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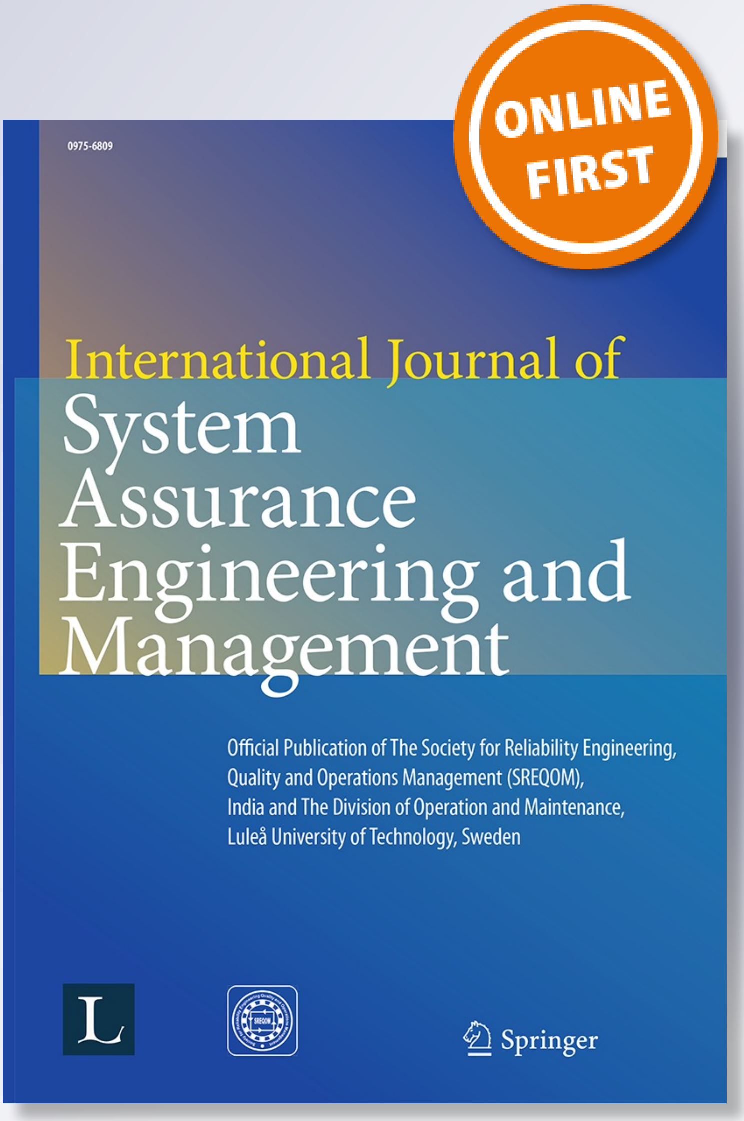

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CASE STUDY

Analysis of system reliability with control, dependent failures, and arbitrary repair times

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Abstract This work is motivated by modelling of real information systems. Parallel and series reliability models or their combinations are usually used for these tasks. Common assumptions for such models are independent failures, exponentially distributed failures and recoveries. These assumptions simplify a system modelling significantly, but often give a very rude approximation for it. So there are a lot of restrictions for an application of these models to practical tasks. This study presents a system with more general assumptions: dependent failures, arbitrary failures and repairs, and a system with control. We apply a continuous-time semi-Markov process to evaluate the reliability and the mean time to system failure (MTTF) for a system under these assumption. The repair time of each component is assumed to have an arbitrary distribution function (e.g., Weibull, Poisson or exponential). Kolmogorov equations method and the Laplace transform are used to derive generalised expressions for system state probabilities, reliability and MTTF. A numerical example is presented in order to illustrate the performance analysis of the model.

Keywords Reliability · Dependent failures · Semi-markov model · Laplace transform

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1 Introduction

Commonly used reliability models of information and computer systems are based on the following assumptions (Siewiorek and Swarz 2014; Kumar and Malik 2012; Munday and Malik 2014; Thanakornworakij et al. 2012; Xin et al. 2014; Zhou et al. 2014; Fiondella and Xing 2015; Huang et al. 2015; Li et al. 2016):

- independent failures,
- the exponential distribution of a failure time,
- the exponential distribution of a recovery time,
- identity of elements in a model of reliability,
- non repairable components.

Exact solutions for availability and reliability coefficients derived under these assumptions are well-known. These results are classical and can be found in a lot of textbooks on mathematical theory of reliability, for example in Barlow and Proschan (1996), Barlow (1996), Ushakov (1994), Blischke and Prabhakar (2000), Ramakumar (2000), or in Birolini (2013), Bazovsky (2013), Abdel-Mohamed (2012).

The study in Kumar and Malik (2012) deals with the reliability modelling of a computer system of two identical units-one is operative and other is kept as spare in cold standby. In each unit h/w and s/w components work together and fail independently. The failure time distribution of the components follow negative exponential whereas the distributions of preventive maintenance, repair and replacement times are taken as arbitrary with different probability density functions. Reliability measures of a computer system with independent constant failure of hardware and software components have been evaluated in Munday and Malik (2014). In this paper the random variables are assumed to be statistically independent. The

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failure times of hardware and software components also follow negative exponential distribution.

