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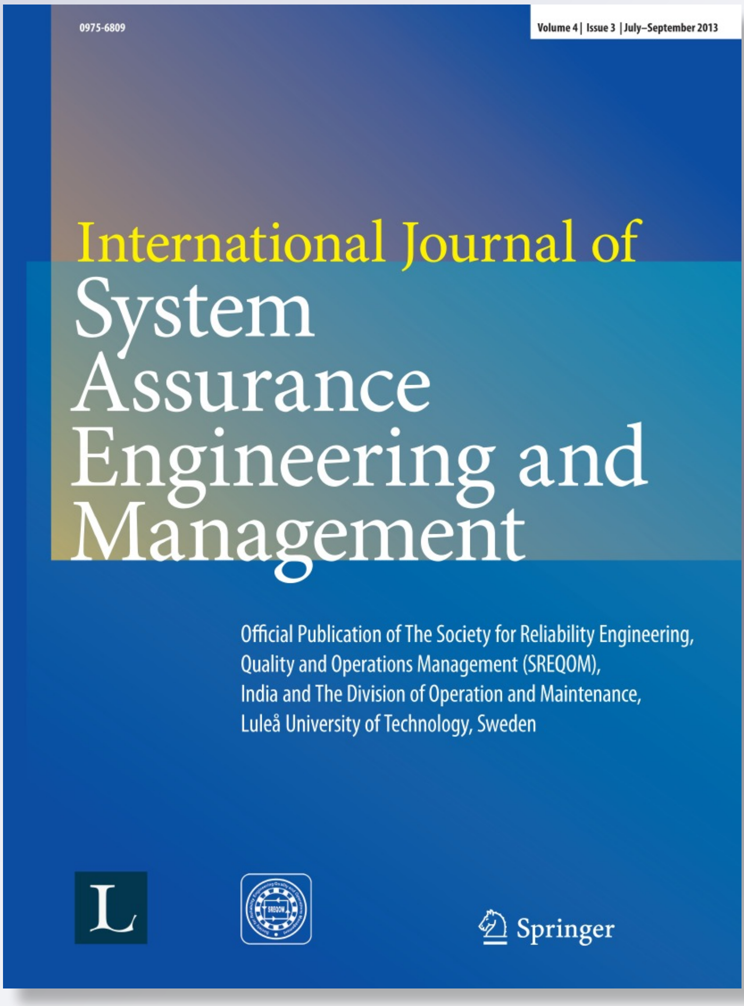

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ORIGINAL ARTICLE

A new approach for dependability planning of network systems

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Abstract In this paper, a new approach to managing reliability of network systems is proposed. In contrast to the well-known problem formulation of reliability optimization it takes into account many parameters of a system such as costs, operational reliability, the profit from system operation, penalties for delays and failures. The objective function and boundary conditions for this problem formulation are derived. Explicit analytical expressions for an optimization of network systems are based on its performance metrics. The method can be used while designing of telecommunication systems, computer networks, network-centric and service-oriented systems.

Keywords Reliability model · Network systems · Reliability optimization · Redundancy

1 Introduction

The current stage of development of network systems is characterized by the growth of complexity of network infrastructure, and by the continuous increase of requirements to quality of services, and there is the need to consider a network not only as complex technical system but also as economic one. The problem of reliability of network systems still remains one of the most important. The low network reliability leads to the loss of customers and profit and increases penalties. Network costs for reliability and penalties, paid under Service Level Agreements (SLA) (Verma 2004), may exceed the profit from providing telecommunication services. The International Telecommunication Union issued the Recommendation E.862 "Planning the reliability of communication networks", that mentioned that during the planning, design, operation and maintenance of networks the economic losses of users and network owners due to unreliability must be considered (1992) . This document is a recommendation and does not include clear procedures and models needed to solve this problem. In this regard, new approaches and new mathematical models for calculating and optimizing reliability of computer networks must be developed.

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2 Problem statement

2.1 Current problem formulation

In the most detailed form the problem of network reliability optimization is considered, for example, in Myers (2010), Shooman (2002), Ushakov (1994). Also the most interesting theoretical and practical results on modern methods of network reliability optimization, can be found in works (Claudio 2000; Xiaoli and Smith 2010; Sooktip et al. 2011; Myers 2010; Blischke and Prabhakar 2000; Tillman et al. 1980; Kuo and Zuo 2003). However, in these works the reliability optimization problem was formulated as the problem of optimal reservation or redundancy and mainly for unrecoverable systems which comprised of identical elements (Barlow and Proschan 1996).

