1719	75%
34	1123	$\overline{1123}_{br}, \overline{1124}_t, 1256_\lambda, 1257_\lambda, 1258_{m\cdot S}-1293_{m\cdot S},$ $1294_{r\cdot T}, 1295_{m\cdot r}-1429_{m\cdot r}, 1430_{r\cdot T}-1434_{r\cdot T},$ $1634_W, 1800_X, 1435_{m\cdot S}-1876_{m\cdot S}$	1256	1877	82%

Key to Table 7.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $f \to (2.5)$, $g \to (2.6)$, $h \to (2.7)$, $j \to (2.8)$, $k \to (2.9)$, $\lambda \to (2.13)-(2.16)$, $m \to (5.1), P \to (4.3), r \to (5.4), S \to (5.5)$, $T \to (5.6), W \to$ Table 4.1, $X \to$Table 4.2, $y \to$ Table 6.1 with $k \leq 15$, $Z \to$ Table 6.1 with $k > 15$, $br \to$Bruck-Ryser Theorem, $t \to$Theorem 6.1

Table 7.1 (continue 3). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \le v \le G(k)-1$	$E(k) \le$	$G(k)$	filling
35	1191	$\overline{\mathit{1192}}_t, 1293_\lambda, 1294_\lambda, 1295_{m\cdot S}, 1296_{m\cdot r} - 1407_{m\cdot r}$, $1433_\lambda, 1434_\lambda, 1435_{m\cdot S} - 1471_{m\cdot S}$, $1472_{r\cdot T} - 1475_{r\cdot T}, 1800_P, 1476_{m\cdot S} - 1974_{m\cdot S}$	1433	1975	83%
36	1261	$1330_\lambda, 1331_\lambda, 1332_{m\cdot r} - 1407_{m\cdot r}, 1474_\lambda$, $1475_\lambda, 1476_{m\cdot S} - 1513_{m\cdot S}, 1514_{r\cdot T} - 1516_{r\cdot T}$, $1517_{m\cdot S} - 1519_{m\cdot S}, 2000_X, 1520_{m\cdot r\cdot T} - 2010_{m\cdot r\cdot T}$	1474	2011	82%
37	1333	$\overline{\mathit{1334}}_t, 1368_{b\cdot f\cdot r}, 1369_{m\cdot r} - 1407_{m\cdot r}$, $1515_\lambda, 1516_\lambda, 1517_{m\cdot S} - 1555_{m\cdot S}$, $1556_{r\cdot T} - 1557_{r\cdot T}, 1558_{m\cdot r} - 2198_{m\cdot r}$	1515	2199	83%
38	1407	$1407_a, 1556_\lambda, 1557_\lambda, 1558_{m\cdot S}, 1559_{m\cdot S}$, $1560_{m\cdot r\cdot S\cdot T} - 1597_{m\cdot r\cdot S\cdot T}, 1598_{r\cdot T}$, $1599_{m\cdot r\cdot T} - 1761_{m\cdot r\cdot T}, 1762_{r\cdot T}, 1763_{m\cdot r} - 2292_{m\cdot r}$	1556	2293	83%
39	1483	$\overline{\mathit{1483}}_{br}, \overline{\mathit{1484}}_t, 1597_\lambda, 1598_\lambda, 1599_{m\cdot S}$, $1600_{m\cdot r\cdot T} - 1761_{m\cdot r\cdot T}, 1762_{\mathbf{r\cdot T}}, 1763_{m\cdot r} - 2504_{m\cdot r}$	1597	2505	89%
40	1561	$\overline{\mathit{1562}}_t, 1638_\lambda, 1639_\lambda, 1640_{m\cdot r} - 1761_{m\cdot r}$, $1762\ _{r\cdot T}, 1763_{m\cdot r} - 1921_{m\cdot r}, 1922_{r\cdot T} - 1926_{r\cdot T}$, $1927_m - 1931_m, 1932_{m\cdot r} - 2564_{m\cdot r}$	1638	2565	92%
41	1641	$\overline{\mathit{1642}}_t, 1680_{b\cdot r}, 1681_{m\cdot r} - 1723_{m\cdot r}, 1761_\lambda, 1762_\lambda$, $1763_m, 1764_{m\cdot r} - 1893_{m\cdot r}, 1925_\lambda, 1926_\lambda$, $1927_{m\cdot S} - 1969_{m\cdot S}, 1970_{r\cdot T} - 1973_{r\cdot T}, 1974_m - 2610_m$	1925	2611	92%
42	1723	$1723_a, \overline{\mathit{1724}}_t, 1804_\lambda, 1805_\lambda, 1806_{m\cdot r} - 1893_{m\cdot r}$, $1972_\lambda, 1973_\lambda, 1974_{m\cdot S} - 2017_{m\cdot S}$, $2018_{r\cdot T} - 2020_{r\cdot T}, 2021_m - 2794_m$	1972	2795	85%
43	1807	$\overline{\mathit{1807}}_{br}, \overline{\mathit{1808}}_t, 1848_{b\cdot r}, 1849_{m\cdot r} - 1893_{m\cdot r}, 2019_\lambda$, $2020_\lambda, 2021_{m\cdot S} - 2065_{m\cdot S}, 2066_{r\cdot T} - 2067_{r\cdot T}$, $2068_{m\cdot r} - 2485_{m\cdot r}, 2486_{r\cdot T} - 2490_{r\cdot T}$, $2491_{m\cdot S} - 2593_{m\cdot S}, 2594_{r\cdot T} - 2595_{r\cdot T}$, $2596_{m\cdot S} - 3014_{m\cdot S}$	2019	3015	86%

