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Sequential decoding on syndrome trellises for nonparametric detection

Dmitry Osipov[1] d_osipov@iitp.ru
Institute for Information Transmission Problems RAS
19 bld. 1 Bolshoy Karetny lane, Moscow, Russian Federation
National Research University Higher School of Economics,
3 Kochnovsky Proezd, Moscow, Russian Federation
Dmitry Titov titovdim93@gmail.com
National Research University Higher School of Economics,
3 Kochnovsky Proezd, Moscow, Russian Federation

Abstract.
The following paper adapts the classical Zigangirov-Jelinek algorithm to the decoding of nonbinary block codes under severe mixed jamming. To ensure reliable communications in this scenario we combine reception techniques based on distribution free statistical tests with sequential decoding on syndrome trellises. It will be shown that the proposed approach can ensure relatively high transmission rate with reasonable complexity.

1 Introduction

Reception techniques using nonparametric or distribution free test can be considered to be promising candidates for communication systems operating under severe jamming. However decoding algorithms that were proposed for communication systems employing those techniques have relatively high computational complexity. This paper deals with the possibility of using sequential decoding on syndrome trellises to solve this problem.

2 A DHA FH CDMA system: Transmission and Reception

Let us consider a multiple access system in which K active users transmit information via a channel split into Q identical nonoverlapping subchannels. In

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what follows it will be assumed that information to be transmitted is encoded into a codeword of a q-ary (n, k, d) block code $(q < Q)$. Whenever a user is to transmit a q-ary symbol it places 1 in the position of the vector $\bar{x}_g$ corresponding to the symbol in question within the scope of the mapping in use (in what follows it will be assumed that all the positions of the vector are enumerated from 1 to Q, moreover for the sake of simplicity and without loss of generality we shall assume that the 1st subchannel corresponds to 0, the 2nd subchannel corresponds to 1 and so on). Thus each q-ary symbol to be transmitted is mapped into a weight 1 binary vector (the construction under consideration is the Kautz-Singleton construction for binary superimposed codes [1]). Then a random permutation of the aforesaid vector is performed and the resulting vector $\bar{\chi}_g = \pi_g(\bar{x}_g)$ is then transmitted by employing Q-ary frequency shift keying (FSK) (permutations are selected equiprobably from the set of all possible permutations and the choice is performed whenever a symbol is to be transmitted).

Within the scope of a certain codeword reception the receiver is to receive n signals corresponding to the codeword in question. Note that the receiver is assumed to be synchronized with transmitters of all users. Therefore all the permutations done within the scope of transmission of the codeword in question are known to the user. The receiver measures energies at the outputs of all subchannels (let us designate the vector of the measurements corresponding to the g-th symbol as $\bar{\beta}_g$) and applies inverse permutation to each vector $\bar{\beta}_g$ corresponding to the respective symbol thus reconstructing the initial order of elements and obtaining vector $\bar{b}_g = \pi_g^{-1}\left(\bar{\beta}_g\right)$. Let us consider a matrix $B = [\bar{b_1}, \bar{b_2}, \ldots, \bar{b_n}]$ corresponding to the codeword of the inner code . Let us consider the submatrix $Y = [\bar{y_1}, \bar{y_2}, \ldots, \bar{y_n}]$ (here Y is the submatrix corresponding to the q first rows of the matrix B and each vector $\bar{y_g}$ is the height q column vector corresponding to the g-th symbol of the codeword). Please note that Y provides all the information necessary to decode the codeword of the inner code.

3 Nonparametric detection

Let us now consider the detection problem. This problem can be decomposed into two successive stages: reliability values computation and decoding. First and foremost let us consider the first problem.

The aim of the first stage is to compute the decision reliability values for each symbol. The latter are to be used by the decoder at the second stage. Since the a priori presupposition is that each symbol can take any of the q possible values for each symbol the corresponding vector of reliability values will be computed for each symbol.Thus a matrix of reliably values corresponds to each codeword. Since the matrix in question is used to make a decision on the transmitted codeword it will be further on referred to as decision statistics matrix M^D.

A number of reception techniques tolerant to severe jamming were developed in recent decades. In normalized envelope detection (NED) method the decision statistics matrix is obtained by dividing each column of the matrix of envelopes by the sum of the respective column.

However this method is hardly suitable for the case when the interfering users' signals have power much higher than that of the user under consideration. For this case a more robust method is needed. Hereinafter we shall consider some methods based on ordered statistics calculation. For simplicity let us assume that all elements of Y are distinct and consider the indicator function

$$\Im\left(x,y\right)=\begin{cases}1 & x<y\\ 0 & x\geq y\end{cases} \tag{1}$$

For each element of the matrix Y its rank is given by

$$R\left(t,z\right)=\sum_{t'\neq t}\sum_{z\neq z'}\Im\left(\left(Y_M\left(t,z\right),Y_M\left(t',z'\right)\right)\right)+1 \tag{2}$$

In "rank sum method" [6] the matrix of the rank matrix is the matrix of the decision statistics. Hereinafter we propose to use a combination of the two previously considered methods. In the resulting method that will be further on referred to as Normalized Rank Sum (NRS) the matrix of the normalized ranks is given by

$$\widetilde{R}\left(t,z\right)=R\left(t,z\right)/\sum_{z=1}^{q}R\left(t,z\right) \tag{3}$$

and its logarithmic version is given by

$$\Lambda\left(t,z\right)=\log\left(\widetilde{R}\left(t,z\right)\right) \tag{4}$$

As far as the second stage of the detection process (i.e. the decoding) is concerned the majority of papers that consider ordered statistics-based reception techniques (e.g. [6]) use the decoding algorithms that boil down to exhaustive search. Unfortunately due to complexity considerations this approach leads to low rate codes. In [7] another approach is proposed that is based on employing convolutional inner codes and Viterbi decoding. However the effectiveness of this approach is limited since the complexity of non-binary convolutional codes Viterbi decoding depends exponentially on the overall constraint length. Thus only codes with relatively small overall constraint length are practical. In this paper another approach is proposed. In what follows we shall consider the perspective of decoding block MDS codes on a syndrome trellis.