Let's consider several examples of reliability optimization problems from Barlow and Proschan (1996), Tillman et al. (1980), Kuo and Zuo (2003)]. Figure 1 shows the system consisting of s series-connected subsystems, each subsystem consists of n_j parallel-connected elements (hot or active redundancy). All elements of the subsystem with number i have the same reliability and price (cost), and cannot be recovered during the operation period. Each subsystem is characterized by some reliability index, usually it may be the probability of non-failure operation. The numerical value of reliability depends on how many elements there are in a given subsystem, a reliability index is a function of the number of redundant elements. Function is denoted by $R_i(n_i)$, where $(n_i - 1)$ is the number of redundant elements in the subsystem with number i.

Reliability of the system is a function $R(\cdot)$ which depends on the values of reliability indices of the individual subsystems $R_i(n_i)$. So $R(\cdot)$ may be considered as a function of variables $n_1, n_2, ..., n_s$. In Shooman (1994), Mettas (2007), Ramakumar (2000) the reliability indices depend on the reliability of individual subsystems (if all of them are functioning independently from each other) and were expressed as

$$R(n_1, n_2, \ldots, n_s) = \prod_{i=1}^{s} R_i(n_i). \tag{1}$$

Non-repairable systems are considering in the well-known reliability optimization problem formulations, see for example Claudio (2000), Blischke and Prabhakar (2000). The number of elements determines the total costs for a system. It is assumed that the total cost for the whole system $C(\cdot)$ rises linearly on the number of elements

$$C(n_1, n_2, \ldots, n_s) = \sum_{i=1}^{s} c_i(n_i) = \sum_{i=1}^{s} c_i * n_i, \tag{2}$$

where c_i - the cost (price) of one element in subsystem with the number i.

As an example here there will be given two problem statements, direct problem statement and backward problem statement from Xiaoli and Smith (2010), Ushakov (1994).

2.1.1 Direct problem

In the direct problem a system consists of s elements connected in series, c_i is the cost of one element in the

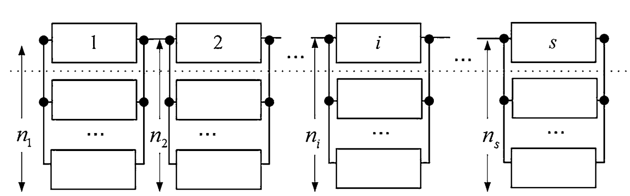

Fig. 1 Block reliability diagram of a system for optimal redundancy problem

subsystem i, $i = 1, 2, ..., s$, r_i is the probability of failure of one element in the subsystem i, $i = 1, 2, ... s$, R_0 is the target value of reliability for the entire system. The number of elements in each subsystem $N = (n_1, n_2, ..., , n_s)$, that gives the minimum of the cost function must be defined:

$$C(n_1^*, n_2^*, \ldots, n_s^*) = \min_{n_1, n_2, \ldots, n_s} C(n_1, n_2, \ldots, n_s) = \min_{n_1, n_2, \ldots, n_s} \sum_{i=1}^{s} c_i n_i.$$

And the boundary condition must be satisfied, the system reliability mustn't be less than the defined value

$$R(n_1^*, n_2^*, \ldots, n_s^*) = \prod_{i=1}^{s} (1 - (1 - r_i)^{n_i}) \geq R_0,$$

where $n_1, n_2, ..., n_s > 0$, $i = 1, 2, ..., s$.

2.1.2 Backward problem statement

In the backward problem statement a system consists of s elements connected in series, c_i and r_i is the cost and the probability of failure of one element in the subsystem i, $i = 1, 2, ..., s$, C_0 is the target value of total costs for the entire system. The number of elements in each subsystem $N = (n_1, n_2, ..., , n_s)$, that gives the maximum for the reliability function must be defined:

$$R(n_1^*, n_2^*, \ldots, n_s^*) = \max_{n_1, n_2, \ldots, n_s} R(n_1, n_2, \ldots, n_s) = \max_{n_1, n_2, \ldots, n_s} \prod_{i=1}^{s} (1 - (1 - r_i)^{n_i}).$$

And the boundary condition must be satisfied, the total system costs mustn't be greater than the defined value.

$$C(n_1^*, n_2^*, \ldots, n_s^*) = \sum_{i=1}^{s} c_i n_i \leq C_0.$$

In these problem formulations the profit from the network functioning, the investments needed to achieve a certain level of reliability, maintenance costs for network and losses due to network failures are not considered (Sooktip et al. 2011). The aim of this paper is to formulate the problem so that all these factors will be taken into account. This will allow to plan and manage the network reliability more accurately and adequately.

