Bifurcation to unbounded sequence of cyclic branches of solutions

Krasnosel'skii A.M. and Rachinskii D.I.

Institute for Information Transmission Problems, Moscow, Russia;
University College Cork, Cork, Ireland
sashaamk@iitp.ru and D.Rachinskii@ucc.ie

We consider operator equations $F(x;\lambda) = 0,\ x \in X$ with a scalar parameter λ. Generically, the topological structure of the set of solutions of the equations do not vary in λ except at bifurcation points. In the vicinity of such points, generically isolated, branches of solutions may intersect, stick together, and go to infinity.

We consider the case where the bifurcation point is not isolated, the set of solutions in the space $X \times \mathbb{R}$ is unbounded and makes up an infinite sequence of bounded cyclic continuous branches.

The example is given for the differential equation

$$\mathcal{L}(p;\lambda)x = b(t;\lambda) + f(x;\lambda), \qquad p = \frac{d}{dt},$$

here $\mathcal{L} = L(p) + (\lambda - \lambda_0)M(p;\lambda)$ is a real polynomial, the function f is continuous, 2π-periodic, and bounded. Let the linear part is degenerate: $L(\pm i) = 0$, $L(ki) \neq 0$, $k \neq \pm 1$, $k \in \mathbb{Z}$. If

$$F^* = \limsup_{r\to\infty} F(r) > F_* = \liminf_{r\to\infty} F(r), \qquad F(r) = \int_0^{2\pi} \sin t\, f\big(r\sin(t);\lambda_0\big)\, dt,$$

then such unbounded sequence of cyclic branches may generically appear.

Authors are supported by Grants 06-01-00256, 06-01-72552 of RFBR.