

Towards cluster edge editing problems (literature survey, models; preliminary material)

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Mark Sh. Levin

The paper addresses the well-known cluster edge editing problem. A literature survey of various cluster editing problems and corresponding solving approaches are presented. Further mathematical problem formulations of cluster edge editing problems are described including some new multicriteria problems. Some future research directions are pointed out. The problems are illustrated by numerical examples.

Keywords: cluster editing problem, cluster edge editing problem, combinatorial clustering, combinatorial optimization, multicriteria problems

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1. Introduction

1.1. Preliminaries

In recent years a special attention is targeted to various combinatorial clustering problems, approaches and frameworks (e.g., [10,28,30,35,43,59,62,77,78,81,82,88,90,96,118,119,129,134–136]). The paper addresses a special type of combinatorial clustering as cluster editing problem (e.g., [1,9,14,22,23,25]).

Note the classical cluster editing problem is often studied as correlation clustering (transformation of a given graph into a disjoint union of cliques-clusters by small number of edge modifications) (e.g., [6,12,15,42,43,51,86,120]). On the other hand the cluster editing problem is close to clique partitioning problem (or clique clustering) (e.g., [54,79,108]).

Here the cluster editing problem is considered as a component of the author’s combinatorial clustering engineering approach. Some recent author publications on the research approach are listed in Table 1.

Table 1. Some publications on combinatorial clustering engineering approach

No.	Publication	Source(s)
1.	Survey on combinatorial clustering	[95,96]
2.	Clique-based partitioning/clustering of graph	[107,108]
3.	Balanced clustering (indices, models, examples)	[99,100]
4.	Balanced clustering with tree-like structures over clusters	[102,104]
5.	Clustering models based on graph edge coloring	[106]
6.	Some clustering problems based on dominating sets problems	[103]
7.	Capacitated clustering: clustering with maximization of cluster elements connections while taking into account two constraints on cluster size	[109,110]
8.	Dynamic combinatorial clustering	[98]
9.	Interval-balanced multiprocessor scheduling on modular jobs	[101,105]

Various versions of the cluster editing problem are often used as significant components of combinatorial frameworks in theoretical and application studies, for example: (i) networking (e.g., networking (e.g., network partitioning, network transformations/modifications); (ii) studies in computational biology (e.g., clustering gene expression patterns, examination of phylogenetic trees, genome editing); (iii) manufacturing systems (manufacturing cell formation). The cluster editing problem and its many variants are NP-hard (e.g., [87,93,128]). A simplified (preliminary) illustration for the cluster editing problems domain is depicted in Fig. 1. The basic Cluster Edge Editing problem is the following (e.g., [21,22,25,50,65,66,68]):

Transform an initial graph by at most k edge modifications (i.e., edge addition and edge deletion) into a disjoint union of cliques.

In the Integer-Weighted Cluster Edge Editing problem, integer modification costs for each edge or non-edge are defined (e.g., [21]). The problem is:

Transform an initial graph into a disjoint union of cliques and the set of edge modifications with total weight is at most k .

Some basic related problems are:

1. Cluster deletion problem [24,47,63,65,66].
2. Cluster vertex deletion problem (or unweighted case of cluster vertex deletion problem) [52,76,93];
3. k -cluster editing problem (or p -cluster editing problem - p -CEP) (e.g., [31]): A given input graph should be edited by adding and/or removing edges in such a way that p vertex-disjoint cliques (clusters) are generated with the minimum number of editions.
4. Bicliaster editing problem [7,49,53,67,75,91,122,131,133]: To add or remove at most k edges of a bipartite graph to obtain a vertex-disjoint union of complete bipartite subgraphs).

In Fig. 2 a numerical illustration of the problem is depicted. Here the following modifications are used: 5 edge deletions, 5 edge addition.

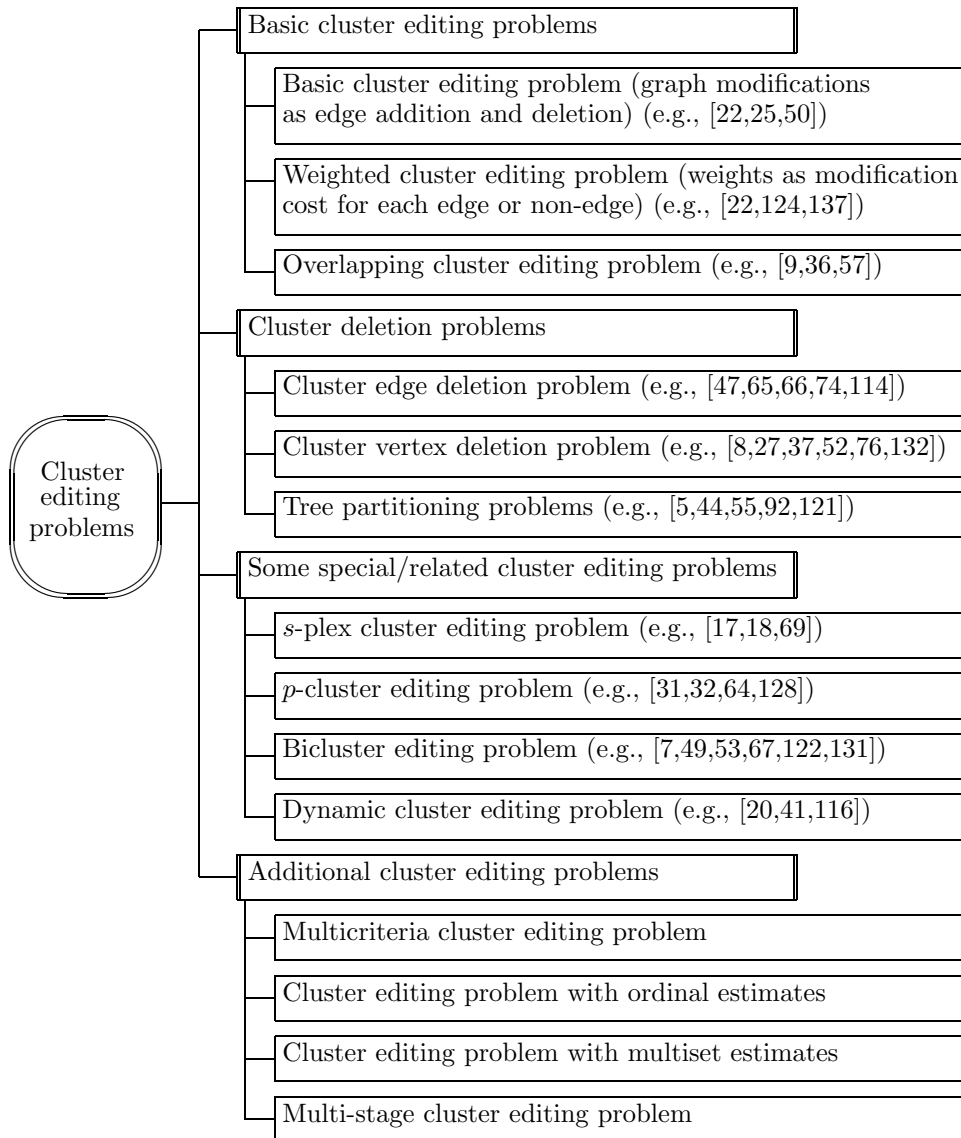


Fig. 1. Illustration for cluster editing problem domain

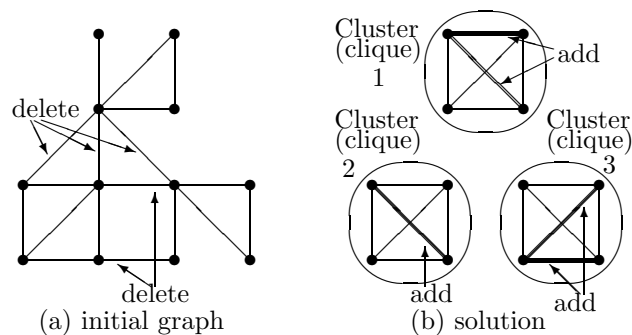


Fig. 2. Illustration for cluster editing problem

1.2. Literature survey

Basic well-known types of cluster editing problems are listed in Table 2. Table 3 contains a list of basic solving approaches. Note recently parameterized algorithms are intensively studied.