There are many examples of reliability calculations for elements of computer systems in Siewiorek and Swarz (2014, Part I, Chapter 5). The general assumption for the most of them is an exponential distribution of failure and recovery times.

The authors of paper Xin et al. (2014) consider a load-sharing computer systems with redundant structure and provide their research of the optimal maintenance policy for high reliability under the assumption that the components are identical distributed with exponential failure times. The paper Zhou et al. (2014) analyses the system reliability of on board computer system by considering a two-state Markov model with independent units and constant intensity rates.

Fiondella and Xing proposed in Fiondella and Xing (2015) discrete and continuous models with k-out-of-n structure. They refused from the assumption that components of a system fail in a statistically independent manner and consider correlated failures. But their reliability model is restricted by identical components, where components possess the same reliability and also exhibit a common failure correlation parameter.

A new reliability model and an analytical solution are developed for a warm standby redundancy with two sets of identical units with exponential distribution in Huang et al. (2015). The complicated reliability model for non-repairable dependent multi-state k-out-of-n systems with identical components, nonidentical components, and partially dependent components are formulated in Li et al. (2016) using copula functions and minimal paths of system states.

But real computer systems have more complex structure. A time of recovery in modern systems differs from exponential and may be, for example, lognormally distributed (for collections of IT services) (Franke et al. 2014), may have a Weibull and q-Weibull distribution (Assis et al. 2013), or may have the other modified distribution function, see, for example, the statistical research paper (Khan and Jan 2015). Non-exponential delays are a standard assumption for reliability models of electronic funds transfer systems (Arajo et al. 2011), mechanical systems (Kumar et al. 2014).

Elements in modern telecommunication networks, computer systems are functioning dependently, redundant components have different reliability characteristics, and usually there is a control system, which affects the total reliability in a nonlinear way (Nader 2014). There are a lot of applied and practical textbooks on reliability modelling, but they are more about general methods and ideas than real practical examples, see Shooman (2003), Alfa (2010), Ushakov (2012). There are no practical reliability models for systems with an automatic control.

In this way the developing of new reliability models for analysis of modern systems (distributed networks, computer systems, information systems and etc.) still remains the actual task. But in difference from classical reliability models the new ones should take into account special features such as non-exponential distributed failures and recovery times, the complex redundancy scheme, the dependency of elements in a reliability model.

A computer simulation is a standard technique for analysing complex systems, but the main disadvantage of simulation is the absence of exact analytical formulae for reliability, which is important for the future analysis of systems, for example, for tasks of reliability planning and optimisation (Kalimulina 2011; Kalimulina et al. 2012; Kalimulina 2013).

The aim of this paper is to develop mathematical models of reliability of complex systems with an application to distributed networks with a control. We achieve this goal by applying semi-Markov models of reliability. We construct a state graph, derive Kolmogorov equations for system state probabilities and solve them using the Laplace transform. Formulae for non-stationary and stationary reliability coefficients with assumptions about arbitrary distributions of recovery times are derived.

Then we estimate the efficiency of control of distributed network, analyse the dependency of availability coefficient from the distribution of the recovery time of network control centre. Also the example illustrating the difference between classical models and models developed here is given. It shows the importance to take into consideration the dependency between elements while reliability planning.

The idea of application of semi-Markov processes in reliability theory is well known. There are a lot of good textbooks on this subject with an introduction to stochastic processes and renewal theory. But they are more theoretical than practical, and sometimes too complicated for engineering problems (Oprisan and Limnios 2001). So developing of models for every special case and adaptation of theoretical models for practical usage are actual tasks. For example, in Titman (2014) the author gives phase-type approximations for estimation of parametric semi-Markov models. In Norros et al. (2014) authors construct a stochastic model for repairable system with dependent components.

In this paper we develop semi-Markov reliability models for the system with dependent components, complex transitions between states, with a control and different schemes of redundancy (active and standby) consisting of n components with different reliability parameters.

2 Functional model of reliability

This work is being done mainly with an application to distributed information system (network) with control. So firstly we'll give a short description of them. We consider a distributed system as a collection of functions (or services). We distinguish three main functions:

- data transmission,
- data processing and storage,
- network control and recovery.

Data transmission functions are performed by an access transport networks (ATN). Processing and storage functions are realised by a central processing complex (CPC). In the case of failure of one of these components, the system stops performing basic functions. For more effective operation the network control centre (NCC) is organised. It controls the failures detection and prompt recovery of network components. Thus, we consider a model of reliability consisting of three main components (as services): ATN, CPC, NCC.

Let's consider a mathematical model of reliability and analytical expressions of availability coefficients of this system. In general, the consideration of reliability model as a serial model consisting of three independent subsystems is not correct, because as we can see from the description, a reliability of transmission and processing depends on the control system.

