Application of multi-valued logic models in traffic aggregation problems in mobile networks

1st Elmira Yu. Kalimulina
V. A. Trapeznikov Institute of Control Sciences
Russian Academy of Sciences
and
Moscow Technical University of Communications and Informatics
Moscow, Russia
elmira.yu.k@gmail.com

***Abstract*—This paper discusses some aspects of providing an Internet-access for users on high-speed trains via aggregation of available cellular network channels. Standard aggregation schemes have limitations of bandwidth increase in areas with unstable network coverage. The conditions and parameters of data transmission are constantly changing and depend on a large number of factors. In this paper a non-standard solution based on models of multi-valued logic has been considered that will allow to increase the time for analysis of these rapidly changing factors. Aggregation schemes based on fuzzy sets or multi-valued logic models potentially can help to decrease the latency and increase of transmission reliability due the decrease in the number of computational operations on an aggregation server. However, the development of new aggregation protocols requires some theoretical problems from multi-valued logic to be solved.**

From a theoretical point of view, such a development means that it is possible to construct a combination of predicates on the set of values of k-valued logic functions. We solved the completeness problem for a closure operator on the set of values of k-valued logic functions and proved the existence of a finite lattice for this set. The lattice construction is given explicitly. Thus the electronic circuits and all possible functional diagrams for a new aggregation protocol may be realized.

***Index Terms*—multi-valued logic, multi-valued models, traffic aggregation, cellular networks, channels aggregation**

I. Introduction

A ternary system is the most optimal from the point of view of information density [9]. The generalization for multi-valued logic is the ternary logic [2], [3]. Further, without loss of generality instead of multivalued case a ternary logic model may be considered. In ternary logic, a statement is assigned one of three values: ”true”, ”false”, ”undefined” [2], [4], [9]; in binary logic – two: either ”true” or ”false”. Symmetric form of number representation based on three-valued logic simplifies a processing of negative numbers, since it requires an extra bit to store the sign [4].

Some features of the operation logic of a ternary computer, for example, the representation of negative numbers, give possibilities for design more reliable and high-performance modern systems, that will be useful for many modern applications. Mathematically, ternary logic is more efficient than binary logic [2], [4], [9]. Research and development of algorithms based on three-valued logic are very relevant [8], for example, in telecommunications [7], [10], in the field of artificial intelligence (AI) [6], quantum computing [7], [11]–[13], medicine, physics [14]. This is confirmed by a significant increase of the number of scientific publications in leading scientific journals related to various applications of three-valued logic over the past few years [17].

The publication has been prepared with the support of the Russian Foundation for Basic Research according to the research project No.20-01-00575 A.

A. A brief overview of modern applications of multivalued logic

Here are examples of several applications where the construction of algorithms based on three-valued logic provides greater efficiency and turns out to be preferable in comparison with two-valued logic. For more detailed overview, you can read references.

1) Availability analysis of technical systems and factors sensitivity assessment : Multi-valued logic allows to consider qualitative variables instead of quantitative ones. Quantitative indicators (factors) are discretized by mapping into a certain m-interval scale. This approach allows you to combine quantitative and qualitative indicators within the single model. The reliability of the factors decreases minimally with such discretisation. This allows to investigate the model as fully as possible. This is especially effective in situations where there is no way to quantify the impact of a particular factor on the process. The use of qualitative variables provides additional opportunities for assessing factors.

2) Simulation of processes and modern design languages: Simulation is the only available way to check the quality and reliability of complicated and expensive technical systems at their design stage. Automated design tools allow you to assess quality based on real-world operating conditions. Temporary simulation of circuits in an automated simulation system is often based on the principles of three-valued logic.

3) Design of data transmission and processing systems: Ternary logic is effective in constructing computing units for equipment of data transmission networks. Potentially, the transmission of three states instead of two bits at a time can increase the data transfer rate by 1.5 times. With an increase of the number of trits (instead of bit) the speed can grow exponentially [10], [18], [19]. Data aggregation and transmission schemes may be released on multivalued logic models. These solutions will provide a single high-dimensional space for network addressing. There are applications not only for for standard purposes of data transmission [15]. Also it will be effective in new tasks for controlling and protecting robotic devices, for the Internet Of Things [7]. Multi-valued logic is also effective both for solving problems of image processing [5], problems of cryptography for quantum computing, and security of cloud web-services. That also uses multi-valued logic models. With the rapid growth of quantum computers, ternary computing has become relevant again [5], [12], [13]. The leading IT companies have introduced their quantum computers operating on several dozen of qubits in the last decade: IBM quantum processors consist of 65 qubits, Google has 72 [20]. The developers plan to release a 1112-qubit processor called ”Condor” by 2023, that should bring quantum technologies to a new commercial level [20].