Table 2. Cluster editing problems

No.	Problem/study	Source(s)
1.	Some basic problems:	
1.1.	Basic cluster editing problems	[22,25,50,66,68]
1.2.	Cluster editing problem on planar graphs	[16]
1.3.	Weighted cluster editing problem	[22,124,137]
1.4.	Integer weighted cluster editing problem	[21]
1.5.	Cluster editing problem (enumeration version)	[45,46]
1.6.	Overlapping cluster editing problem	[9,36,57]
2.	Cluster deletion problems:	
2.1.	Cluster deletion problem	[47,65,66]
2.2.	Cluster deletion problem for cographs	[63]
2.3.	Parameterized cluster deletion problem	[24]
2.4.	Cluster deletion on interval graphs	[37,85]
2.5.	Various cluster vertex deletion problems	[8,27,37,52,76,93,132]
2.6.	Weighted cluster deletion problem on subclasses of chordal graphs	[26]
2.7.	Tree partitioning problems	[5,44,55,92,121]
3.	Parameterized cluster editing problems:	
3.1.	Parameterized cluster editing problems	[22,24,56,123]
3.2.	Parameterized mixed cluster editing via modular decomposition	[130]
3.3.	On the parameterized cluster editing with vertex splitting problem	[3,4]
3.4.	Parameterized dynamic cluster editing	[116]
3.5.	Multi-parameterized cluster editing	[1]
4.	Special cluster editing problems:	
4.1.	Bicluster editing problem	[7,49,53,67,122,131]
4.2.	p -cluster editing problem (cluster graph modification)	[31,32,64,128]
4.3.	Cluster editing with vertex splitting	[2–4]
4.4.	Cluster editing: kernelization based on edge cuts	[33]
4.5.	$2k$ kernel for the cluster editing problem	[40]
4.6.	Cluster editing with a small number of clusters	[60]
4.7.	Cluster editing with locally bounded modifications	[83]
4.8.	Preprocessing techniques for cluster editing via integer linear programming	[115]
4.9.	Biclique edge deletion, general biclique edge deletion problems	[58]
4.10.	Cluster editing problem with clusters of small sizes	[84]
4.11.	Modification-fair cluster editing	[61]
4.12.	Cluster editing parameterized above modification-disjoint P_3 -packings	[111]
4.13.	Cluster editing in multilayer graphs	[41]
5.	s -plex cluster editing problems:	
5.1.	s -plex cluster editing problem	[69]
5.2.	Model for graph-based clustering: s -plex editing	[69]
5.3.	s -plex cluster vertex deletion problem	[17,18]
6.	Dynamic cluster editing problems:	
6.1.	Basic dynamic cluster editing problem	[116]
6.2.	Cluster editing for temporal graphs	[20,41]
7.	Related problems:	
7.1.	Cograph editing problem	[113]
7.2.	Fixed-parameter enumerability of cluster editing and related problems	[48]
7.3.	Vertices removal for feasibility of clustered spanning trees	[71,72]
7.4.	Optimal clustering of multipartite graphs (clique partitioning by adding or deletion of minimum edge set)	[39]
7.5.	Graph deletion problem (node deletion problems)	[89]
7.6.	Skeleton-based clustering by quasi-threshold editing	[29]
7.7.	Balanced cluster edge modification problems	[117]
8.	Additional prospective cluster editing problems:	
8.1.	Multicriteria cluster editing problem	[95,96,126]
8.2.	Cluster editing problem with ordinal estimates	
8.3.	Cluster editing problem with multiset estimates	[94]
8.4.	Restructuring in multi-stage cluster editing problems	[97]

Table 3. Basic solving approaches

No.	Study	Source(s)
1.	Some surveys:	
1.1.	Theoretical study of cluster editing problems (equivalent problems to cluster editing problem)	[12]
1.2.	Efficient algorithms for cluster editing	[14]
1.3.	Exact algorithms for cluster editing (evaluation and experiments)	[23]
2.	Enumerative exact methods:	
2.1.	Branch-and-bound algorithm for cluster editing	[19]
2.2.	Branch-and-cut approaches for p -cluster editing	[23,31]
2.3.	Branch-and-price algorithm for p -cluster editing	[32]
2.4.	Exact methods for weighted cluster editing	[124]
3.	Heuristics, metaheuristics, approximation algorithms:	
3.1.	Heuristics for weighted cluster editing	[124]
3.2.	Layout based heuristic for weighted cluster editing	[137]
3.3.	2-approximation algorithm for cluster vertex deletion problem	[8]
3.4.	Hybrid heuristic for the overlapping cluster editing problem	[36]
3.5.	Heuristics/metaheuristics for bicluster editing problem	[49]
3.6.	Greedy heuristic for cluster editing with vertex splitting	[4]
4.	Polynomial time algorithms:	
4.1.	Polynomial algorithm for cluster vertex deletion problem on block graphs	[34]
4.2.	Polynomial algorithm for cluster vertex deletion problem on split graphs	[34]
4.3.	Polynomial algorithm for cluster vertex deletion problem on bounded tree width graphs	[125]
4.4.	Polynomial algorithm for cluster deletion on cographs (i.e., P_4 -free graphs)	[63]
4.5.	Polynomial algorithm for (1, 1)-Cluster Editing problem	[70]
4.6.	Polynomial time algorithm for cluster editing problem for points on the real line (dynamic programming algorithm)	[13]
4.7.	Polynomial time algorithm for cluster editing problem on proper interval graphs	[112]
4.8.	Cut-based data reduction rule for weighted cluster editing in polynomial time	[127]
5.	Polynomial time approximation schemes (PTAS):	
5.1.	PTAS for cluster editing problem on planar graphs	[16]
5.2.	PTAS for p -Cluster Editing problem	[64]
5.3.	Polynomial time approximation algorithms for cluster editing problem with clusters of small sizes	[84]
6.	Parameterized algorithms:	
6.1.	Modular decomposition to parameterized cluster editing problems	[123,130]
6.2.	Parameterized algorithms for cluster editing	[22,80]
6.3.	Golden ratio parameterized algorithm for cluster editing	[21]
6.4.	Fixed-parameter enumerability of cluster editing and related problems	[45,46]
6.5.	Programming by optimization meets parameterized algorithms: a case study for cluster editing	[73]
6.6.	Faster parameterized algorithm for cluster vertex deletion	[132]
6.7.	Faster parameterized algorithms for bicluster editing	[131,133]
6.8.	Fixed-parameter algorithms for bicluster editing	[91]
6.9.	Dual parameterization for fast biclustering	[53]
6.10.	Fast branching algorithm for cluster vertex deletion (fixed-parameter algorithm)	[27]
7.	Special approaches:	
7.1.	Graph-modeled data clustering (exact algorithm for clique generation)	[66]
7.2.	Effective linear kernelization for cluster editing	[68]
7.3.	Two-phase solving strategy for CEP based on set partitioning	[14]
7.4.	Combinatorial approximations for cluster deletion: simpler, faster, and better	[11]

2. Basic cluster edge editing problem formulations

2.1. Designations

The finite undirected graph $G = (A, E)$ (or $G = (A(G), E(G))$) is examined:

- (i) $A(G) = \{a_1, \dots, a_i, \dots, a_n\}$ is the set of vertices;
- (ii) $E(G) = \{e_{a_i, a_j} \mid (a_i, a_j) \in A\}$ is the set of edges $E(G) \subseteq \{A \times A\}$;
- (iii) $\overline{E}(G)$ is the set of vertex pair which have no edges ($\overline{E}(G) \subseteq \{A \times A\}$): $E \cup \overline{E} = \{A \times A\}$ and $|E \cap \overline{E}| = 0$.

