List 1 – deadline 06.11.23

1. Prove/disprove that $\exists x \in [0,1]$, whose forward trajectory under the action of the map $Tx := 4x(1-x)$ is everywhere dense.
2. Prove/disprove that the map $Tx := 4x(1-x)$ from the unit interval into itself has a trajectory of period 3.
3. Prove/disprove existence of a periodic trajectory of period 3 in any strictly convex billiard.
4. Let $\bar{y}$ be an ε-uniformly perturbed geometric progression, i.e. $\exists \gamma \in \mathbb{R}: \quad |y_{n+1} - \gamma y_n| \le \varepsilon \quad \forall n \in \mathbb{Z}$. Is it true that such $\bar{y}$ can be uniformly shadowed by SOME true geometric progression?
5. Let $\bar{y}$ be an ε-uniformly perturbed generalized Fibonacci sequence, i.e. $|y_{n+2} - y_n - y_{n+1}| \le \varepsilon \quad \forall n \in \mathbb{Z}$ for some given $y_0, y_{-1} \in \mathbb{R}$. Is it true that such $\bar{y}$ can be uniformly shadowed by SOME true generalized Fibonacci sequence?

Only written solutions sent me by e-mail will be checked. Please do not wait until the deadline and try to prepare the solutions in LaTex.