Also, at present, the multi-valued logic toolkit is widely used in tasks related to data analysis and the construction of AI models, for example, in the tasks of hierarchical data clustering for arbitrary complicated data sets [6], [7]. Interpretation models via 3-valued logic allows to overcome exiting limitations on the ability to create fully automatic program-analysis algorithms [1].

At the end of this short overview of multivalued logic models, the application in economic research should be mentioned: models of collective behavior and the problem of collective choice, where “cyclical logic” arises as a special case of k-valued logic [21].

II. Introduction

A. Network channel aggregation technology

An aggregation is a way of bundling a bunch of individual links (channels, frequencies) together so they act like a single logical one in order to improved performance by transmitting several packets simultaneously. Outside large cities the quality of data transmission via cellular networks is very unstable. There are many problems a single user may be faced with in a static position such as a long distance to the mobile phone tower, radio interference, congestion / not enough capacity. So the Internet-access is much more unstable for a moving group of users on high-speed trains. Additionally to the static problems the high speed adds to this list the problems with switching and rapidly changing the distance to towers, when a train moves fast from one coverage zone to another. The heavy load may appear for non-high capacity towers within a small interval of time. So the quality of internet-access is periodically changing from normal to very low. The aggregation of cellular channels allows to organise a stable Internet-connection for users located on high-speed transport units outside the areas with good cellular network coverage. Some operators provide better connection, some very low, or become not available for a period of time. By aggregation data packets are distributed among several available operators and/or by between different frequencies of the same operator at the same time. Thus the data transmission process can be optimised: the latency decreases, the reliability and transmission rate increases by packet distribution between several parallel channels.

There are usually no several operators at once providing the equal good quality of access to the network outside large cities, especially for large groups of users with fast switching from one cellular network tower to the another. An effective way is to constantly monitor (at short intervals) the availability of networks and parameters of the quality of data transmission, and assign some weights to each operator (or channel) in accordance with their availability, and then to distribute the total traffic generated by users in accordance with these weights. A larger volume of traffic can be transmitted via more accessible channels, the less part – via less reliable ones or via channels with a greater delay rate. These functions of monitoring, traffic distributions and collecting from distinct cellular channels are done by an aggregation server (or aggregator). Such a scheme of packets distribution is represented on Fig.1. The solution of channel aggregation is well known. Many hardware vendors support such schemes [26], [27]. The novelty of this work is in a new algorithm of availability monitoring, channel (or operator) selection, and aggregation. Instead of monitoring two parameters we suggest monitoring more possible states for data transmission hardware, links, operators, channels, etc. Thus from the one hand it requires more complicated equipment and software and algorithms have to be produced, and some theoretical problems for data transmission protocols should be solved. On the other hand it decreases the latency and enhances the reliability of traffic transmission. Additionally to the functions mentioned above an aggregation server may perform many other functions such as load balancing, faults monitoring, control redundancy, logging, buffering and queues management, traffic segmentation, tariff and pricing management, etc. In general, the aggregation server may be an internal network comprising several servers that supports databases, data processing and network management systems. The Fig.2 below demonstrates the internal infrastructure on a high-speed train. As it is shown here the most common way to provide a stable access to the Internet on a high speed train is to connect users to the internal Wi-Fi interface. Via this interface the data are transmitted to an aggregation system, that in general case may be a computer network with several servers perform many functions mentioned above. This system distribute traffic and transfer it to an LTE-modem that supports interaction with various cellular operators. Thus, subscribers of various operators, regardless of whether there is a stable connection with their operator at a given time to transmit mobile traffic, receive a guaranteed quality of service. This process of traffic control is completely hide from the particular user.