4 Using ZJ decoding algorithm for nonparametric detection: the proposed approach

Syndrome trellises were introduced by Bahl et al. in [2]. Unfortunately both APP decoding (that has been proposed in [2]) and Viterbi decoding on syndrome trellis [3] have complexity exponential with the number of parity check symbols. Thus only non-binary codes with relatively small distances can be decoded in this way. In what follows we propose another approach based on the classical sequential decoding paradigm. In particular we adapt the classical stack algorithm that has been proposed by Zigangirov [4] and Jelinek [5] for convolutional codes (also known as ZJ algorithm) for block codes decoding. To do so we use the syndrome trellis of the code in use and use the matrix $\widetilde{R}$ (3) as the matrix of conditional probabilities for the respective symbols. The algorithm operates almost in the same way as the ZJ algorithm does and makes use of the conventional Fano metric. The algorithm terminates when the winning path has full length. Then all the paths having full length n are extracted from the stack and exhaustive search is applied to the resulting list. Again the matrix $\widetilde{R}$ is treated as the conditional probabilities matrix, i.e. if I is the set of numbers of the (n, k) block code codewords that were included in the resulting list and X_i $(i \in I)$ are the respective Kautz-Singleton matrixes then the decoder declares the codeword with the number

$$i^* = arg \max_{i \in I} \left(\sum_{t=1}^{n} \sum_{z=1}^{q} (\Lambda \cdot X_i) \right) \tag{5}$$

where $\cdot$ stands for Hadamard product and Λ is given by (4).

5 Simulation

To verify the effectiveness of the proposed algorithm the following scenario will be considered: it will be assumed that the user under consideration transmits in a system with Q orthogonal subcarriers employing the transmission technique considered above and apart from the user under consideration K interfering signals are transmitted in the system under consideration. Hereinafter it will be assumed that each interfering signal has the same form as that of the user under consideration but its power at the receiver end is κ times higher than that of the signal of the user under consideration. Moreover it will be assumed that apart from narrowband interfering signals the received signal is influenced by the wideband interference that will be modeled as an Additive White Gaussian Noise characterized by signal-to-noise ratio $SNR = 10 * log_{10}(\frac{E_s}{E_N})$ where E_s is the energy of the signal transmitted by the user under consideration (at the receiver side) and E_N is noise energy (please note that E_N is noise energy in

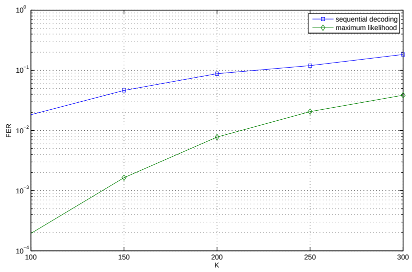

Figure 1: FER for the modified reception strategy

the entire band whereas E_s is the energy in the effective band occupied by the transmitted signal. Since the effective bandwidth is much smaller than the entire one SNR can take great negative values).In particular hereinafter we shall consider the case when $q = 8$,$Q = 4096$, $\kappa = 10^4$,$SNR = -25dB$ and systematic MDS code $C(8, 4)$ over $GF(8)$. In Fig.1 Frame Error Rate for the decoding of this code using the proposed algorithm is shown (curve for the ML decoder employing the same decision statistics is shown for comparison at the respective figures)

As can be seen from the presented curves even though the proposed algorithm shows considerable performance degradation as compared to the exhaustive search algorithm using the matrix Λ the probability of error per block is still low enough to use the proposed decoder as inner decoder in a cascaded coding scheme. It should be noted that the complexity of the exhaustive search even for our example is prohibitively large. Thus unlike the proposed solution its counterpart that uses exhaustive search cannot be used in practical systems. In the next section a strategy using modified decision statistics matrix will be presented.

6 Modified reception strategy

On of the reasons for the fact that that the resulting Frame Error Rate of the code under consideration for the proposed algorithm and the ML decoding algorithm are relatively high is the influence of the interfering narrowband signals. Although the rank calculation makes the decision statistics less sensitive to the distortion introduced by these signals they still have some impact. Therefore we suggest a modified version of the decision statistics matrix that performs some sort of clipping of the rank matrix prior to normalization. The Frame Error Rate for the proposed decoder is shown in Fig. 2 Comparing Fig.2 with Fig .1 one can notice that the rank matrix clipping actually results in a certain performance gain. For instance for the proposed decoder the performance im-

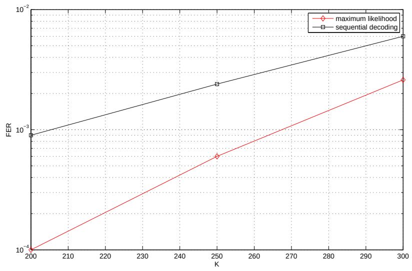

Figure 2: FER for the modified reception strategy

proves more than 30 time even if the number of interfering users is relatively great.

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