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On the Keller-Blank solution to the scattering problem of pulses by wedges

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We prove that the solution of the scattering problem of pulses constructed by Keller and Blank in 1951 coincides with the solution obtained by the method of complex characteristics. The method was developed by Komech and Merzon in 2006-2007. Its main advantage is that it provides the existence and uniqueness of solutions in suitable functional classes, and the limiting amplitude principle. On the other hand, the uniqueness in the Keller-Blank approach was not studied before. Our result means that the Keller-Blank solution belongs to our functional classes. We prove the coincidence for *DD* and *NN*-boundary conditions. Moreover, we obtain the solution for the *DN*-case. Copyright © 2014 John Wiley & Sons, Ltd.

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1. Introduction

In 1934–1937, Sobolev [1–3] obtained an exact solution $u(y,t)$ to the nonstationary scattering of pulses by a two-dimensional wedge. This problem is described by a mixed problem for the wave equation in the complement Q of a convex angle W with suitable boundary conditions on the sides of the angle and the incident wave:

$$u_{in}(y,t) := F(t - n_0 \cdot y), \ y \in Q \tag{1}$$

Here, F is a given profile function, and n_0 is the unit vector in the direction of the incident wave.

Sobolev considered the Heaviside profile function $F(s) = h(s)$ and obtained the solution using the Smirnov–Sobolev representation for general solutions of the wave equation [4]. This approach rises to Sommerfeld method of ramifying solutions [5].

In 1951, Keller and Blank [6] solved independently the same problem for the *DD* and *NN* boundary conditions. They used the Busemann's Conical Flow Method [7], which is in the same spirit as Sobolev's approach.

In these papers by Sobolev, Keller, and Blank, the functional spaces of solutions are not specified, and the uniqueness of the solutions were not analyzed. Moreover, the uniqueness fails if we do not specify the class of singularity at the vertex. On the other hand, the existence breaks down if we require an excessive regularity of the solution. This is why we have developed the rigorous theory [8–13], providing the uniqueness and existence of the solution in suitable functional spaces for the smooth Heaviside-type function profile $F \in C^\infty$, supp $F \subset [0,\infty)$, $F(s) = 1$ for $s > s_0 > 0$ of the incident wave. This approach relies on the method of complex characteristics [14, 15]. In [4], we have considered the case of an arbitrary tempered distribution F. The corresponding general formula is obtained in [4]; see (3)–(7) in the following text.

This solution belongs to a space of distributions $\mathcal{M}_\varepsilon$ with $\varepsilon = 1 - \frac{\pi}{2\Phi}$ for *DD*- and *NN*-problems and $\varepsilon = 1 - \frac{\pi}{\Phi}$ for *DN*-problem, where $\Phi = 2\pi - \phi$ and ϕ is the magnitude of the wedge. Roughly speaking, $\mathcal{M}_\varepsilon$ is the space of functions with the asymptotics $|\nabla u(y,t)| \sim |y|^{-\varepsilon}$ at the vertex, i.e., as $|y| \to 0$.

In [4], we have proved that our solution for the pulse $F = h$ coincides with the Sobolev formula [3]. In present paper, we prove that our solution coincides also with the Keller–Blank formula [6] for the *DD*-problem. The coincidence in the case of the *NN*-problem can be proved similarly.

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2. Formulation of the scattering problem

We consider the scattering of plane wave (1) by two-dimensional wedge $W := \{y = (y_1, y_2) \in \mathbf{R}^2 : y_1 = \rho\cos\theta,\ y_2 = \sin\theta,\ \rho \geq 0,\ 0 \leq \theta \leq \phi\}$ with the magnitude $\phi \in (0,\pi)$; $Q := \mathbf{R}^2 \setminus W$ is the open angle of the magnitude $\Phi := 2\pi - \phi$, $\Phi \in (\pi, 2\pi)$. The boundary $\partial Q = Q_1 \cup Q_2 \cup \{0\}$, where $Q_1 := \{(y_1, 0) : y_1 > 0\}$ and $Q_2 := \{(\rho\cos\phi, \rho\sin\phi) : \rho > 0\}$. The scattering is described by the mixed problem:

$$\begin{cases} \Box u(y,t) = 0,\ \ y \in Q;\ \ Bu(y,t)|_{Q_1\cup Q_2} = 0,\ t \in \mathbf{R} \\ u(y,t) = u_{in}(y,t), \qquad y \in Q,\ t < 0 \end{cases} \tag{2}$$