In general, the quality of communication service depends on the availability of cellular operators’ networks. In situations where no operator provides a stable connection, there may be no service. However, in most cases, several mobile operators (up to 4) are always available. And even at a low data

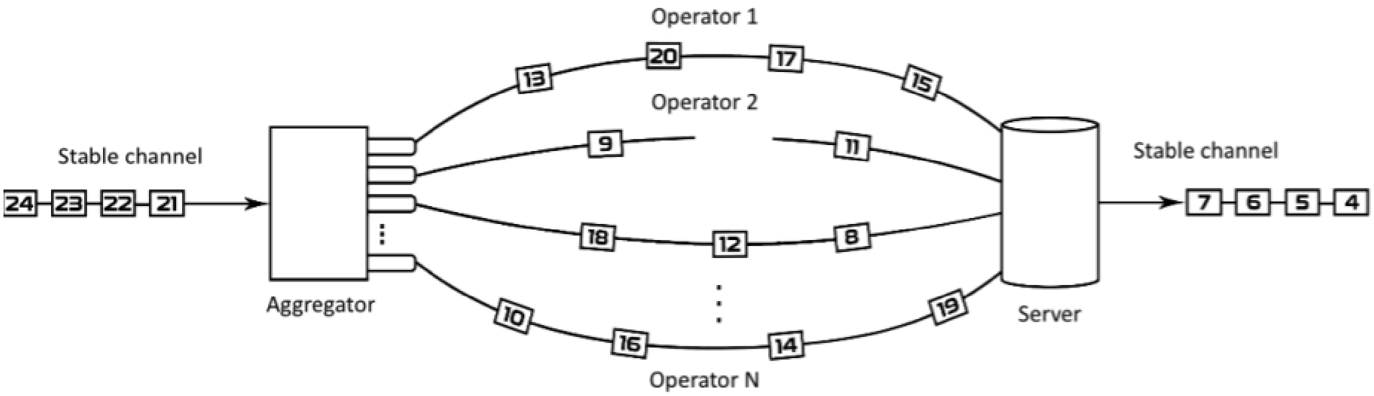

Fig. 1. The traffic distribution depending on the availability of channels.

transfer rate for all of them, aggregation allows you to provide a total capacity acceptable for stable transmission of even video traffic. The frequency of parameters updating for the availability of cellular operators' channels makes it possible to quickly distribute the load, so that the loss of data transmission packets is minimal. In the Fig.2 there are three operators. The highest data speed rate is provided via the second cellular tower in this example, therefore the most data are transmitted via the Operator 2 (on the Fig.2 this is indicated by a bold arrow). The Operator 1 provides the most unstable connection and low transmission rate, so the minimal part of the traffic is redirected through it (this is noted by thin dotted line on the figure). The Operator 3 also provide an enough transmission rate, but less than the Operator 2, so some data packages are also transmitted via them.

Consider a LTE-modem that supports several cellular operators as a part of the aggregation system. Take as an example the following modem states (states are taken arbitrarily):

- state "0" – modem is disabled,
- state "1" – allow roaming,
- state "2" – maximum packet size exceeded,
- state "3" – low transmission rate in the slot with SIM-card 1,
- state "4" – normal transmission rate in the slot with SIM-card 2,
- state "5" – connection lost,
- state "6" – internal error,
- state "7" – no response received from network,
- state "8" – access failure, unknown source,
- state "9" – LTE services not allowed,
- etc...

This approach may decrease the time for analysis of a system state and the speed of transmission and enhance reliability. In practice, such a system can be implemented in software, or (that is more effective) in hardware by designing new electronic circuits with more than two levels of electronic signal. And then a new modem can be realised on these circuits. Theoretically such an implementation means the we can construct explicitly a predicates superposition on the k-valued logic functions values set (see Fig.3). Then on these predicates we can calculate logical functions (right scheme on the Fig.3). In the case with modem example the final result may be answer on this questions:

- "0" – Are there working modems?
- "1" – Is there enough coverage?
- "2" – Is there enough total transmission rate?

Therefore a set of factors with more than two states should be considered to optimise solve a traffic aggregation algorithm. The operation of a modern modem is based on a binary signal. By applying of a non-binary numerical system to coding modem states we may decrease substantially the complexity and the number of internal computation. Our task is to consider a model on k-valued logic. Theoretically, the transition from two-valued logic to multi-valued logic allows you to decrease the amount of computation operations and increase the speed of data processing: the increase in the number of input parameters decreases the number of functions (number of computational levels in logic schemes), the amount of necessary calculations, the number of errors, and increases availability.

In order to realise such a scheme it is necessary to solve the classical mathematical problem of completeness and construct a lattice of closed classes. It's a theoretical problem. And in a general case it has been proved that it hasn't positive solution. But this problem can be solved for the special case of a closure operator. The lattice of closed classes for a closure operator for three-valued logic has been constructed explicitly [16]. In the next section we proved the existence of such a scheme for closure operators for a four-valued case.