Key to Table 7.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $f \to (2.5)$, $g \to (2.6)$, $h \to (2.7)$, $j \to (2.8)$, $k \to (2.9)$, $\lambda \to (2.13) - (2.16)$, $m \to (5.1)$, $P \to (4.3)$, $r \to (5.4)$, $S \to (5.5)$, $T \to (5.6)$, $W \to$ Table 4.1, $X \to$Table 4.2, $y \to$ Table 6.1 with $k \le 15$, $Z \to$ Table 6.1 with $k > 15$, $br \to$Bruck-Ryser Theorem, $t \to$Theorem 6.1

Table 7.1 (continue 4). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \le v \le G(k)-1$	$E(k) \le$	$G(k)$	filling
44	1893	$1893_a, \overline{\mathit{1894}}_t, 2066_\lambda, 2067_\lambda, 2068_{m\cdot S} - 2113_{m\cdot S}$, $2114_{r\cdot T}, 2115_m - 2301_m, 2302_r, 2303_m - 2485_m$, $2486_{r\cdot T} - 2490_{r\cdot T}, 2491_{m\cdot S} - 2539_{m\cdot S}$, $2540_{r\cdot T} - 2543_{r\cdot T}, 2544_{m\cdot S} - 2593_{m\cdot S}$, $2594_{r\cdot T} - 2595_{r\cdot T}, 2596_{m\cdot S} - 2647_{m\cdot S}$, $2648_{r\cdot T} - 2649_{r\cdot T}, 2650_{m\cdot S} - 3192_{m\cdot S}$	2066	3193	86%
45	1981	$\overline{\mathit{1982}}_t, 2113_\lambda, 2114_\lambda, 2115_{m\cdot S}, 2116_{m\cdot r} - 2301_{m\cdot r}$, $2302_r, 2303_{m\cdot r} - 2485_{m\cdot r}$, $2486_{r\cdot T} - 2490_{r\cdot T}$, $2491_{m\cdot S} - 2539_{m\cdot S}, 2540_{r\cdot T} - 2543_{r\cdot T}$, $2544_{m\cdot S} - 2593_{m\cdot S}, 2594_{r\cdot T} - 2596_{r\cdot T}$, $2597_{m\cdot S} - 2647_{m\cdot S}, 2648_{r\cdot T} - 2649_{r\cdot T}$, $2650_{m\cdot r\cdot S\cdot T} - 2701_{m\cdot r\cdot S\cdot T}, 2702_{r\cdot T}$, $2703_{m\cdot r\cdot S\cdot T} - 3374_{m\cdot r\cdot S\cdot T}$	2113	3375	90%
46	2071	$\overline{\mathit{2072}}_t, 2160_\lambda, 2161_\lambda, 2162_{m\cdot r} - 2301_{m\cdot r}, 2302_r$, $2303_{m\cdot r} - 2485_{m\cdot r}, 2486_{r\cdot T} - 2490_{r\cdot T}$, $2491_{m\cdot S} - 2539_{m\cdot S}, 2540_{r\cdot T} - 2543_{r\cdot T}$, $2544_{m\cdot S} - 2593_{m\cdot S}, 2594_{r\cdot T} - 2596_{r\cdot T}$, $2597_{m\cdot S} - 2647_{m\cdot S}, 2648_{r\cdot T} - 2649_{r\cdot T}$, $2650_{m\cdot r\cdot S\cdot T} - 2701_{m\cdot r\cdot S\cdot T}, 2702_{r\cdot T}$, $2703_{m\cdot r\cdot S\cdot T} - 3446_{m\cdot r\cdot S\cdot T}$	2160	3407	93%
47	2163	$\overline{\mathit{2163}}_{br}, \overline{\mathit{2164}}_t, 2208_{b\cdot r}, 2209_{m\cdot r} - 2257_{m\cdot r}$, $2301_\lambda, 2302_\lambda, 2303_m, 2304_{m\cdot r} - 2451_{m\cdot r}, 2489_\lambda$, $2490_\lambda, 2491_{m\cdot S} - 2539_{m\cdot S}, 2540_{r\cdot T} - 2543_{r\cdot T}$, $2544_{m\cdot S} - 2593_{m\cdot S}, 2594_{r\cdot T} - 2596_{r\cdot T}$, $2597_{m\cdot S} - 2647_{m\cdot S}, 2648_{r\cdot T} - 2649_{r\cdot T}$, $2650_{m\cdot r\cdot S} - 2701_{m\cdot r\cdot S}, 2702_{r\cdot T}, 2703_{m\cdot r} - 3608_{m\cdot r}$	2489	3609	91%

Key to Table 7.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $f \to (2.5)$, $g \to (2.6)$, $h \to (2.7)$, $j \to (2.8)$, $k \to (2.9)$, $\lambda \to (2.13) - (2.16)$, $m \to (5.1)$, $P \to (4.3)$, $r \to (5.4)$, $S \to (5.5)$, $T \to (5.6)$, $W \to$ Table 4.1, $X \to$Table 4.2, $y \to$ Table 6.1 with $k \le 15$, $Z \to$ Table 6.1 with $k > 15$, $br \to$Bruck-Ryser Theorem, $t \to$Theorem 6.1

Table 7.1 (continue 5). The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic)