2.2 Suggested problem statement

Now the generalized logical model of reliability of network systems will be constructed. The network will be considered as a system consisting of s subsystems, reliability of s_1 subsystems are enhanced by the installation of more reliable equipment (there is no redundant elements), for s_2

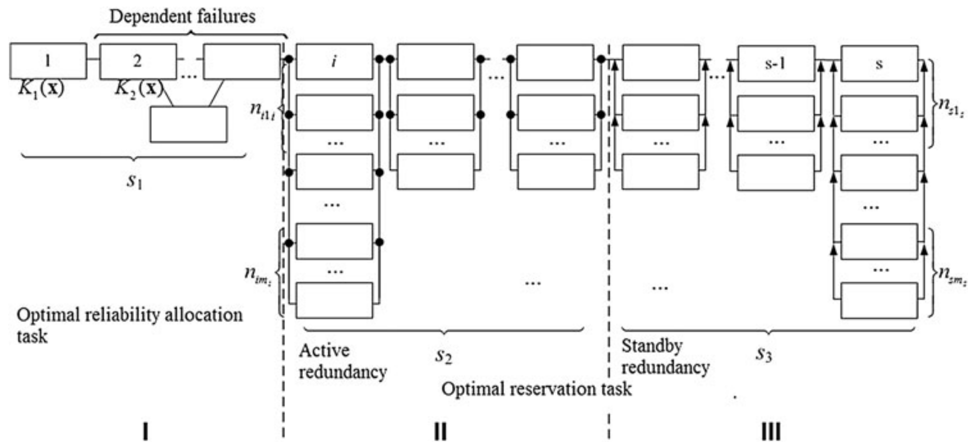

Fig. 2 Generalized model of network reliability

subsystems the active (hot) redundancy is applied, for s_3 subsystems the cold redundancy is applied, so $s = s_1 + s_2 + s_3$. Also, for some network systems there may be subsystems with dependent failures. The numerical value of reliability for subsystems with redundancy depends on the number of standby units. The non-homogeneous redundant units in the network are assumed, so elements in the subsystem i may have different reliability. The number of redundant elements of j type ($j = 1, ..., m_i$) in the subsystem i equals n_{ij}. The reliability $K_i(\cdot)$ of the subsystem i is the function of variables n_{i1}, n_{i2}, ..., n_{im_i}. The generilized logical reliability model for a network system is presented on Fig. 2.

For subsystems without redundancy, the reliability depends on the parameters of the probability distribution of time between failures, recovery time and mode of subsystem and, in general, is defined as the $K_i(x_{i1}, x_{i2}, ..., x_{ik\_i})$, where $X = (x_{i1}, x_{i2}, ..., x_{ik\_i})$ are the model parameters. The availability and steady state availability is used as reliability indices (for short we refer to them as reliability), and defined as $g(.)$ and K, accordingly.

The system reliability K_s is the function of the reliability of individual subsystems. It is the function of the number of redundant elements (for subsystems with redundancy) and parameters of distribution function (for non-redundant subsystems)

$$K_c(\mathbf{N}, \mathbf{X}) = K_s(K_i(n_{i1}, n_{i2}, ..., n_{im_i}), K_l(x_{l1}, x_{l2}, ..., x_{lk_l})), \qquad (3)$$

where $i = 1, 2, ... , s_2, s_2 + 1, ... , s_3$ is the number of redundant subsystems, $l = 1, 2, ..., s_1$ - the number of subsystems without reservation. The total system reliability is defined as

$$K_s(N, X) = K_A(N_A) * K_C(N_C) * K_N(X), \qquad (4)$$

where $K_A(N_A)$ is the availability for the segment with active redundancy, $K_C(N_C)$ is the availability for the segment with cold redundancy, $K_N(X)$ is the availability for segments without redundancy, $X = (x_{ij})$, $i \in N$, $j = 1, 2, ..., k_i$, $N_A = (n_{ij})$, $i \in A$, $j = 1, 2, ..., m_i$, $N_C = (n_{ij})$, $i \in C$.