2.2. Basic mathematical formulation

The mathematical formulation (integer linear programming ILP model) of cluster edge editing problem (CEP) was proposed in [38]. It relies on the simple fact that a graph G is a cluster graph if and only if G does not contain the graph P_3 (a path formed by three vertices) as an induced subgraph.

For each two vertex pair a_i, a_j ($\forall a_i, a_j \in A(G)$) with $i < j$, let x_{ij} be a binary variable such that $x_{ij} = 0$ if and only if vertices a_i and a_j belong to the same clique (cluster) in a final solution.

The minimization problem formulation is:

$$\min [C^d = \sum_{i < j, (a_i, a_j) \in E(G)} x_{ij} + C^a = \sum_{i < j, (a_i, a_j) \notin E(G)} (1 - x_{ij})] \quad (1)$$

$$s.t. \quad x_{ik} \leq x_{ij} + x_{jk}, \quad x_{ij} \leq x_{ik} + x_{jk}, \quad x_{jk} \leq x_{ij} + x_{ik}, \quad i < j < k \quad (2)$$

$$x_{ij} \in \{0, 1\} \quad i < j \quad (3)$$

Note that the objective function (1) minimizes the number of edges that are converted into non-edges (C^d , i.e., deleted edges) plus the number of non-edges that are converted into edges (C^a , i.e., added edges). There are $O(n^3)$ triangle inequalities (2) that eliminate the induced subgraphs isomorphic to P_3 . Two numerical examples of CFP are presented as follows.

Example 1. An illustrative numerical example for CFP is depicted in Fig. 3:

- (a) initial graph (Fig. 3a),
- (b) the solution by 9 editions/modification operations (5 deletion, 4 addition) as vertex-disjoint union of three cliques/clusters (Fig. 3b).

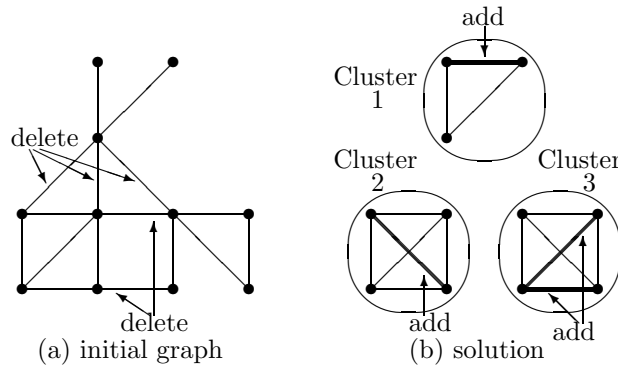


Fig. 3. Numerical example 1 of cluster editing problem

Example 2. Another illustrative numerical example for CFP is depicted in Fig. 4 (for the same initial graph):

- (a) initial graph (Fig. 4a),
- (b) the solution by 8 editions/modification operations (7 deletion, 1 addition) as vertex-disjoint union of four cliques/clusters (Fig. 4b).

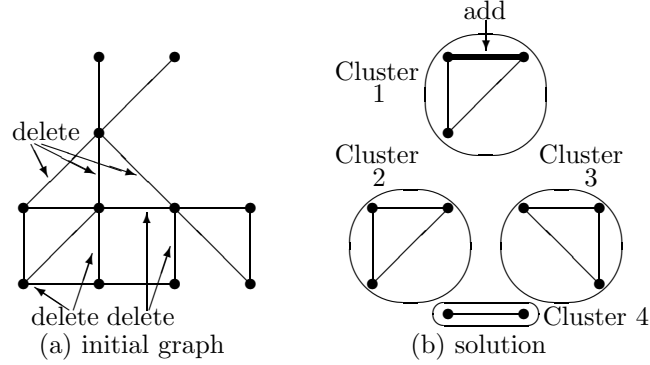


Fig. 4. Numerical example 2 of cluster editing problem

2.3. Bi-objective mathematical formulation

It is possible to examine two objective functions:

$$\min C^d = \sum_{i < j, (a_i, a_j) \in E(G)} x_{ij} \quad (4a)$$

$$\min C^a = \sum_{i < j, (a_i, a_j) \notin E(G)} (1 - x_{ij}) \quad (4b)$$

$$s.t. \quad x_{ik} \leq x_{ij} + x_{jk}, \quad x_{ij} \leq x_{ik} + x_{jk}, \quad x_{jk} \leq x_{ij} + x_{ik}, \quad i < j < k \quad (5)$$

$$x_{ij} \in \{0, 1\} \quad i < j \quad (6)$$

Thus the two-objective function (i.e., two-component vector) is minimized:

$$\min \bar{C}^{da} = (C^d, C^a).$$

2.4. Edge-weighted problem formulations

2.4.1. Weighted basic mathematical formulation

The Weighted Cluster Edge Editing problem is examined as well (e.g., [22,124,137]). Here a non-negative weight $w_{i,j}$ for each vertex pair ($\forall a_i, a_j \in A(G)$, i.e., for edges and for non-edges) is considered:

$$\min [C^{dw} = \sum_{i < j, (a_i, a_j) \in E(G)} w_{i,j} x_{ij} + C^{aw} = \sum_{i < j, (a_i, a_j) \notin E(G)} w_{i,j} (1 - x_{ij})] \quad (7)$$

$$s.t. \quad x_{ik} \leq x_{ij} + x_{jk}, \quad x_{ij} \leq x_{ik} + x_{jk}, \quad x_{jk} \leq x_{ij} + x_{ik}, \quad i < j < k \quad (8)$$

$$x_{ij} \in \{0, 1\} \quad i < j \quad (9)$$

Note that the objective function (1) minimizes the weighted number of edges that are converted into non-edges (C^{dw}) plus the weighted number of non-edges that are converted into edges (C^{aw}).

2.4.2. Bi-objective weighted mathematical formulation

Here the problem formulation is:

$$\min C^{dw} = \sum_{i < j, (a_i, a_j) \in E(G)} w_{ij} x_{ij} \quad (10a)$$

$$\min C^{aw} = \sum_{i < j, (a_i, a_j) \notin E(G)} w_{ij} (1 - x_{ij}) \quad (10b)$$

$$s.t. \quad x_{ik} \leq x_{ij} + x_{jk}, \quad x_{ij} \leq x_{ik} + x_{jk}, \quad x_{jk} \leq x_{ij} + x_{ik}, \quad i < j < k \quad (11)$$

$$x_{ij} \in \{0, 1\} \quad i < j \quad (12)$$

Thus the bi-objective weighted (i.e., two-component vector) objective function is minimized:

$$\min \bar{C}^{daw} = (C^{dw}, C^{aw}).$$

2.4.3. Multi-criteria weighted mathematical formulation

A prospective multicriteria problem formulation is based on using vector-like weights of vertex pairs: $\bar{w}_{ij} = (w_{ij}^1, \dots, w_{ij}^\xi, \dots, w_{ij}^\lambda)$ (for each vertex pair $\forall a_i, a_j \in A(G)$, i.e., for edges and for non-edges).