k	$P(k)$	$P(k) \leq v \leq G(k) - 1$	$E(k) \leq$	$G(k)$	filling
48	2257	$2257_a, \overline{\mathit{2258}}_t, 2350_\lambda, 2351_\lambda, 2352_{m \cdot r} - 2451_{m \cdot r},$ $2542_\lambda, 2543_\lambda, 2544_{m \cdot S} - 2593_{m \cdot S},$ $2594_{r \cdot T} - 2596_{r \cdot T}, 2597_{m \cdot S} - 2647_{m \cdot S},$ $2648_{r \cdot T} - 2649_{r \cdot T}, 2650_{m \cdot r \cdot S \cdot T} - 2701_{m \cdot r \cdot S \cdot T},$ $2702_{r \cdot T}, 2703_{m \cdot r} - 2881_{m \cdot r},$ $2882_{r \cdot T} - 2890_{r \cdot T}, 2891_{m \cdot S} - 3774_{m \cdot S}$	2542	3775	88%
49	2353	$2400_{b \cdot r}, 2401_{m \cdot r} - 2451_{m \cdot r}, 2595_\lambda, 2596_\lambda,$ $2597_{m \cdot S} - 2647_{m \cdot S}, 2648_{r \cdot T} - 2649_{r \cdot T},$ $2650_{m \cdot r \cdot S \cdot T} - 2701_{m \cdot r \cdot S \cdot T}, 2702_{r \cdot T},$ $2703_{m \cdot r} - 2863_{m \cdot r}, 2889_\lambda, 2890_\lambda,$ $2891_{m \cdot S} - 2941_{m \cdot S}, 2942_{r \cdot T} - 2949_{r \cdot T},$ $2950_{m \cdot S} - 3916_{m \cdot S}$	2889	3917	86%
50	2451	$2451_a, \overline{\mathit{2452}}_t, 2648_\lambda, 2649_\lambda, 2650_{m \cdot S} - 2701_{m \cdot S},$ $2702_{r \cdot T}, 2703_{m \cdot r} - 2863_{m \cdot r}, 2948_\lambda, 2949_\lambda,$ $2950_{m \cdot S} - 3001_{m \cdot S}, 3002_{r \cdot T} - 3008_{r \cdot T},$ $3009_{m \cdot S} - 4188_{m \cdot S}$	2948	4189	83%
51	2551	$2701_\lambda, 2702_\lambda, 2703_{m \cdot S}, 2704_{m \cdot r} - 2863_{m \cdot r},$ $3007_\lambda, 3008_\lambda, 3009_{m \cdot S} - 3061_{m \cdot S},$ $3062_{r \cdot T} - 3067_{r \cdot T}, 3068_{m \cdot S} - 4380_{m \cdot S}$	3007	4381	83%

Key to Table 7.1: $a \to (2.1)$, $b \to (2.2)$, $c \to (2.3)$, $f \to (2.5)$, $g \to (2.6)$, $h \to (2.7)$, $j \to (2.8)$, $k \to (2.9)$, $\lambda \to (2.13) - (2.16)$, $m \to (5.1), P \to (4.3), r \to (5.4), S \to (5.5)$, $T \to (5.6), W \to$ Table 4.1, $X \to$Table 4.2, $y \to$ Table 6.1 with $k \leq 15$, $Z \to$ Table 6.1 with $k > 15$, br →Bruck-Ryser Theorem, t →Theorem 6.1