In the paper the network reliability is denoted as K_s and defined as a function of availability indices of subsystems: $K_s = (K_1, K_2, \ldots, K_{s_1+s_2+s_3})$. In the special case K_i is the subsystem steady state availability index, that is the function of subsystem parameters, that must be defined in reliability optimization.

The cost function is associated with each subsystem, it depends on the reliability $C_i(K_i)$ and has the following properties: (1) $C_i(K_i)$ is non-decreasing function of K_i, $K_i \in [0, 1)$; (2) with $K_i = 0$, $C_i(K_i) = 0$, and (3) when $K_i = 1$, $C_i = \infty$. The income in the case of absolutely reliable network ($K_s = 1$) is denoted as U^+. The size of losses in the case of absolutely unreliable network ($K_s = 0$) equals U^-, $U^+ > 0$, $U^- > 0$. The total cost of the system is defined as the sum of costs for all subsystems

$$C_s(K_s) = \sum_{i=1}^{s_1+s_2+s_3} C_i(K_i). \qquad (5)$$

Therefore the profit derived from the using of absolutely reliable network equals $(U^+ - C_s(K_s))$; in the event of failures it is equal to $(U^- + C_s(K_s))$. These assumptions were absent in the classical optimization problem statements. The system reliability can be increased during its planning and designing, but the cost for reliability increments should be less than the income received from the network: $U^+ - C_s(K_s)) > 0$. Given the reliability the total income is

$$K_s(K_1, K_2, ..., K_i, ...)(U^+ - C_s(K_s)). \qquad (6)$$

And the losses are

$$(1 - K_s(K_1, K_2, ..., K_i, ...))(U^- + C_s(K_s)). \qquad (7)$$

Total costs equal to $C_s = \sum_{i=1}^{s_1+s_2+s_3}$, where $C_i(K_i)$ is the cost for subsystem with number i. For network segments without redundancy (with numbers $i = 1, 2, ..., s_1$), total costs needed for increasing the reliability of subsystem i to the level $K_i(x_{i1}, x_{i2}, \ , x_{ik\_i})$ are estimated as:

$$C_i = C_i(K_i(x_{i1}, x_{i2}, , x_{ik_i})). \tag{8}$$

For segments of the network with redundancy, total costs will depend on the number of redundant elements. Assuming that costs for hardware are known and equal to c_{ij}, where
$i = 1, 2, ..., s_2, s_2 + 1, ..., s_3, j = 1, 2, ..., m_i$, the cost for subsystem i is $c_i = \sum_{j=1}^{m_i} c_{ij} n_{ij}$.And the total cost for redundancy equals:

$$\sum_{i=1}^{s_2+s_3} \sum_{j=1}^{m_i} c_{ij} n_{ij}. \tag{9}$$

Total cost for reliability increases:

$$C_s = \sum_{i=1}^{s_1} C_i(K_i(x_{i1}, x_{i2}, , x_{ik_i})) + \sum_{i=1}^{s_2+s_3} \sum_{j=1}^{m_i} c_{ij} n_{ij}. \tag{10}$$

With regard to (6), (7), (8), (9), (10) the objective function, which determines the profit from the operation of the network will be:

$$NPV(K_s) = K_s(K_1, K_2, ..., K_i, ...,)(U^+ - C_s(K_s)) \tag{11}$$

$$-(1 - K_s(K_1, K_2, ..., K_i, ...))(U^- + C_s(K_s)), \tag{12}$$

where NPV (net present value) is a complex financial index of the network system operating, which takes in account the current operating of the network, the increase in incomes derived by improving the reliability, cost for reliability improvements, loss and penalties.

The expected economic effect from the operation of the network can be estimated while planning the structure of the network. The operation of the network generates continuous cash flows: CF_1, CF_2, ..., CF_N. It can be considered as profit in the case of absolutely reliable network, when $K_s = 1$. When network reliability is <1, the company loses the part of the planned revenue due to downtime during repair period of the network. The availability index shows the part of time during which the network is operational. Taking into account the unreliability the volume of cash flows is defined as

$$Total\, cash\, flow_{real} = Total\, cash\, flow_{planned} * K_s.$$

The value $(U^+ + U^-)$ does not depend on the reliability and can be considered as a target profit $\sum_{i=1}^{N} CF_i$ (denoted as TCF). As a result, profit from the operation of the network will be

$$NPV(K_s) = K_s(K_1, K_2, ..., K_{s_1+s_2+s_3}) * (U^+ + U^-) - C_s(K_s) = K_s(K_1, K_2, ..., K_{s_1+s_2+s_3}) * TCF - C_s(K_s).$$