The following multicriteria problem can be examined:

$$\min \bar{C}^{d\bar{w}} = \left(\sum_{i < j, (a_i, a_j) \in E(G)} w_{ij}^1 x_{ij}, \dots, \sum_{i < j, (a_i, a_j) \in E(G)} w_{ij}^\xi x_{ij}, \dots, \sum_{i < j, (a_i, a_j) \in E(G)} w_{ij}^\lambda x_{ij} \right) \quad (13a)$$

$$\min \bar{C}^{a\bar{w}} = \left(\sum_{i < j, (a_i, a_j) \notin E(G)} w_{ij}^1 (1 - x_{ij}), \dots, \sum_{i < j, (a_i, a_j) \notin E(G)} w_{ij}^\xi (1 - x_{ij}), \dots, \sum_{i < j, (a_i, a_j) \notin E(G)} w_{ij}^\lambda (1 - x_{ij}) \right) \quad (13b)$$

$$s.t. \quad x_{ik} \leq x_{ij} + x_{jk}, \quad x_{ij} \leq x_{ik} + x_{jk}, \quad x_{jk} \leq x_{ij} + x_{ik}, \quad i < j < k \quad (14)$$

$$x_{ij} \in \{0, 1\} \quad i < j \quad (15)$$

Here weighted λ -component vector objective functions are used: $\bar{C}^{d\bar{w}}$ and $\bar{C}^{a\bar{w}}$.

Two basic solving approaches can be considered:

(a) analysis of an integrated (e.g., by vector components) additive objective function:

$$\min (\bar{C}^{d\bar{w}} + \bar{C}^{a\bar{w}});$$

(b) consideration of two-part vector objective function: $\{ \min \bar{C}^{d\bar{w}}, \min \bar{C}^{a\bar{w}} \}$.

3. Brief description of other cluster editing problems

3.1. Cluster deletion problem description

The cluster deletion problem is based only on edge deletion operations (via integer partitioning of the initial graph) [65,63].

Example 3. An illustrative numerical example for cluster deletion problem is depicted in Fig. 5:

(a) initial graph (Fig. 5a),

(b) the solution by 6 editions/modification operations (6 deletion) as vertex-disjoint union of four cliques/clusters (Fig. 5b).

Note each cluster contains 3 vertices, i.e., the clustering solution is balanced by cluster size.

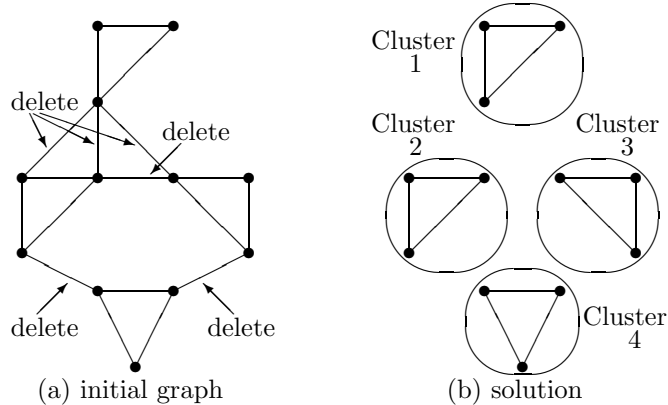


Fig. 5. Numerical example for balanced solution

The min-edge clique partition problem consists in finding a partition of the vertices of a graph into a set of cliques with the fewest edges between cliques. This is a known NP-complete problem and has been studied extensively in the scope of fixed-parameter tractability (FPT) where it is commonly known as the Cluster Deletion problem. Many of the recently-developed FPT algorithms are targeted to solve Cluster Deletion in polynomial time on restricted graph structures.

An illustrative numerical example of cluster deletion problem is depicted in Fig. 6. Here 7 modification (deletion) operations are used. Evidently, edge-weighted cluster deletion problem (i.e., weighted edge deletion problem) is examined as well [26].

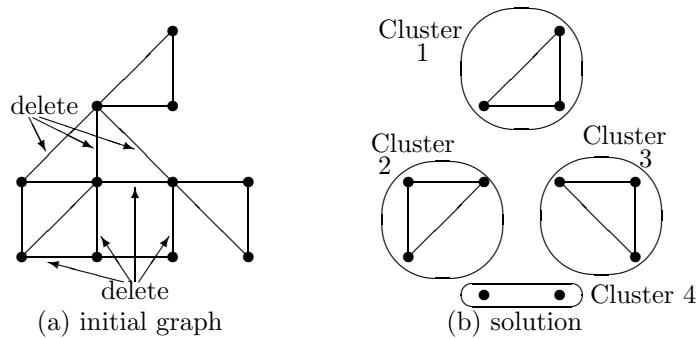


Fig. 6. Illustration for cluster deletion problem

3.2. Cluster editing problem with balance by multi-type elements

The cluster editing problem with balance by multi-type elements can be considered as a special new problem. A numerical example illustrates the problem:

Example 4. The example is (Fig. 7):

(a) initial graph (three types of the vertices) (Fig. 7a),

(b) the solution by 6 editions/modification operations (4 deletion, 2 addition) as vertex-disjoint union of four cliques/clusters (Fig. 7b).

This kind of clustering problem (i.e., with multi-type vertices) has been described in (e.g., [99,100]). Note each cluster contains 3 vertices of three types, i.e., the clustering solution is balanced by cluster size and types of the vertices.

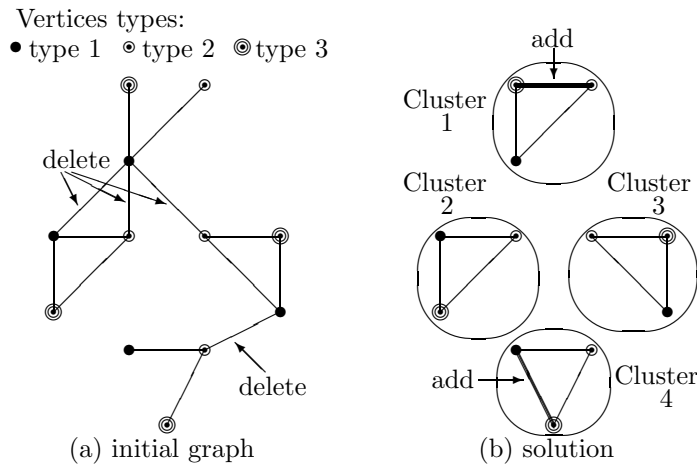


Fig. 7. Cluster solution with balance by multi-type elements

3.3. Optimal clustering of multipartite graphs

Optimal clustering of multipartite graphs (i.e., clique partitioning) is a special cluster editing problem [39]:

Partitioning the vertices of multipartite graph into a disjoint union of cliques by adding or removing a minimum edge set).

This problem belongs to class of NP-hard problems.

3.4. On cluster vertex deletion problem

It is reasonable to point out that cluster vertex deletion problems are very important (from the theoretical viewpoint and from the practical viewpoint) (e.g., [8,27,52,37,76,132]). The following basic problems can be listed: (i) basic cluster vertex deletion problem, (ii) weighted cluster vertex deletion problem, (iii) multicriteria cluster vertex deletion problem, and (iv) s -club cluster vertex deletion.

4. Conclusion

In this paper, cluster editing problems are examined from the viewpoint of combinatorial engineering clustering (e.g., [95,96,98–100,104,106,107]). The paper contains three basic parts (a) literature survey, (b) description of cluster edge editing problem (problems description with numerical examples, problem formulations, some new models, and illustrative applied examples), (c) brief description of future research directions.