III. The structure of closed class in multi-valued logic

Thus, all problems associated with transitions to fuzzy or k-valued sets of factors (parameters) can be reduced to the problem of constructing predicates on the set $\{P_k\}$ of these parameters, where $k \geq 3$. Fuzzy sets in this case are approximated by finite discrete ones [23]: $0, 1, 2, ..., k$. The general state of the system will be determined by a complex

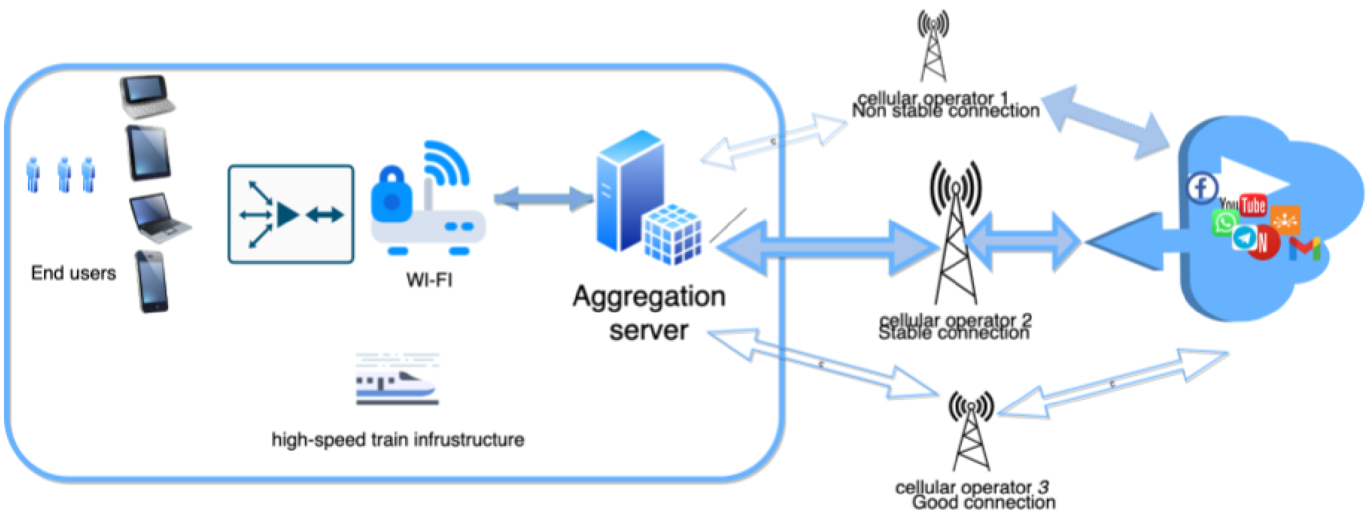

Fig. 2. The aggregation scheme on a high speed train.

