List 1 – deadline 22.02.21

1. Solve the "ant on the rope" problem (see Lecture 1) under the assumption that $L_n(\omega) := L_{n-1}(\omega) \cdot \lambda_n(\omega)$, where $\{\lambda_n\}$ are iid r.v. with $E\lambda_n = \lambda \in (1, 2)$.
2. Let $\{\xi_i\}_{i=1}^n$ be independent r.v. with $E\xi_i = 0, D\xi_i < \infty$ and let $\eta_k := \sum_{i=1}^k \xi_i$. Prove/disprove that $\mathbf{P}(\max_{k\le n} |\eta_k| \ge \varepsilon) \le 2\mathbf{P}(\eta_n \ge \varepsilon - \sqrt{2D\eta_n}) \quad \forall\varepsilon \in \mathbb{R}$.
3. Let $\varphi(x) = \varphi(-x) \ge 0$ be nonincreasing for $x \ge 0$ function, and let ξ, η be r.v. Prove/disprove that $\mathbf{P}(|\xi| \le \varepsilon) \ge \mathbf{P}(|\eta| \le \varepsilon) \quad \forall\varepsilon \ge 0$ $\Longrightarrow E\varphi(\xi) \ge E\varphi(\eta)$ and vice versa.
4. Let $\{\xi_i\}_{i=1}^n$ be independent r.v. with the same distribution function $F(x)$. Let $\xi_- := \min_i \xi_i, \xi_+ := \max_i \xi_i$. Find the distribution function of the vector (ξ_-, ξ_+).
5. Let ξ be a r.v. with median m_ξ. Prove/disprove that $m_{\xi^3} = (m_\xi)^3$.
6. Let $\{\xi_i\}_{i=1}^n$ be iid r.v. with $0 < D\xi_i < \infty$. Find all possible values of the function $\varphi(x) := \lim_{n\to\infty} \mathbf{P}(\sum_{i=1}^n \xi_i < x), \quad x \in \mathbb{R}$.

Please do not wait for the deadline, prepare your solutions in LaTeX, and send by e-mail <blank@iitp.ru> or <npuchkin@gmail.com>.