Table 7.2. The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic). Constructions are not remarked.

k	$P(k)$	$P(k) \leq v \leq G(k) - 1$	$E(k) \leq$	$G(k)$	filling
3.	7	7	7.	7	100%
4.	13	13	13.	13	100%
5.	21	$21, \overline{22}$	23.	23	100%
6.	31	$31, \overline{32}, \overline{33}, 34$	34.	35	100%
7	43	$\overline{43}, \overline{44}, 45, 48, 49, 50$	48	51	75%
8	57	$57, \overline{58}, 63 - 68$	63	69	67%
9	73	$73, \overline{74}, 78, 80 - 88$	80	89	75%
10	91	$91, \overline{92}, 98, 107 - 110$	107	111	35%
11	111	$\overline{111}, \overline{112}, 120 - 133, 135 - 144$	135	145	76%
12	133	$133, \overline{134}, 135, 154 - 170$	154	171	52%
13	157	$\overline{158}, 168 - 183, 189, 193 - 212$	193	213	68%
14	183	$183, \overline{184}, 210, 222 - 254$	222	255	50%
15	211	$\overline{211}, \overline{212}, 231, 238 - 302$	238	303	73%
16	241	$252, 255 - 354$	255	355	89%
17	273	$273, \overline{274}, 288 - 307, 321 - 398$	**321**	399	79%
18	307	$307, 340 - 381, 389, 391, 395 - 398, 401, 403 - 432$	**403**	433	63%
19	343	$\overline{344}, 360 - 381, 434 - 492$	434	493	54%
20	381	$381, \overline{382}, 458 - 566$	458	567	59%
21	421	$\overline{422}, 481 - 666$	481	667	76%
22	463	$\overline{463}, \overline{464}, 504 - 712$	504	713	84%
23	507	$\overline{507}, \overline{508}, 528 - 553, 558, 573 - 744$	573	745	84%
24	553	$553, \overline{554}, 589, 598 - 850$	598	851	85%
25	601	$620, 624 - 651, 673 - 960$	673	961	88%
26	651	$651, \overline{652}, 700 - 984$	700	985	85%
27	703	$728 - 757, 781 - 1106$	781	1107	88%
28	757	$757, \overline{758}, 810 - 1170$	810	1171	87%
29	813	$\overline{814}, 840 - 871, 897 - 1057, 1071 - 1246$	1071	1247	85%
30	871	$871, \overline{872}, 928 - 1057, 1108 - 1360$	1108	1361	78%
31	931	$\overline{931}, \overline{932}, 960 - 1057, 1145 - 1494$	1145	1495	79%
32	993	$993, \overline{994}, 1023 - 1057, 1182 - 1568$	1182	1569	73%

