Seminar – 13/05/2019

Ex. 1 Let $\varphi \in C^\infty(\mathbb{R};\mathbb{R})$ be compactly supported. Consider the problem for $t \geq 0$

$$\partial_t u = \partial_{xx} u + 5i\partial_x u$$
$$u(0,x) = \varphi(x)$$

Explain why it does admit a solution. Write it explicitly in terms of φ.

Ex. 2 Consider the equation in $\mathbb{R}^d$

$$i\partial_t \psi = \Delta \psi$$

(a) Explain why it is well-posed.
(b) Write $\psi = u + iv$. Find closed equations for the real and imaginary parts of ψ.
(c) Write $\psi = r(t,x)e^{is(t,x)}$. Write the equations for r and s.

Ex. 3 Consider the equation

$$i\partial_t \psi = \Delta \psi$$

in a box $[0,L]^d$, with condition $\partial_{\hat{n}}\psi = 0$. Solve the Sturm-Liouville problem, and find the solution with general initial data $u(t=0,x) = u_0(x)$.

Ex. 4 Suppose that the function $u \in C^2(\mathbb{R}^2)$ satisfies $u_{xx} + u_{yy} = 1$ in the set $\{(x,y) \in \mathbb{R}^2 \,:\, 1 < x^2 + y^2 < 4\}$. Which is bigger, the integral of u along the boundary $x^2 + y^2 = 1$ or along the boundary $x^2 + y^2 = 4$?

Ex. 5 Solve the problem in $\mathbb{R}^3$

$$\partial_{tt} u = \Delta u$$
$$u(0,x) = 0$$
$$\partial_t u(0,x) = \|x\|^4$$