Here, $\Box = \partial_t^2 - \triangle$, $B = (B_1, B_2)$ and $Bu|_{Q_1\cup Q_2} = \left(B_1 u|_{Q_1}, B_2 u|_{Q_2}\right)$ where $B_{1,2}$ are equal either to the identity operator I or to $\partial/\partial n$, with the outward normal n to Q. We will consider the directions $n_0 = (\cos\alpha, \sin\alpha)$ with $\max\ (0, \phi - \pi/2) < \alpha < \min\ (\pi/2, \phi)$. The extension of our result to another angles Φ and α is straightforward.

In [4, Sec.3], we have established the existence and uniqueness of the solution u to problem (2) in a suitable space of distributions $\mathcal{M}_\varepsilon$ with $\varepsilon = 1 - \frac{\pi}{2\phi}$. The solution u admits the splitting

$$u = u_{in} + u_r + u_d \tag{3}$$

For the Heaviside profile function $F = h$, the diffracted wave is given by

$$u_d(\rho,\theta,t) = \int_{-l(t/\rho)}^{l(t/\rho)} Z(\beta + i\theta)d\beta,\ \ \theta \in \Theta = [\phi, 2\pi] \setminus \{\theta_1, \theta_2\},\quad Z(\beta) = -H(-i\pi/2 + \beta) + H(-i\pi/2 + \beta),\ \ \beta \in \mathbf{C} \tag{4}$$

with $H(\beta) = \coth(q(\beta + i\pi/2 - i\alpha)) \mp \coth(q(\beta - 3i\pi/2 + i\alpha))$ for the *DD* and *NN*-problems, respectively. Here,

$$\theta_1 = 2\phi - \alpha,\quad \theta_2 = 2\pi - \alpha \tag{5}$$

$$l(\lambda) = \begin{cases} \ln\left(\lambda + \sqrt{\lambda^2 - 1}\right),\ \lambda \geq 1 \\ 0, \qquad\qquad\qquad\quad \lambda \in (0,1) \end{cases} \tag{6}$$

For *DN*-problem, the expression for H can be found in [11, formula (2.16)]. The reflected wave u_r is given by

$$u_r = \left\{ \begin{array}{r} \mp h(t - \rho\cos(\theta - \theta_1)),\quad \varphi \leq \theta \leq \theta_1 \\ 0,\ \ \theta_1 < \theta < \theta_2 \\ \mp h(t - \rho\cos(\theta - \theta_2)),\ \theta_2 \leq \theta \leq 2\pi \end{array} \right| \ t, \rho \geq 0 \tag{7}$$

for the *DD*- and *NN*-problems, respectively (for the *DN*-problem the reflected wave is given in [4, formula (7.11)]).

Note that formulas (4) imply that u_d is a continuous function of $0 \leq \rho \leq t$ and $\theta \in \Theta$, and

$$u_d(\rho,\theta,t) = 0,\quad \theta \in \Theta,\quad \rho \geq t$$

because $l(t/\rho) = 0$ by (6). For $\rho < t$, the integral (4) can be calculated in all cases of the *DD*, *NN*, and *DN* boundary conditions. For the *DD*-problem, we obtain

$$u_d(\rho,\theta,t) = \frac{i}{2\pi}\left[-\ln U_0 - \ln U_1 + \ln U_2 + \ln U_3\right],\quad \rho \in (0,t),\quad \theta \in \Theta \tag{8}$$

Here,

$$U_k = \frac{b^q e^{ic_k} - b^{-q} e^{-ic_k}}{-\left(b^q e^{-ic_k} - b^{-q} e^{ic_k}\right)},\quad k = \overline{0,3};\quad b = \frac{t}{\rho} + \sqrt{\left(\frac{t}{\rho}\right)^2 - 1},\ 0 < \rho \leq t \tag{9}$$

$$c_0 = q(\theta - \alpha),\quad c_1 = q(\theta - \theta_1),\quad c_2 = q(\theta - \theta_2),\quad c_3 = q(\theta - (2\pi + \alpha)) \tag{10}$$

and

$$\operatorname{Im}\ln(\cdot) \in (-\pi, \pi).$$

The formulas for *NN*- and *DN*-boundary conditions can be obtained similarly.