Therefore, in the general case, we cannot consider a serial reliability model, as it's usually done in Barlow and Proschan (1996); Ushakov (1994); Ramakumar (2000); Shooman (2003); Ushakov (2012) and compute a reliability coefficient (availability) as a product of reliabilities of each component:

$$g_s(t) = g_1(t) * g_2(t) * g_3(t), \quad (1)$$

where $g_i(t)$—reliability of component i, t - time.

The operation of components in a distributed network is regulated by the central control system (network control centre or NCC), a NCC is used to detect failures and recover components. At the same time, if a NCC is in failure mode, an ATN and CPC is continuing to work. For the reliability analysis we consider a functional model with dependent components (Fig. 1).

We make the following assumptions for the model in Fig. 1. Each subsystem can be in one of two states: working (up) or failed (down). In the case of ATN failure or/and CPC failure the system is failed. In the case of NCC failure and operational state of CPC and ATN the system is in the operational state. If the ATN, CPC, NCC are failed, NCC must be recovered before ATN and CPC, then the ATN and CPC can be recovered. When NCC is failed, the ATN and CPC failures cannot be detected until NCC is recovered.

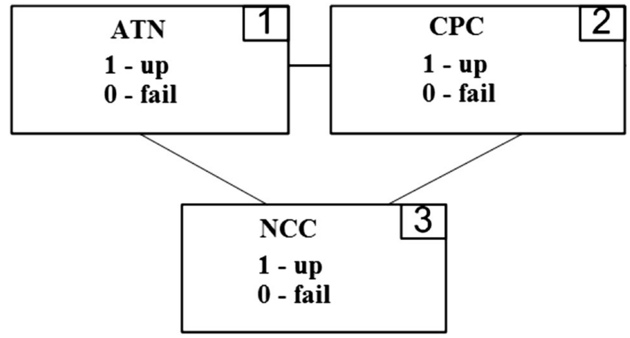

Fig. 1 Reliability model with dependent failures

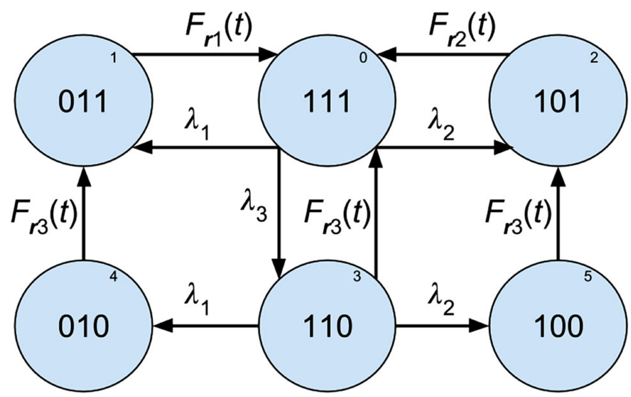

Fig. 2 State graph of the system

When CPC and ATN are failed, then NCC still can be in the operational state. We assume that all network subsystems are recoverable completely (there are no absorbing states on the state graph, which is shown in Fig. 2, where λ_i means the failure rate and F_{ri} means the distribution functions, $i = 1, 2, 3$). The description of states is given in the Table 1, where "0" means that the subsystem is in a failure state, "1" the subsystem is in the operational state for ATN, CPC, NCC, accordingly.

To obtain the explicit solution, we assume the event consisting in the simultaneous failure of ATN and CPC while NCC working to have a zero probability. If NCC is in the failure state, ATN and CPC cannot fail. So, the state graph of network model consists of six states (Fig. 2).

3 Problem solution

We derive formulae for the availability coefficient and the mean time between failures of a system, taking into consideration the following assumptions:

- failure rates of ATN, CPC, NCC equal to $\lambda_1, \lambda_2, \lambda_3$,
- recovery times of subsystems "1", "2", "3", have arbitrary distribution functions $F_{r1}(t)$, $F_{r2}(t)$, $F_{r3}(t)$.

Table 1 Description of states

ATN	CPC	NCC	State number	System decription
1	1	1	0	ATN is up, CPC is up, NCC is up, system is up
0	1	1	1	ATN is down, CPC is up; NCC is up, system is down and recovering
1	0	1	2	ATN is up, CPC is down, NCC is up, system is down and recovering
1	1	0	3	ATN is up, CPC is up, NCC is down, system is up, NCC is recovering
0	1	0	4	ATN is down, CPC is up; NCC is down, system is down
1	0	0	5	ATN is up, CPC is down, NCC is down and recovering, system is down

We denote the stationary probabilities of system states $0, 1, 2, 3, 4, 5$ as $p_0, p_1, p_2, p_3, p_4, p_5$. The network is working, if ATN and CPC are working. The availability coefficient of a system equals to

$$A = p_0 + p_3. \tag{2}$$

We want to find probabilities of network states $p_i(t), i = 0, \ldots, 5$ or their limit values in order $\lim_{t\to\infty} p_i(t)$ to derive the coefficient of availability. The theory of semi-Markov random processes is used to analyse the reliability of a system with non-exponential distributed time of recovery. Methods of reliability analysis of systems described by semi-Markov and non-Markov processes are presented in Oprisan and Limnios (2001). For the reliability estimation we firstly have to find the functional matrix of transition probabilities for the state graph in Fig. 2 (Barlow and Proschan 1996; Oprisan and Limnios 2001). Denote the transition probability from the state i to the state j at Markov moments of time as $P_{ij}(t)$. The system can move from the state 0 to the state 1 (see Fig. 2) only in the case of ATN failure during a small interval of time $(\tau, \tau + d\tau)$ under the condition that CPC and NCC have not failed till time t.