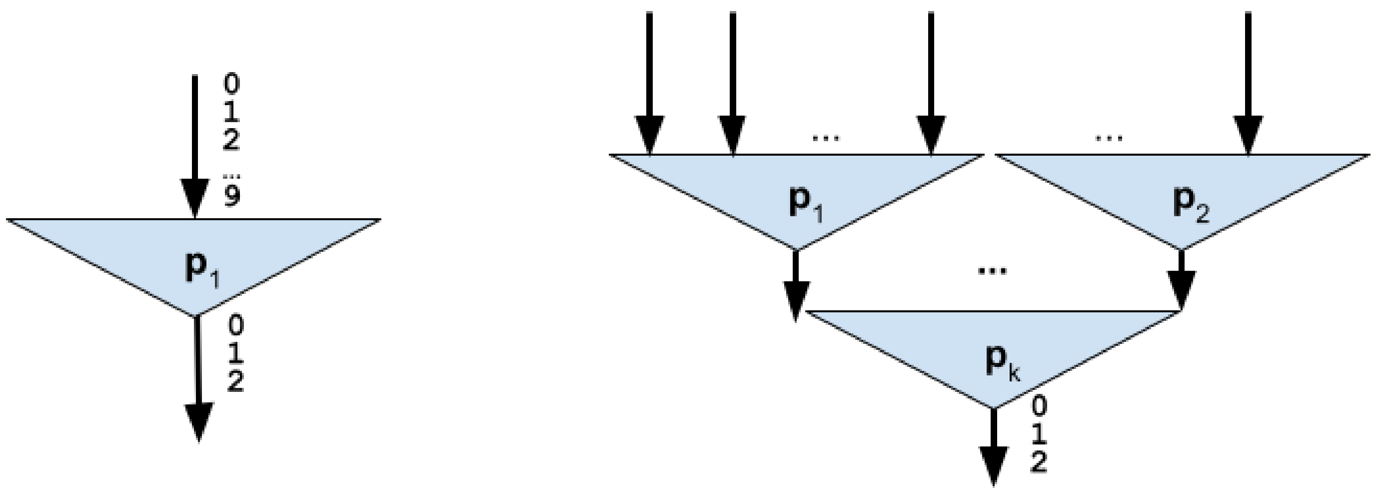

Fig. 3. Example of a predicate for modem state.

predicate $Y(\cdot)$ as a superposition of other predicates on the original set $\{P_k\}$ [22]. In the future, these predicates can be implemented both software and hardware from digital circuits based on multi-valued logic.

A. *The lattice structure and a completeness problem for functions classes of multi-valued logic*

In order to design such a computer program or hardware it is necessary to solve the *the problem of completeness of classes of multi-valued logic functions* and construct a lattice of closed classes [22]. For applications, solving a completeness problem and constructing a lattice means the ability to implement the necessary microcircuits and assemble arbitrary chipsets based on them. Also it means that we can solve an inverse task – to assemble the necessary chipset from available microcircuits for a specific task. There are well known Post's results, where five precomplete classes on the set of functions of two-valued logic are explicitly constructed [24]. The rapid growth and development of computer technology in the second half of the 20th century is due, among other things, to Post's theorem [24]. Later in 1958 S.V. Yablonsky proved the existence of 18 precomplete classes for a three-valued case. He considered the closure of the set of functions with respect to the substitution operator was considered [22], [23]. Also it has been proved that there is no positive solution for a multi-valued case [23].

The lattice of closed classes is countable only for a binary logic case, and it is continuous for other cases. Thus the lattice construction is impossible for a three-valued logic case. However, we can consider **its closure operators** on the set of k-valued logic functions, which are a strength of the common substitution operator. Solving the completeness problems for this new closure operator and finding the structure of the lattice of closed classes will help not only to restore the sublattice of closed classes in the general case of closure of functions with respect to the classical superposition operator, but also will optimize the possible production of chips for functional circuits for solving the problem of traffic aggregation described above.

IV. The structure of closed classes in P_k, where $k \geq 4$

Let M be a set a set of functions from $P_k, k \geq 4$. Denote $\mathcal{R}(M)$ the result of the closure of a set of functions M relative to the operation of substitution and transition from the function g to the equivalent function $f \sim g$, where

$$f \sim g \Leftrightarrow \forall \vec{x} \, [\, (f(\vec{x}) = g(\vec{x})) \vee (f(\vec{x}), g(\vec{x}) \in \{0,1\}) \,].$$

Theorem 1. For all $k(k \geq 4)$ there is a R-closed in P_k class with countable basis.

Proof: Consider the set of functions (1):

$$f_i = f_i(x_1, \ldots, x_i) = \begin{cases} 2, \text{ if } x_1 = \ldots = x_{j-1} = \\ = x_{j+1} = \ldots = x_i == 0, \\ x_j = 1, \\ 3, \text{ otherwise,} \end{cases}$$

where $i = 2, 3, \ldots$.

Let $f_m \in \mathcal{R}(\{f_2, \ldots, f_{m-1}, f_{m+1}, \ldots\})$. Then from the definition of the system (1) it follows that

$f_m \in [\{f_2, \ldots, f_{m-1}, f_{m+1}, \ldots\}]$,

and hence there exists r and formulas $\mathcal{U}_1, \ldots, \mathcal{U}_r$ such that

$$\begin{aligned} f_m(x_1, \ldots, x_m) = \\ f_r(\mathcal{U}_1[f_2, \ldots, f_{m-1}, f_{m+1}, \ldots], \\ \ldots, \mathcal{U}_r[f_2, \ldots, f_{m-1}, f_{m+1}, \ldots]), \\ r \neq m. \end{aligned}$$

Suppose that all formulas $\mathcal{U}_1, \ldots, \mathcal{U}_r$ are variables. Possible cases:

If $r < m$, then $f_m(x_1, \ldots, x_m) = f_r(x_1, \ldots, x_r)$, and therefore a function f_m depends essentially on the r variables, but that contradicts the definition f_m.