Table 7.2 (continue)**.** The currently known parameters of symmetric configurations v_k (cyclic and non-cyclic). Constructions are not remarked.

k	$P(k)$	$P(k) \le v \le G(k) - 1$	$E(k) \le$	$G(k)$	filling
33	1057	$1057, \overline{1058}, 1219 - 1718$	1219	1719	75%
34	1123	$\overline{1123}, \overline{1124}, 1256 - 1876$	1256	1877	82%
35	1191	$\overline{1192}, 1293 - 1407, 1433 - 1974$	1433	1975	83%
36	1261	$1330 - 1407, 1474 - 2010$	1474	2011	82%
37	1333	$\overline{1334}, 1368 - 1407, 1515 - 2198$	1515	2199	83%
38	1407	$1407, 1556 - 2292$	1556	2293	83%
39	1483	$\overline{1483}, \overline{1484}, 1597 - 2504$	1597	2505	89%
40	1561	$\overline{1562}, 1638 - 2564$	1638	2565	92%
41	1641	$\overline{1642}, 1680 - 1723, 1761 - 1893, 1925 - 2610$	1925	2611	92%
42	1723	$1723, \overline{1724}, 1804 - 1893, 1972 - 2794$	1972	2795	85%
43	1807	$\overline{1807}, \overline{1808}, 1848 - 1893, 2019 - 3014$	2019	3015	86%
44	1893	$1893, \overline{1894}, 2066 - 3192$	2066	3193	86%
45	1981	$\overline{1982}, 2113 - 3374$	2113	3375	90%
46	2071	$\overline{2072}, 2160 - 3446$	2160	3407	93%
47	2163	$\overline{2163}, \overline{2164}, 2208 - 2257, 2301 - 2451, 2489 - 3608$	2489	3609	91%
48	2257	$2257, \overline{2258}, 2350 - 2451, 2542 - 3774$	2542	3775	88%
49	2353	$2400 - 2451, 2595 - 2863, 2889 - 3916$	2889	3917	86%
50	2451	$2451, \overline{2452}, 2648 - 2863, 2948 - 4188$	2948	4189	83%
51	2551	$2701 - 2863, 3007 - 4380$	3007	4381	83%
52	2653	$\overline{2654}, 2754 - 2863, 3066 - 4540$	3066	4541	84%
53	2757	$\overline{2758}, 2808 - 2863, 3125 - 4694$	3125	4695	83%
54	2863	$2863, \overline{2864}, 3184 - 4746$	3184	4747	83%
55	2971	$\overline{2971}, \overline{2972}, 3243 - 5196$	3243	5197	88%
56	3081	$\overline{3082}, 3302 - 5450$	3302	5451	91%
57	3193	$\overline{3194}, 3361 - 5546$	3361	5547	93%
58	3307	$\overline{3307}, \overline{3308}, 3420 - 5702$	3420	5703	95%
59	3423	$\overline{3424}, 3480 - 3541, 3597 - 5822$	3597	5823	95%
60	3541	$3541, \overline{3542}, 3658 - 3783, 3838 - 6038$	3838	6039	93%
61	3661	$\overline{3662}, 3720 - 3783, 3902 - 6268$	3902	6269	93%
62	3783	$3783, 3966 - 6430$	3966	6431	93%
63	3907	$\overline{3907}, \overline{3908}, 4030 - 4161, 4219 - 6782$	4219	6783	94%
64	4033	$4095 - 4161, 4286 - 7054$	4286	7055	94%