Thus, transition probabilities equal:

$$P_{0j}(t) = \int_0^t \lambda_j e^{-(\lambda_1+\lambda_2+\lambda_3)\tau} d\tau, \quad j = 1, 2, 3, \tag{3}$$

$$P_{i0}(t) = F_{ri}(t), \quad i = 1, 2, \tag{4}$$

$$P_{30}(t) = \int_0^t e^{-(\lambda_1+\lambda_2)\tau} dF_{r3}(\tau), \tag{5}$$

$$P_{34}(t) = \int_0^t \lambda_1 e^{-(\lambda_1+\lambda_2)\tau}(1 - F_{r3}\tau)d\tau, \tag{6}$$

$$P_{35}(t) = \int_0^t \lambda_2 e^{-(\lambda_1+\lambda_2)\tau}(1 - F_{r3}\tau)d\tau, \tag{7}$$

$$P_{41}(t) = P_{52}(t) = F_{r3}(t), \tag{8}$$

$$p_{0j} = \frac{\lambda_j}{\lambda_1 + \lambda_2 + \lambda_3}, \quad j = 1, 2, 3, \tag{9}$$

$$p_{30} = F^*_{r3}(\lambda_1 + \lambda_2), \tag{10}$$

$$p_{34} = \frac{\lambda_1}{\lambda_1 + \lambda_2}\left(1 - F^*_{r3}(\lambda_1 + \lambda_2)\right), \tag{11}$$

$$p_{35} = \frac{\lambda_2}{\lambda_1 + \lambda_2}\left(1 - F^*_{r3}(\lambda_1 + \lambda_2)\right), \tag{12}$$

where $F^*_{r3}(\lambda_1 + \lambda_2)$ is the Laplace transform of a function $F_{r3}(t)$ with $s = \lambda_1 + \lambda_2$.

The distribution functions for unconditional mean times of staying of a random process in the state i $(i = 0, 1, \ldots, 5)$:

$$\begin{aligned} F_0(t) &= 1 - e^{-(\lambda_1+\lambda_2+\lambda_3)t}, \\ F_1(t) &= F_{r1}(t), \\ F_2(t) &= F_{r2}(t), \\ F_0(t) &= 1 - (1 - F_{r3}(t))e^{-(\lambda_1+\lambda_2)t}, \\ F_4(t) &= F_5(t) = 1 - (1 - F_{r3}(t)). \end{aligned} \tag{13}$$

The unconditional expectations of time of staying in the state i:

$$\overline{\eta}_0 = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3}, \tag{14}$$

$$\overline{\eta}_1 = \int_0^{+\infty} t dF_{r1}(t), \tag{15}$$

$$\overline{\eta}_2 = \int_0^{+\infty} t dF_{r2}(t), \tag{16}$$

$$\overline{\eta}_3 = \frac{1}{\lambda_1 + \lambda_2}(1 - F^*_{r3}(\lambda_1 + \lambda_2)), \tag{17}$$

$$\overline{\eta}_4 = \int_0^{+\infty} t dF_{r3}(t), \tag{18}$$

$$\overline{\eta}_5 = \int_0^{+\infty} t dF_{r3}(t). \tag{19}$$

The stationary probabilities of embedded markov chain are defined by the following system of equations:

$$\begin{aligned}\pi_0 &= \pi_1 p_{10} + \pi_2 p_{20} + \pi_3 p_{30},\\ \pi_1 &= \pi_0 p_{01} + \pi_4 p_{41},\\ \pi_2 &= \pi_0 p_{02} + \pi_5 p_{52},\\ \pi_3 &= \pi_0 p_{03},\\ \pi_4 &= \pi_3 p_{34},\\ \pi_5 &= \pi_3 p_{35}.\end{aligned} \tag{20}$$

The solution is:

$$\begin{aligned}\pi_1 &= \pi_0 (p_{01} + p_{03} p_{34} p_{41}),\\ \pi_2 &= \pi_0 (p_{02} + p_{03} p_{35} p_{52}),\\ \pi_3 &= \pi_0 p_{03},\\ \pi_4 &= \pi_0 p_{03} p_{34},\\ \pi_5 &= \pi_0 p_{03} p_{35}.\end{aligned} \tag{21}$$

From (20, 21) and the condition $\sum_{i=0}^{5} \pi_i = 1$, we can derive all stationary probabilities:

$$\pi_0 = (1 + a_1 + a_2 + a_3 + a_4 + a_5)^{-1}, \tag{22}$$

where

$$\begin{aligned}a_1 &= p_{01} + p_{03} p_{34} p_{41},\\ a_2 &= p_{02} + p_{03} p_{35} p_{52},\\ a_3 &= p_{03},\\ a_4 &= p_{03} p_{34},\\ a_5 &= p_{03} p_{35},\end{aligned}$$

and p_{ij} are derived from (3–12).