Using these expressions the system reliability optimization problem can be formulated in the following form: find subsystems reliability $K_S = (K_1, K_2, ..., K_{s_1+s_2+s_3}), K_i \in [0, 1]$, which gives the maximum to profit from operation of network:

$$NPV(K_1^*, K_2^*, ..., K_{s_1+s_2+s_3}^*) - \left(\sum_{i=1}^{s_1} C_i(K_i(x_{i1}, x_{i2}, , x_{ik_i})) + \sum_{i=1}^{s_2+s_3} \sum_{j=1}^{m_i} c_{ij} n_{ij}. \right). \tag{13}$$

With boundary conditions

$K_i \in [0, 1], i = 1, 2, ..., s_1 + s_2 + s_3.$

2.3 Examples of problem formulations for reliability optimization task

This problem statement (13) is flexible; also it can take into account additional losses due to network downtime. The defined above losses $TCF^*(1 - K_s(N, X))$ can be considered as inevitable (or natural) losses due to downtime. In practice, there are situations when for network downtimes the network owner has to pay a "penalty", whose size is usually specified in the contract (SLA) between the client and the company. Its calculation is based on the total network downtime for a certain period of time. Therefore, penalties for failures for a certain period may be considered as extra losses. The penalty size for time unit (usually per hour) can be considered as a constant and is denoted by L. The network downtime depends on reliability indices of subsystems. The idle time depends on the structure of the network and can be defined as the mathematical expectation of the recovery time

$$\overline{T}_{recovery} = E[\max_{T_i} T_1, T_2, ..., T_k], \tag{14}$$

where $\overline{T}_{recovery}$ is the mean recovery time of subsystem i, which depends on the reliability, for example, on the probability of failure-free operation or pointwise availability, i.e., $T_{recovery\quad i} = T_{recovery\quad i}(K_i)$, $i = 1, 2, , k$. Thus, the penalties of network downtime can be estimated as

$$L * E[\max_{T_i(K_i)} T_1(K_1), T_2(K_2), ..., T_k(K_k)]. \tag{15}$$

And the profit can be one of the following three functions of reliability:

(1) given the time t:

$$NPV = NPV(r, t, g_s(g_i(n_{i1}, n_{i2}, ..., n_{im_i}), g_l(t, x_{l1}, x_{l2}, ..., x_{lk_l}))) = \sum_{i=1}^{n} \frac{CF_i}{(1+r)^{i-1}} g_s((i-1)\Delta t, i\Delta t) - \left(\sum_{i=1}^{s_1} c_i(g_i(x_{i1}, x_{i2}, ..., x_{ik_i})) + \sum_{i=1}^{s_2+s_3} \sum_{j=1}^{m_i} c_{ij} n_{ij} \right), \tag{16}$$

where $i = 1, 2, ..., s_2, s_2 + 1, ..., s_3, l = 1, 2, ...s_1$.
(2) with $t \to \infty$:

$$NPV = NPV(g_s(g_i(n_{i1}, n_{i2}, ..., n_{im_i}), g_l(x_{l1}, x_{l2}, ..., x_{lk_l}))) = g_s(N, X)TCF - \left(\sum_{i=1}^{s_1} c_i(g_i(x_{i1}, x_{i2}, ..., x_{ik_i})) + \sum_{i=1}^{s_2+s_3} \sum_{j=1}^{m_i} c_{ij} n_{ij} \right) \quad (17)$$

(3) given the penalties:

$$NPV = NPV(g_s(g_i(n_{i1}, n_{i2}, ..., n_{im_i}), g_l(x_{l1}, x_{l2}, ..., x_{lk_l}))) = g_s(N, X)TCF - \left(\sum_{i=1}^{s_1} c_i(g_i(x_{i1}, x_{i2}, ..., x_{ik_i})) + \sum_{i=1}^{s_2+s_3} \sum_{j=1}^{m_i} c_{ij} n_{ij} \right) + L * E[\max\{T_{recovery1}(g_1), T_{rec.2}(g_2), ..., T_{rec.k}(g_k)\}]. \quad (18)$$

As defined above in models (16)–(18) optimal values for reliability of subsystems which give the maximum the profit must be found:

$$NPV = NPV\left(g_s(g_i(n^*_{i1}, n^*_{i2}, ..., n^*_{im_i}), g_l(x^*_{l1}, x^*_{l2}, ..., x^*_{lk_l}))\right) = \max_{\substack{n_{i1}, n_{i2}, ..., n_{im_i}, \\ x_{l1}, x_{l2}, ..., x_{lk_l}}} NPV(g_s(g_i(n_{i1}, n_{i2}, ..., n_{im_i}), g_l(x_{l1}, x_{l2}, ..., x_{lk_l}))).$$

3 The generalized reliability optimization technique

Based on this problem formulation the step by step method of reliability optimization has been developed. The method comprises of the following steps.

