Problem of conformal invariance in early vision

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Within the framework of geometric optics, we analyze which input information about external world comes to the retina and how the input function changes under eye movements. Under some assumptions, the change of the input function (density of the light energy on retina) and contours (its level curves with large gradient) is described by a conformal transformation of the retina $R \subset S^2$ which is a part of the eye sphere S^2. Contours and their infinitesimal versions - directions are main objects which are detected in early vision (in primary visual cortex V1 and V2).

Since the eye is always participated in different types of involuntary movements, there must be a universal neural mechanism of identification of conformally equivalent contours, which represent for example the edge of a stationary object at different positions of the eye.

We discuss two possible versions of such mechanism : first, based on the classical invariant description of curves on conformal sphere in terms of differential invariants and signature, and second, based on a modification of Sarti-Citti-Petitot model of V1 cortex (2007), described in terms of the cotangent bundle T^*S^2 of the eye sphere, and on proposed by Hubel and Wiezel concept of multicolumn in V1 cortex (recently found also in V2 area).
Also we shortly review the basic facts about fixational eye movements, architecture of retina and primary visual cortex.