In general case a stationary and non-stationary reliability coefficients are derived from Barlow (1996); Ushakov (1994); Ramakumar (2000)

$$g(t) = \sum_{k \in E_+} P_k(t), A = \sum_{k \in E_+} p_k,$$

where

$$p_i = \pi_i \overline{\eta}_i / \sum_{j \in E} \pi_j \overline{\eta}_j, \tag{23}$$

E, the system states set, E_+, the subset of operating states, $P_i(t)$, a probability of no failure operation during the time t under the condition, that at the initial moment the system was in the state k, $\overline{\eta}_i$, is taken from (14 to 19).

The system is working if it's in the state **0** or **3**, so the availability coefficient is a sum of probabilities, that system in the state **0** or **3**

$$A_s = p_0 + p_3 = \left(1 + \frac{\lambda_3}{\lambda_1 + \lambda_2}\left(1 - F_{r3}^*(\lambda_1 + \lambda_2)\right)\right) / \Delta \tag{24}$$

where

$$\begin{aligned}\Delta = 1 &+ (\lambda_1 + \lambda_3 p_{34} p_{41}) \int_0^{+\infty} t dF_{r1}(t)\\ &+ (\lambda_2 + \lambda_3 p_{35} p_{52}) \int_0^{+\infty} t dF_{r2}(t)\\ &+ \lambda_3 (1 - F_{r3}(\lambda_1 + \lambda_2)) \left(\frac{1}{\lambda_1 + \lambda_2} + \int_0^{+\infty} t dF_{r3}(t)\right).\end{aligned}$$

In the special exponential case if we set $F_{ri}, i = 1, 2, 3$ equal:

$$F_{r1}(t) = 1 - e^{-\mu_1 t}, F_{r2}(t) = 1 - e^{-\mu_2 t}, F_{r3}(t) = 1 - e^{-\mu_3 t}, \tag{25}$$

the availability coefficient equals:

$$A_s = \frac{1 + \frac{\lambda_3}{\lambda_1 + \lambda_2 + \mu_3}}{1 + \frac{\lambda_3}{\mu_3} + \left(1 + \frac{\lambda_3}{\lambda_1 + \lambda_2 + \mu_3}\right)\left(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}\right)}. \tag{26}$$

4 Efficiency of a control centre of network

In this section we analyse the dependency of the system reliability from subsystems parameters. To evaluate the influence of a recovery time distribution of the NCC on system reliability we consider the widely used for renewal models distributions (O'Connor 2011): exponential, uniform, gamma-distribution.

As it was mentioned in Sect. 3, if we set F_{r3} equal to $(1 - e^{-\mu_3(t)})$, the availability coefficient is defined by the formula (26).

For a uniform distribution:

$$F_{r3}(t) = \begin{cases} 0, & if \quad t < a, \\ \dfrac{(t-a)}{(b-a)}, & if \quad a \le t \le b, \\ 1, & if \quad b \le t, \end{cases}$$

the availability coefficient equals

$$\begin{aligned}F_{r3}^*(\lambda_1 + \lambda_2) &= \frac{a(\lambda_1 + \lambda_2) - 1}{(\lambda_1 + \lambda_2)(a - b)},\\ \overline{\eta}_4 = \overline{\eta}_5 &= \frac{a + b}{2},\\ A_s &= \frac{1 + x}{1 + \rho_1(1 + x) + \rho_1(1 + x) + x\left(1 + \frac{(a+b)(\lambda_1 + \lambda 2)}{2}\right)},\end{aligned} \tag{27}$$

where

$$\begin{aligned}x &= \frac{\lambda_3(1 - b(\lambda_1 + \lambda_2))}{(a - b)(\lambda_1 + \lambda_2)^2},\\ \rho_1 &= \lambda_1 / \mu_1,\\ \rho_2 &= \lambda_2 / \mu_2.\end{aligned}$$