It may be reasonable to consider the following future research directions:

- (1) cluster editing problems with joint edge edition and vertex editing (e.g., [75]);
- (2) vertex cluster editing problems (e.g., [8,132]);
- (3) cluster editing problems under uncertainty;
- (4) cluster editing problems with various types of used parameter estimates (e.g., ordinal estimates, vector-like estimates, multiset-like estimates);
- (5) dynamic cluster editing problems (e.g., [20,41,116]);
- (6) relations between the cluster editing problems and close/related problems;
- (7) cluster editing problems as basic components of graph modification problems (e.g., [128]);
- (8) versions of the cluster editing problems as basic auxiliary components for reconfiguration (restructuring) of cluster solutions in dynamical clustering or design of cluster trajectories (e.g., [97,98,107]);
- (9) special new solving strategies (e.g., special heuristics and metaheuristics);
- (10) application of cluster editing problems in dynamical communication network design and management; and
- (11) educational applications of cluster editing problems.

The author states that there is no conflict of interest.

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REFERENCES

1. F.N. Abu-Khzam, On the complexity of multi-parameterized cluster editing. *J. Discrete Algorithms*, 45, 26–34, 2017.
2. F.N. Abu-Khzam, J. Egan, S. Caspers, A. Show, P. Shaw, Cluster editing with vertex splitting. In: *Proc. of the 5th Int. Symp. on Combinatorial Optimization ISCO 2018*, LNCS 10856, Springer, pp. 1–13, 2018.
3. F.N. Abu-Khzam, J. Egan, S. Caspers, A. Shaw, P. Shaw, On the parameterized cluster editing with vertex splitting problem. *Electr. prepr.*, 23 p., Jan. 1, 2019. <http://arxiv.org/abs/1901.00156> [cs.CC]
4. F.N. Abu-Khzam, J.R. Barr, A. Fakhereldine, P. Shaw, A greedy heuristic for cluster editing with vertex splitting. In: *2021 4th Int. Conf. on Artif. Intell. for Industries (AI4I)*, pp. 38–41, 2021.
5. E. Agasi, R.I. Becker, Y. Perl, A shifting algorithm for constrained min-max partition on trees. *Discr. Appl. Math.*, 45, 1–28, 1993.

6. S. Ahmadian, A. Epasto, R. Kumar, M. Mahdian, Fair correlation clustering. *Electr. prepr.*, 17 p., Mar. 2, 2020. <http://arxiv.org/abs/2002.02274> [cs.DS]
7. N. Amit, The bicluster graph editing problem. PhD thesis, Tel Aviv University, 2004.
8. M. Aprile, M. Drescher, S. Florini, T. Huynh, A tight approximation algorithm for the cluster vertex deletion problem. *Electr. prepr.*, 26 p., Oct. 18, 2021. <http://arxiv.org/abs/2007.08057> [math.CO]
9. E. Arrighi, M. Bentert, P.G. Drange, B.D. Sullivan, P. Wolf, Cluster editing with overlapping communities. In: 18th Int. Symp. on Parameterized and Exact Computation IPEC 2023, vol. 285, pp. 2:1–2:12, 2023.
10. D.A. Bader, H. Meyerhenke, P. Sanders, D. Wagner (eds), Graph Partitioning and Graph Clustering. ser. Contemporary Mathematics, AMS and DIMACS, no. 588, 258 p. 2013.
11. V. Balmaseda, Y. Xu, Y. Cao, N. Veldt, Combinatorial approximations for cluster deletion: simpler, faster, and better. *Electr. prepr.*, 29 p., Apr. 24, 2024. <http://arxiv.org/abs/2404.16131> [cs.DS]
12. N. Bansal, A. Blum, S. Chawla, Correlation clustering. *Machine Learning*, 56(1–3), 89–113, 2004.
13. M. Bassel, Cluster editing problem for points on the real line: a polynomial time algorithm. *Inf. Process. Lett.*, 110(21), 961–965, 2010.
14. L. Bastos, L.S. Ochi, F. Protti, A. Subramanian, L.C. Martins, R.G.S. Pinheiro, Efficient algorithms for cluster editing. *J. Combin. Optim.*, 31(1), 347–371, 2016.
15. A. Ben-Dor, R. Shamir, Z. Yakhini, Clustering gene expression patterns. *J. Comput. Biol.*, 6(3–4), 281–297, 1999.
16. A. Berger, A. Grigoriev, A. Winokurow, A PTAS for the Cluster Editing Problem on Planar Graphs. In: K. Jansen, M. Mastrolilli (eds), WAOA 2016, LNCS 10138, Springer, pp. 27–39, 2017.
17. R. van Bevern, A quadratic-vertex problem kernel for s -plex cluster vertex deletion. *Electr. prepr.*, 41 p., Sep. 15, 2009. <http://arxiv.org/abs/0909.2814> [cs.DM]
18. R. van Bevern, H. Moser, R. Niedermeier, Approximation and tidying – a problem kernel for s -plex cluster vertex editing. *Algorithmica*, 62, 930–950, 2011.
19. T. Blasius, P. Fischbeck, L. Gottesburen, M. Hamann, T. Heuer, J. Spinner, C. Weyand, M. Wilhelm, A branch-and-bound algorithm for cluster editing. In: C. Schulz, B. Ucar (eds), 20th Int. Symp. on Experimental Algorithms (SEA 2022), art. no. 13, pp. 13:1–13:19, 2022.
20. C. Bocci, C. Capresi, K. Meeks, J. Sylvester, A new temporal interpretation of cluster editing. *J. of Computer and System Sciences*, 144, art. 103551, 1–21, 2024.
21. S. Bocker, A golden ratio parameterized algorithm for Cluster Editing. *J. of Discrete Algorithms*, 16, 79–89, 2012.
22. S. Bocker, S. Briesemeister, Q.B.A. Bui, A. Truss, Going weighted: parameterized algorithms for cluster editing. *Theor. Computer Science*, 410(52), 5467–5480, 2009.
23. S. Bocker, S. Briesemeister, G.W. Klau, Exact algorithms for cluster editing: evaluation and experiments. *Algorithmica*, 60, 316–334, 2011.
24. S. Bocker, P. Damaschke, Even faster parameterized cluster deletion and cluster editing. *Inform. Process. Lett.*, 111(14), 717–721 2011.
25. S. Bocker, J. Baumbach, Cluster editing. In: P. Bonizzoni, V. Brattka, B. Lowe (eds), Proc. of the 9th Conf. on Computability in Europe CiE 2013, LNCS 7921, Springer, pp. 33–44, 2013.
26. F. Bonomo, G. Duran, A. Napoli, M. Velencia-Pabon, Complexity of the cluster deletion problem on subclasses of chordal graphs. *Theor. Comput. Sci.*, 600, 59–69, 2015.
27. A. Boral, M. Cygan, T. Kociumaka, M. Pilipczuk, A fast branching algorithm for cluster vertex deletion. *Theory Comput. Syst.*, 58, 357–376, 2016.
28. S.A. Borgwardt, A Combinatorial Optimization Approach to Constrained Clustering. Doctor dissertation, Faculty of Mathematics, Technical Univ. of Munchen, 2010.
29. U. Brandes, M. Hamann, L. Hauser, D. Wagner, Skeleton-based clustering by quasi-threshold editing. In: H. Bast, C. Korzen, U. Meyer, M. Penschuk (eds), Algorithm for Big Data. LNCS 13201, Springer, pp. 134–151, 2022.
30. T. Broderick, L. Mackey, J. Paisley, M.I. Jordan, Combinatorial clustering and the beta negative binomial process. *IEEE Trans. PAMI*, 37(2), 290–306, 2015.
31. T. Bulhoes, G.F. de Sousa Filho, A. Subramanian, L. dos Anjos F. Cabral, Branch-and-cut approaches for p -Cluster Editing. *Discr. Appl. Math.*, 219, 51–64, 2017.
32. T. Bulhoes, A. Subramanian, G.F. de Sousa Filho, L. dos Anjos F. Cabral, Branch-and-price for

