List 2 – deadline 22.04

1. Construct uncorrelated but dependent normally distributed r.v.
2. Prove/disprove existence of a Gaussian process $\xi_t,\ 0 \le t \le 1$ with $E\xi_t \equiv 0$ and a correlation function $K(t,s) := t \wedge s - ts$, such that almost all its realizations are continuous.
3. Let w_t be a standard Wiener process, and let $t_i := \frac{i}{n},\ 0 \le i \le n$. Calculate $\lim\limits_{n\to\infty} \mathbf{P}(\sum_{i=0}^{n-1} |w_{t_{i+1}} - w_{t_i}| > n^\alpha)$ as a function of $\alpha \in \mathbb{R}$.
4. Let $\{\xi_i\}_{i=1}^n$ be iid r.v. with $E\xi_i = 0,\ D\xi_i = 1$. Let $\eta_n := \sqrt{n}\frac{\sum_{i=1}^n \xi_i}{\sum_{i=1}^n \xi_i^2}$. Prove that η_n is asymptotically normal as $n \to \infty$.
5. Let $\{\xi_i\}_{i=1}^n$ be iid r.v. and let $\frac{1}{n}\sum_{i=1}^n \xi_i \overset{n\to\infty}{\longrightarrow} 1$ almost surely. Prove that $E|\xi_1| < \infty$ and calculate $E\xi_1$.
6. Let $\{\xi_i\}_{i=1}^n$ be iid r.v. with $E\xi_i = 0, D\xi_i = \sigma^2 > 0$ and let $\eta_n := \frac{1}{\sigma\sqrt{n}}\sum_{i=1}^n \xi_i$. Prove/disprove existence of $(P)\lim_{n\to\infty} \eta_n$.
7. Find ALL stationary processes $\xi_t,\ t \ge 0$ such that $\exists (P)\lim_{t\to\infty} \xi_t$.

Do not wait until the deadline, and send written solutions (preferably in LaTex) by e-mail.