Formulas (9) imply that for $k = \overline{0,3}$

$$U_k = \frac{1 - B\cos(2c_k) + iB\sin(2c_k)}{-1 + B\cos(2c_k) + iB\sin(2c_k)}, \quad e^{2ic_k}, \quad B := b^{-2q} = \left(\frac{t - \sqrt{t^2 - \rho^2}}{\rho}\right)^{2q}, \quad 0 < \rho \leq t, \tag{11}$$

so

$$|u_k| = 1, \quad k = \overline{0,3}, \quad 0 < B < 1 \tag{12}$$

In the following section we express (8) in $\arg U_k$, as in [6].

Let us note that

$$u(\rho, \theta, t) = \left.\begin{cases} 1, & \theta \in [\theta_1, \theta_2] \\ 0, & \theta \notin [\theta_1, \theta_2] \end{cases}\right| \rho = t \tag{13}$$

by (3), (7), and (1).

3. Solution in Keller–Blank variables

Formulas (11), (12) imply that

$$\arg U_k = 2\arctan\frac{B\sin 2c_k}{1 - B\cos 2c_k} - \pi + 2c_k, \quad k = \overline{0,3}, \tag{14}$$

$$\arctan(\cdot) \in (0, \pi) \tag{15}$$

By (8) and (12), we obtain

$$u_d(\rho, \theta, t) = \frac{1}{2\pi}\left[\arg U_0 + \arg U_1 - \arg U_2 - \arg U_3\right], \quad \theta \in \Theta, \; 0 < \rho \leq t$$

Hence,

$$u_d(\rho, \theta, t) = \frac{1}{\pi}\left[\arctan\frac{B\sin 2c_0}{1 - B\cos 2c_0} + \arctan\frac{B\sin 2c_1}{1 - B\cos 2c_1} - \arctan\frac{B\sin 2c_2}{1 - B\cos 2c_2} - \arctan\frac{B\sin 2c_3}{1 - B\cos 2c_3}\right] \tag{16}$$

$\theta \in \Theta$, $0 < \rho \leq t$ by (14). Let us introduce the Keller–Blank [6] variables: for $\rho < t$

$$q_1 := \frac{t}{\sqrt{t^2 - \rho^2}}, \quad \rho_1 := \left(\frac{q_1 - 1}{q_1 + 1}\right)^{1/2} = \frac{t - \sqrt{t^2 - \rho^2}}{\rho}, \quad \lambda := \frac{\pi}{\Phi} = 2q$$

Note that

$$0 < \rho \leq t \Longleftrightarrow 0 < \rho_1 \leq 1 \tag{17}$$

Formula (11) implies that

$$B = \rho_1^{\lambda}$$

Moreover, we introduce the angle variable $\overline{\theta}$ and the incidence angle ψ (see [6, Sec.2]) by

$$\theta := \overline{\theta} + \phi/2, \quad \alpha := \psi + \phi/2 \tag{18}$$

The condition $\theta \in \Theta$ is equivalent to

$$\overline{\theta} \in \overline{\Theta} := (\phi/2, 2\pi - \phi/2) \setminus \{\phi - \psi, 2\pi - \psi - \phi\} \tag{19}$$

By (10),

$$\arctan\frac{B\sin 2c_0}{1 - B\cos 2c_0} = \arctan\frac{\rho_1^{\lambda}\sin\lambda\left(\overline{\theta} - \psi\right)}{1 - \rho_1^{\lambda}\cos\lambda\left(\overline{\theta} - \psi\right)} \tag{20}$$

From (5) and (18), we obtain $\theta_1 = 3\pi - \frac{3}{2}\Phi - \psi$, and hence,

$$\sin 2q(\theta - \theta_1) = -\sin\lambda\left(\overline{\theta} - 2\pi + \psi\right); \quad \cos 2q(\theta - \theta_1) = -\cos\lambda\left(\overline{\theta} - 2\pi + \psi\right)$$