Table 2 Reliability coefficients for different distributions of recovery time

μ_3	a	b	k	A_s Exp	A_s Uniform	A_s Gamma
0.1	10	20	0.5	0.921095	0.855504	0.922826
0.11	9.09091	19.0909	0.55	0.921424	0.860728	0.922799
0.12	8.33333	18.3333	0.6	0.921677	0.865127	0.922772
0.13	7.69231	17.6923	0.65	0.921876	0.868883	0.922744
0.14	7.14286	17.1429	0.7	0.922035	0.872127	0.922717
0.15	6.56777	16.6667	0.75	0.922165	0.874958	0.92269
0.16	6.25	16.25	0.8	0.922272	0.877449	0.922662
0.17	5.88235	15.8824	0.85	0.922361	0.879658	0.922635
0.18	5.55556	15.5556	0.9	0.922436	0.881631	0.922608
0.19	5.26316	15.2632	0.95	0.9225	0.883403	0.922581
0.2	5	15	1	0.922555	0.885004	0.922555
0.21	4.7619	14.7619	1.05	0.922602	0.886458	0.922529
0.22	4.54545	14.5455	1.1	0.922643	0.887783	0.922503
0.23	4.34783	14.3478	1.15	0.922679	0.888996	0.922478
0.24	4.16667	14.1667	1.2	0.922711	0.890111	0.922454
0.25	4	14	1.25	0.922739	0.891139	0.92243
0.26	3.84615	13.8462	1.3	0.922764	0.89209	0.922407
0.27	3.7037	13.7037	1.35	0.922787	0.892972	0.922385
0.28	3.57143	13.5714	1.4	0.922807	0.893793	0.922363
0.29	3.44828	13.4483	1.45	0.922824	0.894558	0.922342
0.3	3.33333	13.3333	1.5	0.922841	0.895274	0.922322
0.31	3.22581	13.2258	1.55	0.922855	0.895944	0.922303
0.32	3.125	13.125	1.6	0.922869	0.896574	0.922284
0.33	3.0303	13.0303	1.65	0.922881	0.897166	0.922267
0.34	2.94118	12.9412	1.7	0.922892	0.897723	0.92225
0.35	2.85714	12.8571	1.75	0.922903	0.89825	0.922235
0.36	2.77778	12.7778	1.8	0.922912	0.898748	0.92222
0.37	2.7027	12.7027	1.85	0.922921	0.899219	0.922206
0.38	2.63158	12.6316	1.9	0.922929	0.899667	0.922193
0.39	2.5641	12.5641	1.95	0.922936	0.900091	0.922181
0.4	2.5	12.5	2	0.922943	0.900495	0.92217

For a gamma-distribution:

$$F_{r3}(t) = 1 - \sum_{i=0}^{k-1} \frac{\mu_3 t}{i!} e^{-\mu_3 t},$$

$$F^*_{r3}(\lambda_1 + \lambda_2) = 1 - \sum_{i=0}^{k-1} \frac{\lambda_1 + \lambda_2}{(\lambda_1 + \lambda_2 + \mu_3)^{i+1}},$$

$$\bar{\eta}_4 = \bar{\eta}_5 = \frac{k}{\mu_3},$$

$$A_s = \frac{1 + x}{1 + \rho_1(1 + x) + \rho_1(1 + x) + x\left(1 + \frac{k(\lambda_1 + \lambda 2)}{\mu_3}\right)},$$

$$x = \frac{\lambda_3(1 - (\lambda_1 + \lambda_2 + \mu_3)^{-k})}{\lambda_1 + \lambda_2 + \mu_3 - 1},$$

$$\rho_1 = \lambda_1/\mu_1,$$

$$\rho_2 = \lambda_2/\mu_2. \quad (28)$$

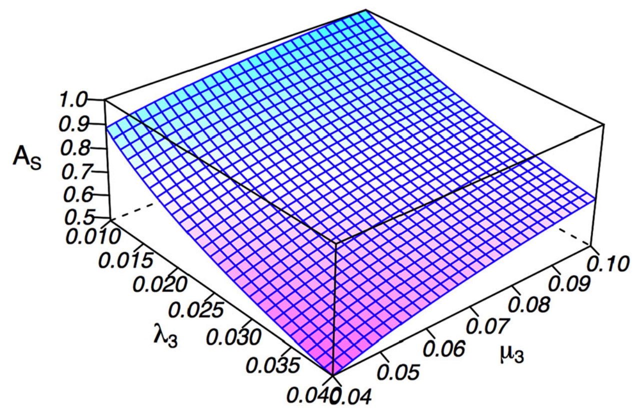

Fig. 3 Graph of the dependency coefficient of reliability from NCC parameters

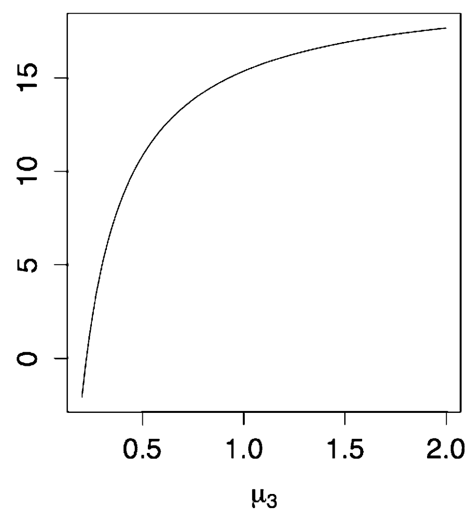

Fig. 4 Graph of $\frac{1-A_{s_{series}}}{1-A_s}$ as a function of μ_3

Now set:

$$\mu_1 = 1/11, \mu_2 = 1/10, \lambda_1 = 1/240, \lambda_2 = 1/300, \lambda_3 = 1/290.$$

and calculate reliabilities on formulae (26), (27), (28). The calculation results are in the Table 2.

Using formula (26) we can estimate the dependency of the system reliability from the reliability of NCC. The system reliability increases at least by 7 % with NCC, and the mean time between failures increases 10 times. Figure 3 shows the dependency of the system reliability from reliability parameters of NCC. The initial data: $\mu_1 = 1/11, \mu_2 = 1/20, \lambda_1 = 1/220, \lambda_2 = 1/250$.

