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A new iterative decoder for product codes

Kreshchuk Alexey krsch@iitp.ru
IITP RAS
Zyablov Victor zyablov@iitp.ru
IITP RAS
Ryabinkin Eygene rea@grid.kiae.ru
NRC KI

Abstract. The most popular decoder for product codes is an iterative decoder. It has low complexity and can correct many errors in average case, but it also has an uncorrectable errors of weight equal to one quoter code distance. To overcome this limitation we propose a new decoder algorithm, that utilizes iterative decoder, but has better error correction efficiency at low error rates. The computer simulation showed that the increase in error correction efficiency is larger for high-rate codes.

1 Introduction

Product codes also known as iterative codes are a specific instance of the concatenated codes.

Product codes as a technique to construct good codes were first introduced by Elias in [2]. They were later explored as a way to combine multiple codes in [3]. In section 2 we will describe this code and its iterative decoding algorithm.

The length of a product codes is a product of lengths of its component codes, the number of information symbols — product of number of information symbols and the code distance is a product of code distances.

There is a concatenated codes decoder (and so it is also a product code decoder) that corrects all errors of weigh less than one half of the code distance [1]. Another one is described in [4].

In this work we will consider iterative decoder. Its main disadvantage is low effective code distance. We will introduce an extended version of iterative decoder that improves effective code distance, and therefore error correcting performance on low error rate, by inserting erasures during decoding.

The aim of this work is to improve error correcting performance of iterative decoder of product codes by inserting erasures during decoding.

The results of the work were obtained using computational resources of MCC NRC "Kurchatov Institute" (http://computing.kiae.ru/).

2 Product codes

Let us describe the design of a product of codes $[n_r, k_r, d_r]_q$ and $[n_c, k_c, d_c]_q$. We will call the first component code the row code and the second component code the column code. We will represent codewords of this product codes as a matrix, where each row is a codeword of the row code and each column is a codeword of the column code. As the minimum weight of each row is d_r (or zero) and the minimum weight of each column is d_c (or zero), the code distance is $d_r d_c$.

The coder of this code can be described as:

1. Place all information symbols in the top left corner of the matrix.
2. Encode all rows with a systematic coder of the row code and write the parity check symbols in the top right corner of the codeword matrix.
3. Encode all columns with a systematic coder of the column code and write the parity check symbols in the bottom of the codeword matrix.

In this work we will only consider Reed-Solomon component codes, but the results can be applied for any linear component codes.

3 Iterative Decoder

Let us first describe the iterative decoder.

1. Decode all columns of the received matrix with the column code and correct all errors and erasures in-place.
2. Decode all rows of the received matrix with the row code and correct all errors and erasures in-place.
3. If the word differs from the one on step 1, repeat from step 1.

This decoder can correct some errors of high weight even if the component codes decoder only corrects errors inside $\frac{d_{(\cdot)}}{2}$ sphere ($d_{(\cdot)}$ is code distance of a component code). Unfortunately this decoder has an uncorrectable error of weight $\left\lceil \frac{d_r}{2} \right\rceil \left\lceil \frac{d_c}{2} \right\rceil$, which leads to bad error correction efficiency at low error rates. Let us construct this error matrix. For simplicity we will only consider component code decoder that refuse to correct more than $\left\lfloor \frac{d_{(\cdot)}}{2} \right\rfloor$ errors.

For simplicity let us consider only odd d_r and d_c.

1. Let us fill a submatrix of size $\frac{d_c+1}{2} \times \frac{d_r+1}{2}$ with nonzero values. Let us denote this submatrix as $\mathbf{A}$, its columns as A_c and its rows as A_r.

2. Each column will either be rejected or decoded into a codeword of weight d_c. In either case no elements $\mathbf{A}$ will be corrected.

3. Likewise each row will either be rejected or decoded into a codeword of weight d_r. No errors can be inserted into the columns A_c, as either a row has weight less than $\frac{d_r+1}{2}$ or all columns A_c already have errors in this row.

4. On later steps some of rejected rows or columns could be decoded into some codeword, but none of the incorrectly decoded could be corrected.

4 Inserting erasures

During computer simulation of the iterative decoder we have noticed that the decoder correct all errors outside an error submatrix and then stops. That means if we can insert some erasures we could continue decoding. So in this paper we propose new decoder.

1. Try to decode the received word with iterative decoder.

2. If it succeeds, return its result. Otherwise continue.

3. Denote all rows that either changed in last iteration or were rejected by row code as set E_r.

4. Denote all columns that either changed in last iteration or were rejected by column code as set E_c.

5. Insert erasures at positions $\{(r, c), r \in E_r, c \in E_c\}$

6. Run iterative decoder again with its previous result and erasures from the last step.

7. If the decoder rejects again, reject this word. If it succeeds, return its result.

5 Simulation

To estimate error correction efficiency of the proposed decoder we have performed a computer simulation. We have studied the product of $[32, 30, 3]_{256}$ and $[32, 28, 5]_{256}$ Reed-Solomon codes. This $[1024, 840, 15]_{256}$ code has rate 0.82. The channel simulated was q-ary symmetric channel.

The results of the simulation are display on fig. 1. It shows that the frame error rate has decreased by two factors of 10, or equivalent channel symbol rate has improved twofold.

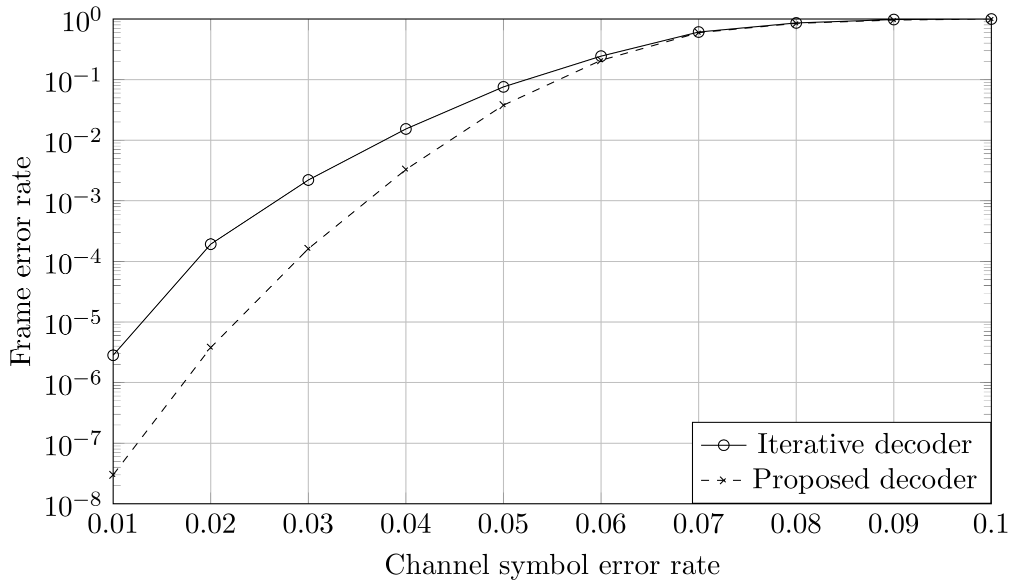

Figure 1: Simulation results

6 Conclusion

In this paper we have proposed a new decoder for product codes based on the iterative decoder. This decoder differs from other erasure-inserting decoders as it first completes the iterative decoding and only then insert erasures. Computer simulations show that it has better error correction efficiency on low error rates but makes no difference on high error rates.

References

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