- p -cluster editing. Computational Optimization and Applications, 67, 293–316, 2017.
33. Y. Cao, J. Chen, Cluster editing: kernelization based on edge cuts. *Algorithmica*, 64(1), 152–169, 2012.
 34. Y. Cao, Y. Ke, Y. Otachi, J. You, Vertex deletion problems on chordal graphs. *Theor. Comput. Sci.*, 745, 75–86, 2018.
 35. Y. Cao, Z. Wang, Combinatorial optimization-based clustering algorithm for wireless sensor networks. *Mathematical Problems in Engineering*, art. ID 613704, 1–13, 2020.
 36. G.O. Chagas, L.A.N. Lorena, R.D.C. dos Santos, A hybrid heuristic for the overlapping cluster editing problem. *Appl. Soft Computing*, 81, art. 105482, 1–12, 2019.
 37. D. Chakraborty, L.S. Chandrau, S. Padinhatteeri, s -Club Cluster Vertex Deletion on interval and well-partitioned chordal graphs. *Electr. prepr.*, 30 p., Oct. 14, 2022. <http://arxiv.org/abs/2210.07699> [cs.DS]
 38. M. Charikar, V. Guruswami, A. Wirth, Clustering with qualitative information. *J. Comput. Syst. Sci.*, 71, 360–383, 2005.
 39. I. Charon, O. Hundry, Optimal clustering of multipartite graphs. *Discrete Applied Math.*, 156(8), 1330–1347, 2008.
 40. J. Chen, J. Meng, A $2k$ kernel for the cluster editing problem. *J. Comput. Syst. Sci.*, 78(1), 211–220, 2012.
 41. J. Chen, H. Molter, M. Sorge, O. Suchy, Cluster editing for multi-layer and temporal graphs. *Theory Comput. Syst.*, 2024.in press.
 42. V. Cohen-Addad, E. Lee, A. Newman, Correlation clustering with Sherali-Adams. *Electr. prepr.*, 43 p., May 3, 2023. <http://arxiv.org/abs/2207.10889> [cs.DS]
 43. V. Cohen-Addad, D.R. Lolck, M. Pilipczuk, M. Thorup, S. Yan, H. Zhang, Combinatorial correlation clustering. In: *Proc. of the 56th Annual ACM Symp. on Theory of Computing STOC'24*, pp. 1617–1628, 2024.
 44. R. Cordone, D. Franchi, A. Scozzari, Cardinality constrained connected balanced partitions of trees under different criteria. *Discr. Optim.*, 46, art. 100742, 2022.
 45. P. Damaschke, Fixed-parameter enumerability of cluster editing and related problems. In: *Proc. of Int. Workshop on Graph-Theoretic Concept in Computer Science WG 2005*, LNCS 3778, Springer, pp. 283–294, 2006.
 46. P. Damaschke, Fixed-parameter enumerability of cluster editing and related problems. In: *Proc. 6th Int. Conf. on Algorithms and Complexity CIAC 2006*, LNCS 3998, Springer, pp. 344–355, 2006.
 47. P. Damaschke, Bounded-degree techniques accelerate some parameterized graph algorithms. In: *Proc. of Int. Workshop on Parameterized and Exact Computation (IWPEC 2009)*, LNCS 5917, Springer, pp. 98–109, 2009.
 48. P. Damaschke, Fixed-parameter enumerability of cluster editing and related problems. *Theory of Computing Systems*, 46(2), 261–283, 2010.
 49. G.F. de Sousa Filho, T.L. Bulhoes Junior, L.A.F. Cabral, L.S. Ochi, F. Protti, New heuristics for the bicluster editing problem. *Ann. Oper. Res.*, 258(2), 781–814, 2017.
 50. F. Dehne, M.A. Langston, X. Luo, S. Pitre, P. Shaw, Y. Zhang, The cluster editing problem: implementation and experiments. In: *2nd Int. Workshop on Parameterized and Exact Computation IWPEC 2006*, LNCS 4169, Springer, pp. 13–24, 2006.
 51. E.D. Demaine, D. Emanuel, A. Fiat, N. Immerlica, Correlation clustering in general weighted graphs. *Theor. Comp. Sci.*, 361, 172–187, 2006.
 52. M. Doucha, J. Kratochvil, Cluster vertex deletion: a parameterization between vertex cover and clique-width. In: *Int. Symp. on Mathematical Foundation of Computer Science MFCS 2012*, LNCS 7464, Springer, pp. 348–359, 2012.
 53. P.G. Drange, F. Reidl, F.S. Villaamil, S. Sikdar, Fast biclustering by dual parameterization. *Electr. prepr.*, 16 p., Jul. 29, 2015. <http://arxiv.org/abs/1507.08158> [cs.DS]
 54. Y. Du, G. Kochenberger, F. Glover, H. Wang, M. Lewis, W. Xie, T. Tsuyuguchi, Solving clique partitioning problems: a comparison of models and commercial solvers. *Int. J. of Information Technology & Decision Making*, 21(01), 59–81, 2022.
 55. A.E. Feldmann, L. Foschini, Balanced partitions of trees and applications. *Algorithmica*, 71(2), 354–376, 2015.