Now we estimate the influence of centralised control centre on reliability improvement. To achieve this task we compare the reliability of the model in Fig. 1 with the reliability of series model consisting of two independent components (without control centre). The system reliability

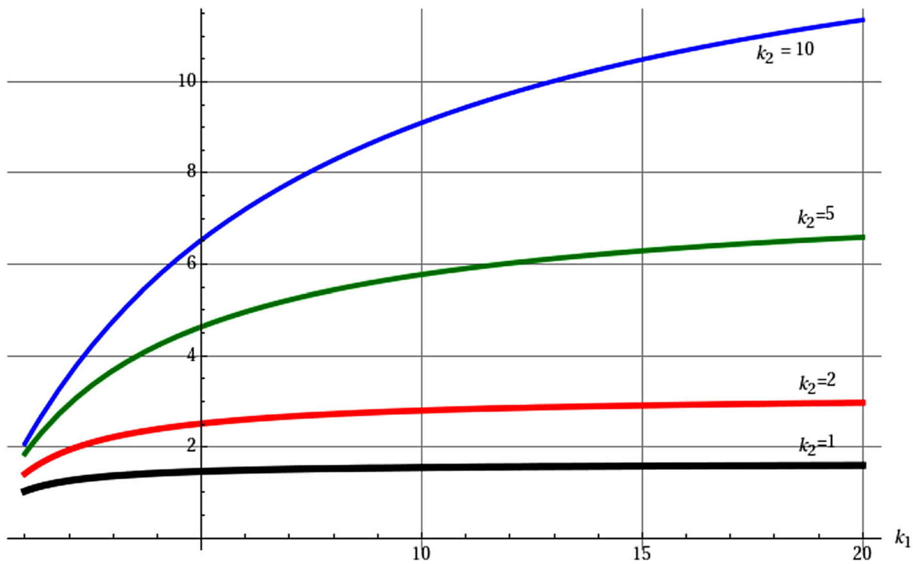

Fig. 5 Graph of $\frac{1-A_{s_{series}}}{1-A_s}$ as a function of k_1 with $k_2 = 1, 2, 5, 10$

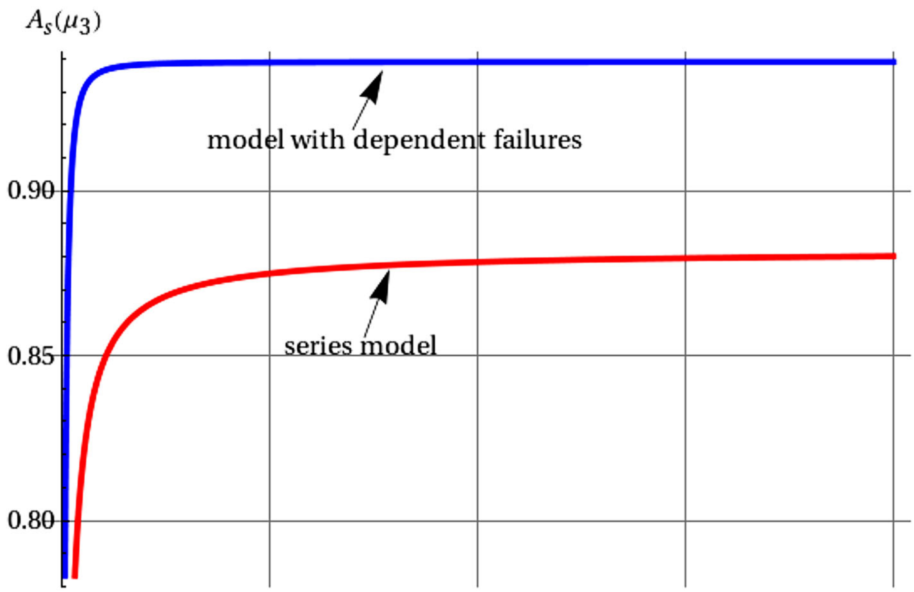

Fig. 6 Graph of the reliability for dependent and series models as functions of μ_3

of two independent components is estimated by the following formula (Ushakov 2012; Shooman 2003; Ushakov 1994):

$$A_{s_{series}} = \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}. \tag{29}$$

Suppose CPC and ATN is recovering faster with NCC. Set recovery rates of CPC and ATN equal to $\mu_1 = \mu_1 * k_1$ and $\mu_2 = \mu_1 * k_2$ $(k_1 > 1, k_2 > 1)$ and consider relation:

$$\frac{1-A_{s_{series}}}{1-A_s} = \frac{k_2\lambda_1\mu_2\mu_3(\lambda_1+\lambda_2+\lambda_3+\mu_3)}{k_1k_2(\lambda_1+\mu_1)(\lambda_2+\mu_2)\mu_3(\lambda_1+\lambda_2+\lambda_3+\mu_1)} + \frac{k_1\mu_1k_2\mu_2(\lambda_1+\lambda_2+\mu_3)(\lambda_3+\mu_3)}{k_1k_2(\lambda_1+\mu_1)(\lambda_2+\mu_2)\mu_3(\lambda_1+\lambda_2+\lambda_3+\mu_1)} + \frac{k_1\mu_1\lambda_2\mu_3(\lambda_1+\lambda_2+\lambda_3+\mu_3)}{k_1k_2(\lambda_1+\mu_1)(\lambda_2+\mu_2)\mu_3(\lambda_1+\lambda_2+\lambda_3+\mu_1)}. \tag{30}$$

The dependency of efficiency (30) from recovery rate μ_3 with fixed k_1 and k_2 is shown in Fig. 4. The dependency of efficiency (30) from k_1 is shown in Fig. 5.