Step 1 Design and construction of a logical model of network reliability. Initial data of a model and the actual definition of what parameters in a model are known and what is necessary to optimize should be defined at this step.

Step 2 Construct the analytical model of network reliability, and derive the reliability function. First construct an initial series model of reliability. This takes into account what subsystems operate independently, but for some subsystems a model of reliability with dependent failures should be used. Selection the subsystems whose reliability can be improved by implementing redundancy: the standby or hot redundant units are added to the main unit for these subsystems. The numbers of redundant units are variables which have to be determined during the optimization problem. Then define subsystems whose reliability cannot be improved through redundancy. Failure rates for these subsystems are variables to be determined in the optimization problem. Derive the formula for calculating availability index $g(t, \mu_1, \mu_2, ..., \mu_n, n_1, n_2, ...n_m)$ (or the steady state availability index for subsystems where the complexity of the calculations doesn't allow to get availability index in explicit form: $K(t, \mu_1, \mu_2, ..., \mu_n, n_1, n_2, ...n_m)$).

Step 3 Define a financial model of a network and net cash flows generated from a network. Based on a reliability model net cash flows $\sum_{i=1}^{N} CF_i$, obtained from a network is calculated. At this step, it's assumed that network is reliable, i.e. $K = 1$ or $g(t) = 1$. Thus, the value of the first summand $\sum_{i=1}^{N} CF_i$ in the formula for $NPV = \sum_{i=1}^{N} CF_i - Costs$ is determined.

Step 4 Define an analytical cost model of a network. Estimate costs needed to improve network reliability, that determines the second summand in the formula $NPV = \sum_{i=1}^{N} CF_i - Costs$. Costs for reliability improvement consist of costs for redundancy (for those subsystems where redundancy is used), and costs for increasing reliability by installing a more reliable (and more expensive) equipment. The value of *costs* is a function of variables $\mu_1, \mu_2, ..., \mu_n, n_1, n_2, ..., n_m$.

Step 5 Formulation of an optimization problem: derive analytical expressions for objective functions in the light of the data obtained at steps 1–4; choose an optimization criterion and restrictions. Substitute the values $\sum_{i=1}^{N} CF_i, K(\cdot)$ or $g(t)$ and $C_s(\cdot)$ obtained respectively at steps 1, 2, 3 in formula (13). Influence of reliability on the objective function is taken into account by multiplying a cash flow on an availability function. Thus, the first summand in the formula for calculating *NPV* becomes: $(\sum_{i=1}^{N} CF_i)* \; g(t, \mu_1, \mu_2, ..., \mu_n, n_1, n_2, ...n_m)$ or $(\sum_{i=1}^{N} CF_i)* \; K_s(\mu_1, \mu_2, ..., \mu_n, n_1, n_2, ...n_m)$. Investments to improve network reliability are deducted from net cash flow and the expression for the net present value is obtained

$$NPV = \sum_{i=1}^{N} CF_i * g(t, \mu_1, \mu_2, , n_1, n_2) - C_s(t, \mu_1, \mu_2, , n_1, n_2).$$

Step 6 An optimization procedure and finding optimal values of model parameters.

Step 7 Issue recommendations for construction of the particular network model taking into account the results of optimization.

