List 1 (Probability 2) deadline 24.02.20

1. Let $\{\xi_i\}_{i=1}^n$ be independent r.v. with $E\xi_i = 0, D\xi_i < \infty$ and let $\eta_k := \sum_{i=1}^k \xi_i$. Prove/disprove that $\mathbb{P}(\max_{k\le n} |\eta_k| \ge \varepsilon) \le 2\mathbb{P}(\eta_n \ge \varepsilon - \sqrt{2D\eta_n})$ $\forall \varepsilon \in \mathbb{R}$.

2. Let $\varphi(x) = \varphi(-x) \ge 0$ be a nonincreasing for $x \ge 0$ function, and let ξ, η be r.v. Prove/disprove $\mathbb{P}(|\xi| \le \varepsilon) \ge \mathbb{P}(|\eta| \le \varepsilon)$ $\forall \varepsilon \ge 0 \Longrightarrow E\varphi(\xi) \ge E\varphi(\eta)$ and vice versa.

3. Let $\{\xi_i\}_{i=1}^n$ be independent r.v. with the same distribution function $F(x)$. Let $\xi^- := \min_i \xi_i, \xi^+ := \max_i \xi_i$. Find the distribution function of the vector (ξ^-, ξ^+).

4. Let ξ be a r.v. with the median m_ξ. Prove/disprove that $m_{\varepsilon\xi} = \varepsilon m_\xi$ $\forall \varepsilon \in \mathbb{R}$.

5. Let $\{\xi_i\}_{i=1}^n$ be independent r.v. with $E\xi_i = a, D\xi_i = \sigma^2 < \infty$ and let $\eta_n := \frac{1}{n}\sum_{i=1}^n \xi_i$. Find a function $\varphi(n)$ such that $E\varphi(n)\sum_{i=1}^n(\xi_i - \eta_n)^2 = \sigma^2$.

6. Let A be a $n \times n$ matrix with independent random entries a_{ij} with $Ea_{ij} \equiv 0,\ Da_{ij} \equiv \sigma^2$. Calculate $D(\det A)$.

7. Let $\{\xi_i\}_{i=1}^n$ be iid r.v. with $0 < D\xi_i < \infty$. Find all possible values of the function $\varphi(x) := \lim_{n\to\infty} \mathbb{P}(\sum_{i=1}^n \xi_i < x)$, $x \in \mathbb{R}$.

Reception of solutions (by prior arrangement):
Michael Lvovich Blank – <blank@iitp.ru>
Mauro Mariani – <mariani.mau@gmail.com>
Nikita Puchkin – <npuchkin@gmail.com>

PS Please prepare written solutions before the reception and do not wait until the deadline.

Lecture plans

Lecture 1. (13.01.20) Basic probabilistic concepts. Probability space; random value; equivalence of random variables; σ -algebra generated by a random variable; its distribution; joint distribution and independence; moments; types of convergence: in probability, on average, weak; Lebesgue integral for an arbitrary measure.