Therefore,

$$\arctan\frac{B\sin 2c_1}{1 - B\cos 2c_1} = \arctan\frac{-\rho_1^\lambda \sin\lambda\left(\overline{\theta} - 2\pi + \psi\right)}{1 + \rho_1^\lambda\cos\lambda\left(\overline{\theta} - 2\pi + \psi\right)} \tag{21}$$

Similarly, $\sin 2q(\theta - \theta_2) = -\sin\lambda\left(\overline{\theta} + \psi\right)$, $\cos 2q(\theta - \theta_2) = -\cos\lambda\left(\overline{\theta} + \psi\right)$. Hence,

$$\arctan\frac{B\sin 2c_2}{1 - B\cos 2c_2} = \arctan\frac{-\rho_1^\lambda \sin\lambda\left(\overline{\theta} + \psi\right)}{1 + \rho_1^\lambda\cos\lambda\left(\overline{\theta} + \psi\right)} \tag{22}$$

Finally, $\sin 2q(\theta - (2\pi + \alpha)) = \sin\lambda\left(\overline{\theta} - 2\pi - \psi\right)$. Hence,

$$\arctan\frac{B\sin 2c_3}{1 - B\cos 2c_3} = \arctan\frac{\rho_1^\lambda \sin\lambda\left(\overline{\theta} - 2\pi - \psi\right)}{1 - \rho_1^\lambda\cos\lambda\left(\overline{\theta} - 2\pi - \psi\right)} \tag{23}$$

Now (16), (19)–(23), and (17) imply that for $\overline{\theta} \in \overline{\Theta}$ and $\rho_1 \in (0, 1]$

$$u_d\left(\rho_1, \overline{\theta}\right) = \frac{1}{\pi}\left(\arctan\frac{\rho_1^\lambda\sin\lambda\left(\overline{\theta} - \psi\right)}{1 - \rho_1^\lambda\cos\lambda\left(\overline{\theta} - \psi\right)} + \arctan\frac{-\rho_1^\lambda\sin\lambda\left(\overline{\theta} - 2\pi + \psi\right)}{1 + \rho_1^\lambda\cos\lambda\left(\overline{\theta} - 2\pi + \psi\right)}\right.$$
$$\left. - \arctan\frac{-\rho_1^\lambda\sin\lambda\left(\overline{\theta} + \psi\right)}{1 + \rho_1^\lambda\cos\lambda\left(\overline{\theta} + \psi\right)} - \arctan\frac{\rho_1^\lambda\sin\lambda\left(\overline{\theta} - 2\pi - \psi\right)}{1 - \rho_1^\lambda\cos\lambda\left(\overline{\theta} - 2\pi - \psi\right)}\right)$$

where arctan is defined by (15).

Formula (1) with $F = h$ and formulas (3), (7), (19) imply that the total solution of (2) for the *DD*-conditions is given by

$$u\left(\rho_1, \overline{\theta}\right) = \begin{cases} 1 + u_d\left(\rho_1, \overline{\theta}\right), & \overline{\theta} \in \left(\overline{\theta}_1, \overline{\theta}_2\right) \\ u_d\left(\rho_1, \overline{\theta}\right), & \overline{\theta} \in \overline{\Theta} \setminus \left[\overline{\theta}_1, \overline{\theta}_2\right] \end{cases} \tag{24}$$

where

$$\overline{\theta}_1 = \phi - \psi; \qquad \overline{\theta}_2 = 2\pi - \psi - \phi$$

according to (18) and (4). Let us compare solution (24) with the Keller–Blank formula [6, (16)]

$$v\left(\rho_1, \overline{\theta}\right) = \frac{1}{\pi}\arctan\left\{\frac{-\left(1 - \rho_1^{2\lambda}\right)\sin\lambda\pi}{2\rho_1^\lambda\cos\lambda\left(\overline{\theta} + \psi - \pi\right) + \left(\rho_1^{2\lambda} + 1\right)\cos\lambda\pi}\right\} - \frac{1}{\pi}\arctan\left\{\frac{\left(1 - \rho_1^{2\lambda}\right)\sin\lambda\pi}{2\rho_1^\lambda\cos\lambda\left(\overline{\theta} - \psi - \pi\right) - \left(\rho_1^{2\lambda} + 1\right)\cos\lambda\pi}\right\} \tag{25}$$