If $r > m$, then the formula contains at least twice some variable x_p. Then with a set of variables $x_1 = \ldots = x_{p-1} = x_{p+1} = \ldots = x_m = 0, x_p = 1$, f_m takes the value two, and f_r takes the value 3, i.e. $f_m \neq f_r$.

There remains the case when there is at least one formula $\mathcal{U}_j$ that differs from the variable symbol. But then on j-th input of function f_r for any input data, either 2 or 3 will be supplied. And then we have $f_m \equiv 3$ that contradicts the definition f_m.

Therefore, this case is also impossible.

Thus, the set of functions (1) forms a countable basis of the class M.

The theorem is proved.

Corollary 1. Due to the fact that if the class is $\mathcal{R}_1$-closed, then it is also $\mathcal{R}_2$-closed, we have that that generalizations of operators $\mathcal{R}_1$ and $\mathcal{R}_2$ on the set $P_k, k \geq 4$ will have a continuous lattice structure of closed classes.

V. Conclusion

As it was already mentioned, the general goal of the research is to find the limit closure $\mathcal{R}r$, that satisfies the property

$$||P_k||_{\ calRr} < \infty,$$

i.e. we would like to construct a strength of superposition sufficient for countable Boolean of closed classes.

This problem has a trivial solution – to construct the operator $\mathcal{I}()$:

$$P_k \to P_k,$$

and if $f \in P_k$, then

$$\mathcal{I}(f) = P_k.$$

Our task is to construct a non-trivial operator that will satisfy, in some sense, the extreme property: namely that the strengthening of the operator of superposition be minimal. In terms of three-valued logic, the expansion of the superposition operator was minimal (original default identity matrix of relations in the operator of superposition has been minimally strengthened by adding only one additional transitive relation). This result based on several Lemmas and Theorems, that was proved earlier and described in a detail in [25].

We have shown that problems of completeness for a closure operator and the lattice construction have a solution. For application it means that if there are several different chipsets producers, then we are able to assemble an electronic circuit from them such that all possible states of our system (modem, routers, etc.) may be described. Also the inverse problem may be solved. Because the lattice may be constructed there is possible to restore a functional diagram from available functional circuits produced by different vendors.

Acknowledgment

The publication has been prepared with the support of the Russian Foundation for Basic Research according to the research project No.20-01-00575 A.

References

[1] Reps T.W., Sagiv M., Wilhelm R. (2004) Static Program Analysis Via 3-valued Logic. In: Alur R., Peled D.A. (Eds) Computer Aided Verification. Cav 2004. Lecture Notes In Computer Science, Vol 3114. Springer, Berlin, Heidelberg. doi:10.1007/978-3-540-27813-9_2

[2] Trogemann, Georg; Nitussov, Alexander Y.; Ernst, Wolfgang (2001), Computing In Russia: The History Of Computer Devices And Information Technology Revealed, Vieweg+Teubner Verlag, Pp. 19, 55, 57, 91, 104–107, Isbn 978-3-528-05757-2.

[3] Rumyantsev, Dmitri. Interviews With The Designer Of The Ternary Computer. Upgrade 33:175 (August 2004). An Interview With Nikolai Brusentsov, Designer Of The Setun Ternary Computer. In Russian.

[4] The Ternary Calculating Machine Of Thomas Fowler. url: www. Mortati.Com/ Glusker/ Fowler/Index.Htm

[5] Abiri Ebrahim, Darabi Abdolreza, Salem Sanaz. Design Of Multiple-valued Logic Gates Using Gate-diffusion Input For Image Processing Applications . Computers Electrical Engineering. 2018. Vol.69. Pages 142-157. doi: 0.1016/J.Compeleceng.2018.05.019

[6] Aizenberg I. Complex-valued Neural Networks With Multi-valued Neurons. Studies In Computational Intelligence (Vol. 353). Springer-verlag Berlin Heidelberg. 2011. 273 C. doi 10.1007/978-3-642-20353-4.

[7] Bykovsky Alexey Yu. Heterogeneous Network Architecture For Integration Of Ai And Quantum Optics By Means Of Multiple-valued Logic . Quantum Rep. 2020. №2. Pp. 126-165. doi:10.3390/Quantum2010010.