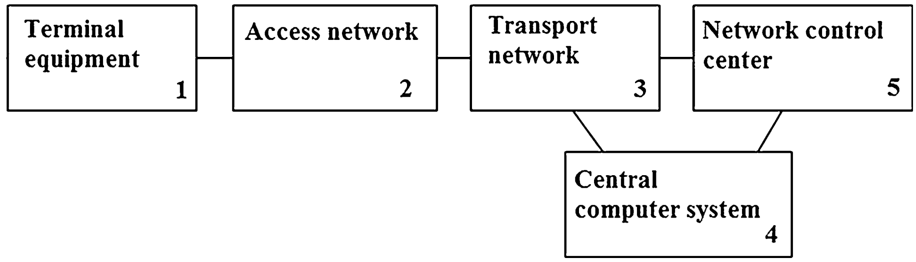

Fig. 3 The example of network reliability model

4 The example of network optimization task

Consider an example of a network optimization problem. Suppose, that reliability of the system shown at Fig. 3 must be optimized. The system is specified by the following data. The reliability of units with numbers 1, 4 and 5 is enhanced by the installation of more expensive and reliable equipment, the reliability of units 2 and 3 is increased by standby redundancy. Non-stationary availability coefficient for the subsystems is given by well-known formula from Barlow and Proschan (1996).

$$g_i(t) = \frac{\mu_i}{\lambda_i + \mu_i} + \frac{\lambda_i}{\lambda_i + \mu_i} e^{-(\lambda_i + \mu_i)t},$$

where t is the operational time of the system, λ_i, μ_i, $i = 2, 3$ are failure and recovery rates for units with number i, c_i are its costs, μ_i are recovery rates. The cost of units with numbers 1, 4 and 5 is the function of reliability and is defined by the following formula:

$$C_i(g_i(\lambda_i)) = a_i \ln\left[\frac{1}{1 - g_i(\lambda_i)}\right], i = 1, 4, 5,$$

where a_i is some constant.

The planned revenue is *TCF* during the time t. The task is to find the number of standby units for blocks 2 and 3, and failure rates for units 1, 4, 5 (n_2^*, n_3^*, λ_1^*, λ_4^*, λ_5^*), that give the maximum to revenue

$$\begin{aligned} NPV(\lambda_1^*, \lambda_4^*, \lambda_5^*, n_2^*, n_3^*) &= \max_{n_i^*, \lambda_i^*} TCF * g_s(\lambda_1, \lambda_4, \lambda_5, n_2, n_3) \\ &- a_1 \ln\left[\frac{1}{1 - \frac{\mu_1}{\lambda_1+\mu_1} - \frac{\lambda_1}{\lambda_1+\mu_1} e^{-(\lambda_1+\mu_1)t}}\right] \\ &- a_4 \ln\left[\frac{1}{1 - \frac{\mu_4}{\lambda_4+\mu_4} - \frac{\lambda_4}{\lambda_4+\mu_4} e^{-(\lambda_4+\mu_4)t}}\right] \\ &- a_5 \ln\left[\frac{1}{1 - \frac{\mu_5}{\lambda_5+\mu_5} - \frac{\lambda_5}{\lambda_5+\mu_5} e^{-(\lambda_5+\mu_5)t}}\right] - (c_2 n_2 + c_3 n_3), \end{aligned}$$

where

$$\begin{aligned} g_s(\lambda_1, \lambda_4, \lambda_5, n_2, n_3) &= \left(\frac{\mu_1}{\lambda_1 + \mu_1} - \frac{\lambda_1}{\lambda_1 + \mu_1} e^{-(\lambda_1+\mu_1)t}\right) \\ &\left(1 - \left(1 - \left(\frac{\mu_2}{\lambda_2 + \mu_2} - \frac{\lambda_2}{\lambda_2 + \mu_2} e^{-(\lambda_2+\mu_2)t}\right)\right)^{n_2}\right) \\ &\left(1 - \left(1 - \left(\frac{\mu_3}{\lambda_3 + \mu_3} - \frac{\lambda_3}{\lambda_3 + \mu_3} e^{-(\lambda_3+\mu_3)t}\right)\right)^{n_3}\right) \\ &\left(\frac{\mu_4}{\lambda_4 + \mu_4} - \frac{\lambda_4}{\lambda_4 + \mu_4} e^{-(\lambda_4+\mu_4)t}\right) \\ &\left(\frac{\mu_5}{\lambda_5 + \mu_5} - \frac{\lambda_5}{\lambda_5 + \mu_5} e^{-(\lambda_5+\mu_5)t}\right), \end{aligned}$$

and with the following conditions:

$$\begin{aligned} &0 < \lambda_1 < \mu_1, g_1 \in (0, 1), \\ &0 < \lambda_4 < \mu_4, g_4 \in (0, 1), \\ &0 < \lambda_5 < \mu_5, g_5 \in (0, 1), \\ &n_2 > 0, n_3 > 0, n_2, n_3 \in \mathbf{N}. \end{aligned}$$

The objective function is non-linear, the variables n_2,n_3 can be only integer, λ_1, λ_4, λ_5 are real and positive. The modern mixed-variables non-linear methods should be used for solving such problems. Some approaches to solving such problems can be found, for example in Johan and Petter (2001), Feoktistov (2006), Kenneth et al. (2005).