In particular, from this representation, we have that

$$v\left(\rho_1, \overline{\theta}\right) = \begin{cases} 1, & \overline{\theta} \in \left(\overline{\theta}_1, \overline{\theta}_2\right) \\ 0, & \overline{\theta} \notin \left(\overline{\theta}_1, \overline{\theta}_2\right) \end{cases} \Bigg|_{\rho_1 = 1 - 0}$$

(see also the discussion in [6, Sect.5]). Hence, this solution coincides with (24) at $\rho_1 = 1$ by (13) and (19):

$$v\left(\rho_1, \overline{\theta}\right) = u\left(\rho_1, \overline{\theta}\right), \quad \overline{\theta} \in \overline{\Theta}, \quad \rho_1 = 1$$

Therefore, by the continuity of u and v, it suffices to prove that

$$\tan(\pi u) = \tan(\pi v) \tag{26}$$

which we will accomplish in the following section.

4. Proof of the identity (26)

Denote

$$a := \frac{\rho_1^\lambda \sin\lambda\left(\overline{\theta} - \psi\right)}{1 - \rho_1^\lambda \cos\lambda\left(\overline{\theta} - \psi\right)}, \quad b := \frac{-\rho_1^\lambda \sin\lambda\left(\overline{\theta} - 2\pi + \psi\right)}{1 + \rho_1^\lambda \cos\lambda\left(\overline{\theta} - 2\pi + \psi\right)}, \quad c := \frac{-\rho_1^\lambda \sin\lambda\left(\overline{\theta} + \psi\right)}{1 + \rho_1^\lambda \cos\lambda\left(\overline{\theta} + \psi\right)}, \quad d := \frac{\rho_1^\lambda \sin\lambda\left(\overline{\theta} - 2\pi - \psi\right)}{1 - \rho_1^\lambda \cos\lambda\left(\overline{\theta} - 2\pi - \psi\right)}$$

$$x := \arctan a, \quad y := \arctan b, \quad z := -\arctan c, \quad t := -\arctan d. \tag{27}$$

Hence,

$$\tan(x + t) = \frac{2\rho_1^\lambda \cos\lambda\left(\overline{\theta} - \psi - \pi\right)\sin\lambda\pi - \rho_1^{2\lambda}\sin 2\lambda\pi}{1 - 2\rho_1^\lambda \cos\lambda\left(\overline{\theta} - \psi - \pi\right)\cos\lambda\pi + \rho_1^{2\lambda}\cos 2\lambda\pi} \tag{28}$$

$$\tan(y + z) = \frac{2\rho_1^\lambda \cos\lambda\left(\overline{\theta} + \psi - \pi\right)\sin\lambda\pi + \rho_1^{2\lambda}\sin 2\lambda\pi}{1 + 2\rho_1^\lambda \cos\lambda\left(\overline{\theta} + \psi - \pi\right)\cos\lambda\pi + \rho_1^{2\lambda}\cos 2\lambda\pi} \tag{29}$$

Lemma 4.1
The following identity holds

$$\tan(\pi u_d) = \frac{D}{G}$$

Here,

$$D = 4\rho_1^\lambda\left(1 - \rho_1^{2\lambda}\right)\sin\lambda\pi\cos\lambda\left(\overline{\theta} - \pi\right)\cos\lambda\psi \tag{30}$$

$$G = \rho_1^{4\lambda} - 4\rho_1^\lambda\left(\rho_1^{2\lambda} + 1\right)\sin\lambda\left(\overline{\theta} - \pi\right)\sin\lambda\psi\cos\lambda\pi - 4\rho_1^{2\lambda}\cos\lambda\left(\overline{\theta} + \psi - \pi\right)\cos\lambda\left(\overline{\theta} - \psi - \pi\right) + 2\rho_1^{2\lambda}\cos 2\lambda\pi + 1 \tag{31}$$