[8] Cobreros Pablo, Egre Paul, Ripley David, Van Rooij Robert. Three-valued Logics And Their Applications. Journal Of Applied Non-classical Logics. 2014. Vol.24, Iss.1-2. P. 1-11. doi: 10.1080/ 11663081.2014. 909631.

[9] Connelly Jeff. Ternary Computing Testbed 3-trit Computer Architecture. Phd Thesis. Computer Engineering Department. California Polytechnic State University. 2008. P.192.

[10] Yi Jin, Huacan He, Yangtian Lü. Ternary Optical Computer Architecture . Physica Scripta. 2005. T118. doi: 10.1238/Physica.Topical.118a00098.

[11] Hu Zhengbing, Deibuk Vitaly. Design Of Ternary Reversible/Quantum Sequential Elements . Journal Of Thermoelectricity. 2018. №1. C. 5-16.

[12] Muthukrishnan Ashok, Stroud C. R. Jr. Multivalued Logic Gates For Quantum Computation . Phys. Rev. A. 2000. Iss. 5 (Vol. 62). doi: doi:10.1103/Physreva.62.052309.

[13] Muthukrishnan Ashok. Classical And Quantum Logic Gates: An Introduction To Quantum Computing. - Rochester Center For Quantum Information (Rcqi). Quantum Information Seminar, 1999. P.22.

[14] Warzecha M., Oszajca M., Pilarczyk K., Szaciłowski K. A Three-valued Photoelectrochemical Logic Device Realising Accept Anything And Consensus Operations . Chemical Communications. 2015. Vol.51, Iss.17. P. 3559-3561. doi: 10.1039/C4cc09980j.

[15] Esin A., Yavorskiy R., Zemtsov N. Brief Announcement Monitoring Of Linear Distributed Computations. In: Dolev S. (Eds) Distributed Computing. Disc 2006. Lecture Notes In Computer Science, Vol 4167. Springer, Berlin, Heidelberg. doi:10.1007/11864219-47

[16] Esin A.A. On Function Classes In P3 Precomplete With Respect To A Strengthened Closure Operator . Math Notes. 2008, 83:5. C. 594603. doi: 10.1134/S0001434608050027

[17] Kak Subhash. On Ternary Coding And Three-valued Logic. 2018. Url: Arxiv.Org/Abs/1807.06419

[18] Gaudet V. A Survey And Tutorial On Contemporary Aspects Of Multiple-valued Logic And Its Application To Microelectronic Circuits . Ieee Journal On Emerging And Selected Topics In Circuits And Systems. 2016. Vol. 6, March, No. 1. Pp. 5-12. doi: 10.1109/Jetcas.2016.2528041.

[19] Wu Haixia, Bai Yilong, Li Xiaoran, Wang Yiming. Design Of High-speed Quaternary D Flip-flop Based On Multiple-valued Current-mode . Journal Of Physics: Conference Series. 2020, October. Vol. 1626. doi: 10.1088/1742-6596/1626/1/012067.

[20] Ibm Quantum Summit 2020: Exploring The Promise Of Quantum Computing For Industry, Www.Ibm.Com/Blogs/Research/2020/09/Quantum-industry/.

[21] J. B. Nation. Logic On Other Planets. Preprint, 2005. url: Www.Math. Hawaii.Edu/ Jb/Planets.Pdf.

[22] S.V. Yablonskiy, G.P. Gavrilov And V.B. Kudryavtsev ”Logical Algebra Functions And Post Classes”. Moscow.: ”Nauka”, 1966.

[23] S.V. Yablonskii, “Functional Constructions In A K-valued Logic”, Collection Of Articles On Mathematical Logic And Its Applications To Some Questions Of Cybernetics, Trudy Mat. Inst. Steklov., 51, Acad. Sci. Ussr, Moscow, 1958, 5–142

[24] Post E.L. Two-valued Iterative Systems Of Mathematical Logic . Annals Of Math. Studies. Princeton Univ. Press. 1941. V. 5.

[25] Kalimulina Elmira Yu., Lattice structure of some closed classes for non-binary logic and its applications. Mathematical Methods for Engineering Applications, Subtitle: ICMASE 2021, Salamanca, Spain, July 1-2.

[26] Configuration Guide - Ethernet Switching. Example for Configuring Link Aggregation in LACP Mode. https://support.huawei.com, 2020.

[27] Cisco guide. Link Aggregation Control Protocol (LACP) (802.3ad) for Gigabit Interfaces. https://www.cisco.com/c/en/us/td/docs.