56. M.R. Fellow, M.A. Langston, F.A. Rosamond, P. Shaw, Efficient parameterized preprocessing for Cluster Editing. In: E. Csuhaj-Varju, Z. Esik (eds), Proc. 16th FCT, Fundamentals of Computation Theory FCT, LNCS 4639, Springer, pp. 312–321, 2009.
57. M.R. Fellow, J. Guo, C. Komusiewicz, R. Niedermeier, J. Uhlmann, Graph-based data clustering with overlaps. *Discr. Optim.*, 8, 2–17, 2011.
58. Q. Feng, S. Li, Z. Zhou, J. Wang, Parameterized algorithms for Edge Biclique and related problems. *Theor. Comp. Sci.*, 734, 105–118, 2018.
59. P. Festa, Combinatorial optimization approaches for data clustering. In: M. Celebi, K. Aydin (eds), *Unsupervised Learning Algorithms*, Springer, pp. 109–134, 2015.
60. F.V. Fomin, S. Kratsch, M. Pilipczuk, M. Pilipczuk, Y. Villanger, Tight bounds for parameterized complexity of cluster editing with a small number of clusters. *J. Comput. Syst. Sci.*, 80(7), 1430–1447, 2014.
61. V. Froese, L. Kellerhals, R. Niedermeier, Modification-fair cluster editing. *Social Network Analysis and Mining*, 14, art. 109, 1–16, 2024.
62. N.J. Fuda, K. Brejc, W.S. Kruesi, E.J. Ralston, R. Bigley, A. Shin, M. Okada, B.J. Meyer, Combinatorial clustering of distinct DNA motifs directs synergistic binding of *Caenorhabditis elegans* dosage compensation complex to X chromosomes. *PNAS*, 119(37), e2211642119, 1–9, 2022.
63. Y. Gao, D.R. Hare, J. Nastos, The cluster deletion problem for cographs. *Discr. Math.*, 313, 2763–2771, 2013.
64. I. Giotis, V. Guruswami, Correlation clustering with a fixed number of clusters. In: Proc. of the Seventeenth Annual ACM-SIAM Symp. on Discrete Algorithm, SODA06, pp. 1167–1176, 2006.
65. J. Gramm, J. Guo, F. Huffner, R. Niedermeier, Automated generation of search tree algorithms for hard graph modification problems. *Algorithmica*, 39(4), 321–347, 2004.
66. J. Gramm, J. Guo, F. Huffner, R. Niedermeier, Graph-modeled data clustering: fixed-parameter algorithms for clique generation. *Theory of Computing Systems*, 38(4), 373–392, 2005.
67. J. Guo, F. Huffner, C. Komusiewicz, Y. Zhang, Improved algorithms for bicluster editing. In: M. Agrawal et al. (eds), Proc. 5th TAMC, LNCS 4978, Springer, pp. 445–456, 2008.
68. J. Guo, A more effective linear kernelization for cluster editing. *Theor. Comput. Sci.*, 401(8–10), 718–726, 2009.
69. J. Guo, C. Komusiewicz, R. Niedermeier, J. Uhlmann, A more relaxed model for graph-based data clustering: s -plex cluster editing. *SIAM J. on Discr. Math.*, 24(4), 1662–1683, 2010.
70. G. Gutin, A. Yeo, $(1, 1)$ -Cluster Editing is polynomial-time solvable. *Discr. Appl. Math.*, 340, 259–271, 2023.
71. N. Guttmann-Beck, Z. Sorek, M. Stern, Clustered spanning tree - conditions for feasibility. *Discrete Mathematics and Theoretical Computer Science*, 21(1), 1–15, 2019.
72. N. Guttmann-Beck, R. Rozen, M. Stern, Vertices removal for feasibility of clustered spanning trees. *Discr. Appl. Math.*, 296, 68–84, 2021.
73. S. Hartung, H.H. Hoos, Programming by optimization meets parameterized algorithms: a case study for cluster editing. In: Proc. of the 9th Int. Conf. on Learning and Intelligent Optimization LION 2015, LNCS 8994, Springer, pp. 43–58, 2015.
74. E. Hartuv, R. Shamir, A clustering algorithm based on graph connectivity. *Inf. Process Lett.*, 76, 175–181, 2000.
75. D. Hochbaum, Approximating clique and biclique problems. *J. of Algorithms*, 29(1), 174–200, 1998.
76. F. Huffner, C. Komusiewicz, H. Moser, R. Niedermaier, Fixed-parameter algorithms for cluster vertex deletion. *Theory of Computing Systems*, 47(1), 196–217, 2010.
77. B. Jarbouri, M. Cheikh, P. Siarry, A. Rebai, Combinatorial particle swarm optimization (CPSO) for partitional clustering problem. *Appl. Math. Comput.*, 192(2), 337–345, 2007.
78. D. Jollyta, S. Efendy, M. Zarlis, H. Mawengkang, A combinatorial optimization approach to determining optimal data in cluster. In: 2021 Int. Conf. on Artificial Intelligence and Mechatronics Systems (AIMS), pp. 1–5, 2021.
79. R. Jovanovic, A.P. Sanfilippo, S. Voss, Fixed set search applied to the clique partitioning problem. *EJOR*, 309(1), 65–81, 2023.
80. L. Kellerhals, T. Koana, A. Nichterlein, P. Zschoche, The PAGE 2021 parameterized algorithms and computational experiments challenge: cluster editing. In: 16th Int. Symp. on Parameterized and

- Exact Computation (IPEC 2021), art. no. 26, pp. 26:1–26:18, 2021.
81. J. Kim, W. Lee, J.J. Song, S.B. Lee, Optimized combinatorial clustering for stochastic processes. *Cluster Computing*, 20(2), 1135–1148, 2017.
 82. K. Kim, M. Kumagai, Y. Yamamoti, Combinatorial clustering with a coherent XY machine. *Optics Express*, 32(19), 33737–33757, 2024.
 83. C. Komusiewicz, J. Uhlmann, Cluster editing with locally bounded modifications. *Discrete Appl. Math.*, 160(15), 2259–2270, 2012.
 84. A. Kononov, V. Il'ev, On cluster editing problem with clusters of small sizes. In: N. Olenev, Y. Evtushenko, M. Jacimovic, M. Khachay, V. Malkova (eds), *Optimization and Applications (OPTIMA 2023)*, LNCS 14395, Springer, pp. 316–328, 2023.
 85. A.L. Konstantinidis, C. Papadopoulos, Cluster deletion on interval graphs and split related graphs. *Algorithmica*, 83(7), 2018–2026, 2021.
 86. H.-P. Kriegel, P. Kroger, A. Zimek, Clustering high-dimensional data: a survey on subspace clustering, pattern-based clustering, and correlation clustering. *ACM Trans. on Knowledge Discovery from Data*, 3(1), art. no. 1, 1–58, 2009.
 87. M. Krivanek, J. Moravek, NP-hard problems in hierarchical-tree clustering. *Acta Inform.*, 23(3), 311–323, 1986.
 88. M. Kumagai, K. Komatsu, F. Takano, T. Araki, M. Sato, H. Kobayashi, An external definition of the one-hot constraint and fast QUBO generation for high-performance combinatorial clustering. *Int. J. of Networking and Computing*, 11(2), 463–491, 2021.
 89. M. Kumar, S. Mishra, N.S. Devi, S. Saurabh, Approximation algorithms for node deletion problems on bipartite graphs with finite forbidden subgraph characterization. *Theor. Comp. Sci.*, 526, 90–86, 2014.
 90. V. Kumar, G. Bass, C. Tomlin, J. Dulny III, Quantum annealing for combinatorial clustering. *Quantum Information Processing*, 17, art. 39, 1–14, 2018.
 91. M. Lafond, Even better fixed-parameter algorithms for bicluster editing. In: *Proc. of 26th Int. Conf. Computing and Combinatorics COCOON 2020*, pp. 578–590, 2020.
 92. L. Lan, A. Zocca, Mixed-integer linear programming approaches for tree partitioning of power networks. *Electr. prepr.*, 10 p., Aug. 21, 2023. <http://arxiv.org/abs/2110.07000> [math.OC]
 93. H.-O. Le, V.B. Le, Complexity of the cluster vertex deletion problem on H -free graphs. In: *47th Int. Symp. on Math. Foundations of CS (MFCS 2022)*, art. no. 68, pp. 68:1–68:10, 2022.
 94. M.Sh. Levin, *Modular System Design and Evaluation*. Springer, 2015.
 95. M.Sh. Levin, Towards combinatorial clustering: preliminary research survey. *Electr. prepr.*, 102 p., May 28, 2015. <http://arxiv.org/abs/1505.07872> [cs.AI]
 96. M.Sh. Levin, On combinatorial clustering: literature review, methods, examples. *J. of Commun. Technol. and Electronics*, 60(12), 1403–1428, 2015.
 97. M.Sh. Levin, Towards integrated glance to restructuring in combinatorial optimization. *Electr. prepr.*, 31 p., Dec. 20, 2015. <http://arxiv.org/abs/1512.06427> [cs.AI]
 98. M.Sh. Levin, On dynamic combinatorial clustering. *J. of Communications Technology and Electronics*, 62(6), 718–730, 2017.
 99. M.Sh. Levin, Towards balanced clustering - part 1 (preliminaries). *Elec. prepr.*, 21 p., Jun. 9, 2017. <http://arxiv.org/abs/1706.03065> [cs.DS]
 100. M.Sh. Levin, On balanced clustering (indices, models, examples). *J. of Communications Technology and Electronics*, 62(12), 1506–1515, 2017.
 101. M.Sh. Levin, Time-interval balancing in multi-processor scheduling of composite modular jobs (preliminary description). *Elec. prepr.*, 37 p., Nov. 11, 2018. <http://arxiv.org/abs/1811.04458> [cs.AI]
 102. M.Sh. Levin, On balanced clustering with tree-like structures over clusters. *Electr. prepr.*, 15 p., Dec. 9, 2018. <http://arxiv.org/abs/1812.03535> [cs.AI]
 103. M.Sh. Levin, Note on dominating sets problems. *J. of Communications Technology and Electronics*, 66(Suppl. 1), S8–S22, 2021.
 104. M.Sh. Levin, Balanced clustering with a tree over clusters. *J. of Communications Technology and Electronics*, 66(Suppl. 1), S23–S34, 2021.
 105. M.Sh. Levin, Interval-balanced multiprocessor scheduling on modular jobs. *J. of Communications Technology and Electronics*, 66(Suppl. 1), S35–S52, 2021.