Proof
Using (27)–(29), we obtain

$$\begin{aligned} D = &\left[2\rho_1^\lambda \cos\left(\lambda\left(\overline{\theta} - \pi - \psi\right)\right)\sin(\lambda\pi) - \rho_1^{2\lambda}\sin(2\lambda\pi)\right]\left[1 + 2\rho_1^\lambda\cos\left(\lambda\left(\overline{\theta} - \pi + \psi\right)\right)\cos(\lambda\pi) + \rho_1^{2\lambda}\cos(2\lambda\pi)\right] \\ &+ \left[2\rho_1^\lambda \cos\left(\lambda\left(\overline{\theta} - \pi + \psi\right)\right)\sin(\lambda\pi) + \rho_1^{2\lambda}\sin(2\lambda\pi)\right]\left[1 - 2\rho_1^\lambda\cos\left(\lambda\left(\overline{\theta} - \pi - \psi\right)\right)\cos(\lambda\pi) + \rho_1^{2\lambda}\cos(2\lambda\pi)\right] \end{aligned}$$

that gives (30). Similarly,

$$\begin{aligned} G = &\left[1 - 2\rho_1^\lambda\cos\lambda\left(\overline{\theta} - \pi - \psi\right)\cos\lambda\pi + \rho_1^{2\lambda}\cos 2\lambda\pi\right]\left[1 + 2\rho_1^\lambda\cos\lambda\left(\overline{\theta} - \pi + \psi\right)\cos\lambda\pi + \rho_1^{2\lambda}\cos 2\lambda\pi\right] \\ &- \left[2\rho_1^\lambda\cos\lambda\left(\overline{\theta} - \pi - \psi\right)\sin\lambda\pi - \rho_1^{2\lambda}\sin 2\lambda\pi\right]\left[2\rho_1^\lambda\cos\lambda\left(\overline{\theta} - \pi + \psi\right)\sin\lambda\pi + \rho_1^{2\lambda}\sin 2\lambda\pi\right] \end{aligned}$$

that gives (31). □

Next corollary implies (26) by (24).

Corollary 4.2
We have

$$\tan(\pi v) = \tan(\pi u_d) \tag{32}$$

Proof
From (25), it follows that $\tan(\pi v) = \frac{W}{Z}$, where

$$\begin{aligned} W = &\left(1-\rho_1^{2\lambda}\right)\sin(\lambda\pi)\left[2\rho_1^{\lambda}\cos\lambda\left(\overline{\theta}-\psi-\pi\right)-\left(\rho_1^{2\lambda}+1\right)\cos\lambda\pi\right] \\ &+\left(1-\rho_1^{2\lambda}\right)\sin(\lambda\pi)\left[2\rho_1^{\lambda}\cos\lambda\left(\overline{\theta}+\psi-\pi\right)+\left(\rho_1^{2\lambda}+1\right)\cos\lambda\pi\right]=D \end{aligned} \tag{33}$$

$$\begin{aligned} Z = &\left(2\rho_1^{\lambda}\cos\lambda\left(\overline{\theta}+\psi-\pi\right)+\left(\rho_1^{2\lambda}+1\right)\cos\lambda\pi\right)\left(2\rho_1^{\lambda}\cos\lambda\left(\overline{\theta}-\psi-\pi\right)-\left(\rho_1^{2\lambda}+1\right)\cos\lambda\pi\right) \\ &+\left[\left(1-\rho_1^{2\lambda}\right)\sin(\lambda\pi)\right]\left[\left(1-\rho_1^{2\lambda}\right)\sin(\lambda\pi)\right] \\ &=\rho_1^{4\lambda}-4\rho_1^{\lambda}\left(\rho_1^{2\lambda}+1\right)\sin\lambda\left(\overline{\theta}-\pi\right)\sin\lambda\psi\cos\lambda\pi-4\rho_1^{2\lambda}\cos\lambda\left(\overline{\theta}+\psi-\pi\right)\cos\lambda\left(\overline{\theta}-\psi-\pi\right) \\ &+2\rho_1^{2\lambda}\cos 2\lambda\pi+1=G \end{aligned} \tag{34}$$

Finally, (32) follows from (33) and (34) by (25). □

5. Conclusion

There are different approaches to non-stationary scattering of plane waves by two-dimensional wedges. Some particular solutions for the pulse incident wave were obtained by Sobolev in 1930 and Keller and Blanc in 1950. However, the uniqueness of the solutions in an appropriate functional class was not established up to now. Moreover, it is well known that the solution is not unique if its singularity is not specified.

Recently, we have proposed a universal approach that gives explicit formulas for the solution with general incident waves and guarantees the existence and uniqueness of solutions in suitable functional classes.

In present paper, we check that for the pulse incident wave, our solution coincides with Keller–Blanc's formula.

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