Solution of this task was obtained numerically in MatLab with the following initial data:

$$\begin{aligned} &\lambda_2 = 1/240, \lambda_3 = 1/190, \\ &\mu_1 = 1/5, \mu_2 = 1/11, \mu_3 = 1/25, \mu_4 = 1/24, \mu_5 = 1/2, \\ &a_1 = 500, a_4 = 200, a_5 = 9600, \\ &c_2 = 3000, c_3 = 1500, \\ &TCF = 960000, \\ &t = 26280. \end{aligned}$$

and the following results were obtained

Table 1 Comparison of two approaches

	Classical aproach	New approach
Costs	180,385	60,135
NPV	779,358	886,208
Reliability	0.9997	0.9858

$$n_2^* = 2, n_3^* = 3,$$
$$\lambda_1^* = 0.00010567, \lambda_4^* = 8.80583 * 10^{-6},$$
$$NPV^* = 886208.$$

Now the reliability and total costs for system can be calculated with obtained results:

$$g_s(\lambda_1^*, \lambda_4^*, \lambda_5^*, n_2^*, n_3^*) = 0.985774,$$
$$Costs = 60134.6.$$

Now consider optimization problem in earlier approach with the same initial data. The number of standby units for blocks 2 and 3 n_2^* and n_3^*, and failure rates for units 1, 4, 5, λ_1^*, λ_4^*, λ_5^*, such that give the maximum to the reliability function g_s (λ_1, λ_4, λ_5, n_2, n_3) must be defined.

For this problem formulation the following results were obtained

$$n_2^* = 3, n_3^* = 4,$$
$$\lambda_1^* = 0.0000076, \lambda_4^* = 9.4038 * 10^{-9}, \lambda_5^* = 0.00000304531,$$
$$g_s(\lambda_1^*, \lambda_4^*, \lambda_5^*, n_2^*, n_3^*) = 0.999733032842512.$$

The *NPV* and total costs for a system in this case:

$$NPV(\lambda_1^*, \lambda_4^*, \lambda_5^*, n_2^*, n_3^*) = 779358,$$
$$Costs = 180385.$$

Now compare the results. The results are presented in Table 1. The last column shows the reliability, cost and profit for sugested approach.

As we can see from Table 1, a new approach gives more optimal result from the economical point of view. The proposed approach allows to find optimal relation between reliability improvement and system cost. It allows avoid extra costs for systems improvement while other doesn't consider costs. So in the above example with classical approach the system costs increased three times (300 %) in comparision with proposed approach while reliability increased <1.5 %.

The explanation is that classical approach to reliability planning originated from radiotechnical tasks, where the efficiency and costs were not so important as reliability, and additional (redundant) components were not so expensive. So the reliability of the system might be rised and tends to 1 via using redundant elements which number is much more than one ($\gg$1). Besides the classical approach doesn't account costs for maintenance. But as mentioned in ITU-T Recommendation E.862 (1992) today extra components are much expensive for designing and requires maintenance. For example in telecommunication tasks it's impossible to provide a lot of redundant data channels, it's usually not much >2. It's impossible to satisfy modern demands to networks designing with classical approaches. These can be done with the suggested approach, which satisfy modern demands (ITU-T Recommendation 1992) and gives the efficient solution over classic one.

5 Conclusion

In this paper the logical model of reliability of network systems with redundant subsystems (cold and standby redundant) and non-redundant subsystems were described. It allows finding such a way of increasing reliability and redundancy, which will ensure maximum revenue from the operation of the network.

The new formulation of reliability optimization problem was proposed, it takes into account a logical model of reliability, the cost of ensuring reliability and economic efficiency of the network. This formulation of the problem allows to quantitatively determine the optimal values of reliability parameters of the network subsystem, taking into account income from the exploitation of the network through the providing of commercial services.

The technique of optimizing the reliability of networks systems based on the analytic approach, which includes a logical modelling of network reliability, economic criteria.

For the method described in this paper the software for reliability of the network optimization is available, it allows research reliability, maximum efficiency and reduce calculation time.

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