- 106.M.Sh. Levin, Clustering models based on graph edge coloring. *J. of Communications Technology and Electronics*, 67(12), 1570–1577, 2022.
- 107.M.Sh. Levin, Towards clique partitioning problem. Preprint, 10 p., Nov. 29, 2022. DOI: 10.13140/RG.2.2.35252.12169 (ResearchGate)
- 108.M.Sh. Levin, On the clique partitioning of a graph. *J. of Communications Technology and Electronics*, 67(S2), S267–S274, 2022.
- 109.M.Sh. Levin, On capacitated clustering problem. Preprint, 14 p., Nov 21, 2023. DOI: 10.13140/RG.2.2.33224.70401 (ResearchGate)
- 110.M.Sh. Levin, The capacitated clustering problem. *J. of Communications Technologies and Electronics*, 69, 1–10, 2024.
- 111.S. Li, M. Pilipczuk, M. Sorge, Cluster editing parameterized above modification-disjoint P_3 -packings. *ACM Trans. on Algorithms*, 20(1), art. 3, 1–43, 2023.
- 112.M.C. Lin, F.J. Soullignac, J.L. Szwarcfiter, A faster algorithm for the cluster editing problem on proper interval graphs. *Inform. Process. Lett.*, 115(12), 913–916, 2015.
- 113.Y. Liu, J. Wang, J. Guo, J. Chen, Complexity and parameterized algorithms for Cograph Editing. *Theor. Comp. Sci.*, 461, 45–54, 2012.
- 114.Y. Liu, J. Wang, J. You, J. Chen, Y. Cao, Edge deletion problem: branching facilitated my modular decomposition. *Theor. Comp. Sci.*, 573, 63–70, 2015.
- 115.L.H.N. Lorena, M.G. Quiles, A.C.P.L.F. de Carvalho, L.A.N. Lorena, Preprocessing techniques for cluster editing via integer linear programming. In: D.-S. Huang, V. Bevilacqua, P. Premarathe, P. Gupta (eds), *Proc. Int. Conf. Intelligent Computing Theories and Application ICCIC 2018*, LNCS 10954, Springer, pp. 287–297, 2018.
- 116.J. Luo, H. Molter, A. Nichterlein, R. Niedermeier, Parameterized dynamic cluster editing. *Algorithmica*, 83(1), 1–44, 2021.
- 117.J. Madathil, K. Meeks, Parameterized algorithms for balanced cluster edge modification problems. *Electr. prepr.*, 66 p., Sep. 4, 2024. <http://arxiv.org/abs/2403.03830> [cs.DS]
- 118.H. Masoud, S. Jalili, S.M.H. Hasheminejad, Dynamic clustering using combinatorial particle swarm optimization. *Appl. Intell.*, 38(3), 289–314, 2013.
- 119.B. Mirkin, I. Muchnik, Combinatorial optimization in clustering. In: D.-Z. Du, P.M. Pardalos (eds), *Handbook of Combinatorial Optimization*. vol. 2, Springer, pp. 261–329, 1999.
- 120.A. Miyauchi, F. Adriaens, F. Bonchi, N. Tatti, Multilayer correlation clustering. *Electr. prepr.*, 24 p., Apr. 25, 2024. <http://arxiv.org/abs/2404.16676> [cs.DS]
- 121.Y. Perl, S.R. Schach, Max-min tree partitioning. *J. of the ACM*, 28(1), 5–15, 1981.
- 122.R.G.S. Pinheiro, I.C. Martins, F. Protti, L.S. Ochi, L.G. Simonetti, A. Subramanian, On solving manufacturing cell formation via bicluster editing. *EJOR*, 254(3), 769–779, 2016.
- 123.F. Protti, M.D. da Silva, J.L. Szwarcfiter, Applying modular decomposition to parameterized cluster editing problems. *Theory Comput. Syst.*, 44(1), 91–104, 2009.
- 124.S. Rahmann, T. Wittkop, J. Baumbach, M. Martin, A. Truss, S. Bocker, Exact and heuristics for weighted cluster editing. In: 6th Annual Int. Conf. on Computational Systems Bioinformatics CSB 2007, 6(1), pp. 391–401, 2007.
- 125.I. Sau, U. dos Santos Souza, Hitting forbidden induced subgraphs on bounded treewidth graphs. *Inf. Comput.*, 281(C), art. 104812, 2021.
- 126.S.A. Sert, H. Bagci, A. Yazici, MOFCA: multi-objective fuzzy clustering algorithm for wireless sensor networks. *Appl. Soft Computing*, 30, 151–165, 2015.
- 127.H. Schulz, A. Nichterlein, R. Niedermeier, C. Weyand, Applying a cut-based data reduction rule for weighted cluster editing in polynomial time. In: H. Dell, J. Nederlof (eds), 17th Int. Symp. on Parameterized and Exact computation (IPEC 2022), art. no. 25, pp. 25:1–25:14, 2022.
- 128.R. Shamir, R. Sharan, D. Tsur, Cluster graph modification problems. *Discr. Appl. Math.*, 144(1), 173–182, 2004.
- 129.M.-Y. Shan, R.-L. Zhang, L.-H. Zhang, Combinatorial clustering algorithm of quantum-behaved particle swam optimization and cloud model. *Math. Problems in Eng.*, art. ID 406047, 1–11, 2013.
- 130.M.D. da Silva, F. Protti, J.L. Szwarcfiter, Parameterized mixed cluster editing via modular decomposition. *Electr. prepr.*, 25 p., June 2, 2015. <http://arxiv.org/abs/1506.00944> [cs.DS]
- 131.D. Tsur, Faster parameterized algorithm for Bicluster Editing. *Inform. Proc. Lett.*, 168, art. 106095,

- 2021.
- 132.D. Tsur, Faster parameterized algorithm for cluster vertex deletion. *Theory Comput. Syst.*, 65, 323–343, 2021.
- 133.D. Tsur, Faster parameterized algorithms for Biclustor Editing and Flip Consensus Tree. *Theor. Comput. Sci.*, 953, art. 113796, 2023.
- 134.B. Vallet, B. Soheilian, M. Bredif, Combinatorial clustering and its application to 3D polygonal traffic sign reconstruction from multiple images. In: *ISORS Annals of the Photogrammetry Remote Sensing and Spatial Information Sciences*, vol. II-3, pp. 165–172, 2014.
- 135.N. Veldt, Optimization Frameworks for Graph Clustering. PhD Thesis, Purdue Univ., 2021.
- 136.S.T. Wierzchon, M.A. Klopotek, Algorithms of combinatorial cluster analysis. In: *Modern Algorithms of Cluster Analysis*, Springer, Cham, pp. 67–161, 2018.
- 137.T. Wittkop, J. Baumbach, F. Lobo, S. Rahmann, Large scale clustering of protein sequences with FORCE – A layout based heuristic for weighted cluster editing. *BMC Bioinformatics*, 8(1), art. 396, 1–12, 2007.

Author work address: Mark Sh. Levin, Inst. for Information Transmission Problem (Kharkevich Institute), Russian Academy of Sciences, 19 Bolshoy Karetny lane, Moscow 127051, Russia
Author home address: Mark Sh. Levin, 5 Sumskoy Proezd, kor. 1, apt. 103, Moscow 117208, Russia
<http://www.mslevin.iitp.ru/> email: mslevin@acm.org