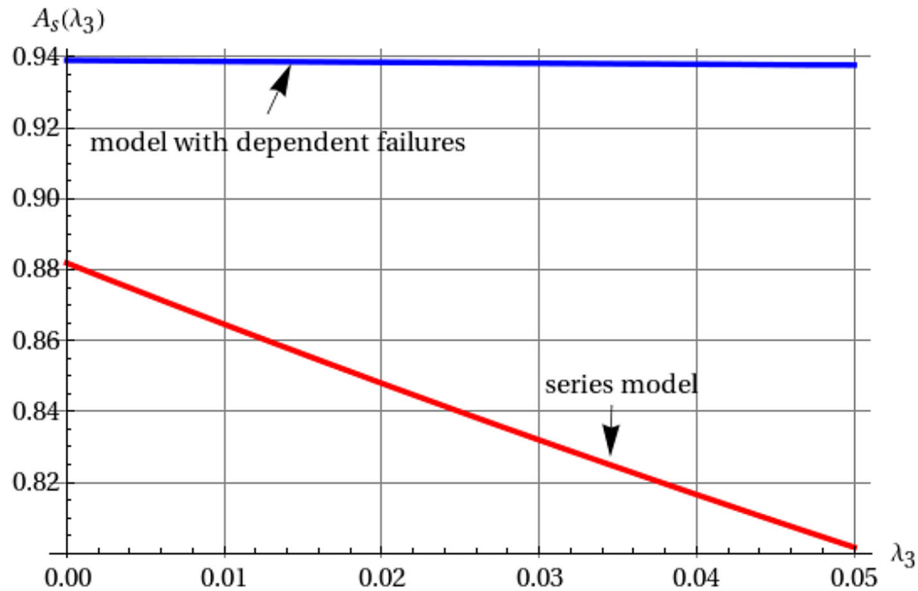

Fig. 7 Graph of the reliability for dependent and series models as functions of λ_3

We have restricted here only by three distributions. But the formula (24) can be applied for more flexible distributions, for example, for a Weibull distribution with the density function $f(x) = k/\lambda * (x/\lambda)^{k-1} e^{-(x/\lambda)^{k-1}}$, $x \geq 0$. We only have to apply the Laplace transform $F^*_{r3}()$ for distribution function, but this problem has been solved. One of the approaches to this task can be found in Rossberg (2008). We don't provide this result here due to the cumbersome formula. And it is worth mentioning the proposed method may perform well in the simulation study for a large system where we can use their approximations instead of exact distributions.

Now we estimate an increase in reliability after NCC implementation. Assume the following initial data: $\mu_1 = 1/11$, $\mu_2 = 1/20$, $\lambda_1 = 1/220$, $\lambda_2 = 1/250$, $k_1 = 2, \mu_3 = 1/2$, $\lambda_3 = 1/250$. Reliability of two-component series system equals 0.881834 (downtime coefficients equals 0.118166), of three-component system with control centre equals 0.93885 (downtime coefficients equals 0.06115). Thus, from calculations and graphs in Figs. 4 and 5 we can make a conclusion that control centre decreases the downtime approximately twice.

As it was mentioned above the reliability of CPC and ATN depends on early failures detection, consequently on the reliability of NCC. Let's compare numerical results of the reliability estimation for series model with the model of dependent components. Set $\mu_1 = 2/11$, $\mu_2 = 1/10$, $\lambda_1 = 1/220, \lambda_2 = 1/250, \mu_3 = 1/2, \lambda_3 = 1/250$ in (26). So reliability for a model with dependent components equals to 0.93855, for independent series system equals to 0.874836. The dependency of reliability for these models from μ_3 and λ_3 is shown in Figs. 6 and 7.

5 Conclusion

We derive analytical formulae for reliability estimation of systems with dependent components with a control based on semi-Markov models. General results were obtained for widely used renewable models such as exponential, uniform, gamma distributions. Also a special case of exponential failure and recovery rates has been considered.

Then we estimated the efficiency of NCC of distributed network, and analysed the dependency of a reliability coefficient from the distribution of recovery time of NCC. The numerical example of reliability estimation for a distributed information system (network) with control has been presented.

The derived formulae may be applied to reliability estimation of different distributed systems (information, telecommunication, energy networks and etc.) with a structure which is more complex than independent and parallel ones. The results show the importance to take into consideration the dependency between elements while reliability planning and efficiency of NCC.

But there may be some small restrictions with the described model. An implementation of extra states may be required in model in Fig. 2, but this problem can be easily solved by using any software for graph modelling.

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